LINEAR FUNCTIONS

Exercise 1: Let M denote the total amount of money to be collected and d denote the number of economically disadvantaged students. Then the linear relation is:

$$M = 500d + 50000.$$

Exercise 2:

$(a)\, f(x) = -2x - 5$ and $(c)\, h(s) = 100(1-2s) = 100 - 200s$ are linear functions since they can be expressed as $y = mx + b$, the others are not.

Exercise 3:

The formula for the rate of change is: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$. Hence,

$$\text{the rate of change} = \frac{(4)^2 - (1)^2}{4-1} = \frac{15}{3} = 5.$$

Since $f(1) = 1$ and $f(4) = 16$, the change in f from $x_1 = 1$ to $x_2 = 4$ is: $f(4) - f(1) = 15$.

For $x_1 = 0$ and $x_2 = 3$, we have: $\text{the rate of change} = \dfrac{(3)^2 - (0)^2}{3-0} = \dfrac{9}{3} = 3$.

Exercise 4:

For any x_1 and x_2,

$$\text{the rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{(x_2)^2 - (x_1)^2}{x_2 - x_1} = \frac{(x_2 - x_1)(x_2 + x_1)}{x_2 - x_1} = x_2 + x_1.$$

Exercise 5:

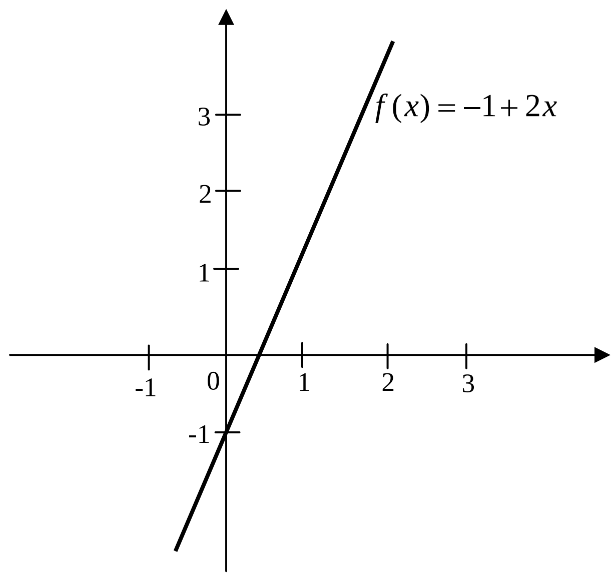
Between the year before last year and the last year the change in the number of e.d. students was $\Delta x = -20$, and the change in the funding is $\Delta f = -24000$. Hence, the rate of change for this linear relation is: $m = \dfrac{-24000}{-20} = 1200$.

From last year to this year we have $\Delta x = 30$, so $\Delta f = m\Delta x = 1200 \cdot 30 = 36000$.

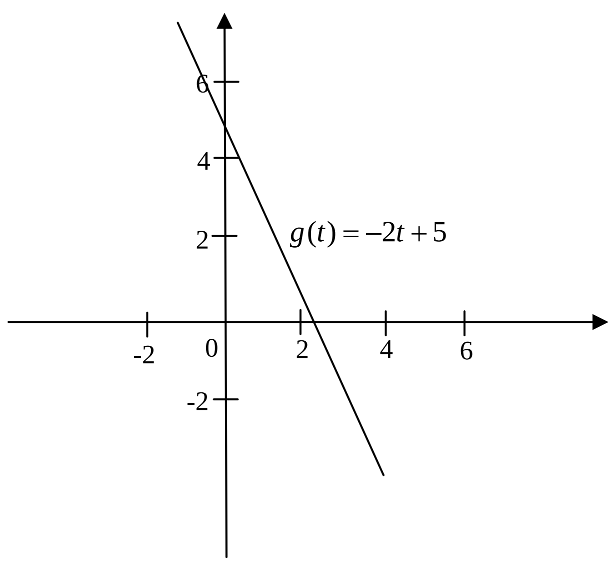
That is, there will be an increase of \$36000 in the school's funding.

Exercise 6:

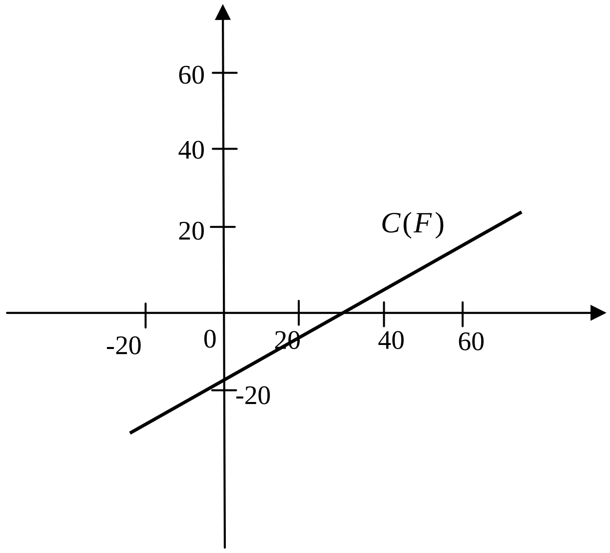
(1) The graph of $f(x) = -1 + 2x$ is given below.



(2) The graph of $g(t) = -2t + 5$ is:



(3) The graph of $C(F) = \frac{5}{9}(F - 32)$ is given below:



Exercise 7:

We know that $(x_0, y_0) = (-1,4)$ and $m = -2$.
If we substitute these in the equation, we get:

$$y = y_0 + m(x - x_0) = 4 + (-2)(x+1) = 4 - 2x - 2 = -2x + 2 .$$

Hence, the slope-intercept form is: $y = -2x + 2$.

Exercise 8:

We know from the slope intercept form that $m = -4$. We need to find (x_0, y_0) where $x_0 = 3$. To find y_0, substitute $x_0 = 3$ in the given equation;

$$y_0 = -4x_0 + 2 = -4(3) + 2 = -10 .$$

Hence, $(x_0, y_0) = (3,-10)$. The point-slope form of the given equation is:

$$y = y_0 + m(x - x_0) = -10 + (-4)(x - 3) .$$

Exercise 9:

Firstly, we need to find the slope by using the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{4-(-1)} = -\frac{1}{5} .$$

Now, pick one of the points to use it in finding the point-slope form of the equation.

Let's say $(x_0, y_0) = (4,0)$. The equation is:

$$y = y_0 + m(x - x_0) = 0 + \left(-\frac{1}{5}\right)(x-4) = -\frac{1}{5}(x-4).$$

Exercise 10:

Let's rearrange the given equation; $2x - 5y = 8 \Rightarrow 5y = 2x - 8 \Rightarrow y = \frac{2}{5}x - \frac{8}{5}$.

So, the rate of change of the function is : $m = \frac{2}{5}$.

To find $f(0)$,we need to find y when $x = 0$.

$$2x - 5y = 8 \Rightarrow -5y = 8 \Rightarrow y = -\frac{8}{5}.$$

That is, $f(0) = -\frac{8}{5}$.

Now, to find $f(-2)$, we need to find y when $x = -2$.

$$2x - 5y = 8 \Rightarrow -4 - 5y = 8 \Rightarrow -5y = 12 \Rightarrow y = -\frac{12}{5}.$$

Hence, $f(-2) = -\frac{12}{5}$.

Exercise 11:

The lines like $y = 2$, $y = 10$, or $y = -\frac{1}{4}$ are horizontal lines. So, in the equation $cx + dy = e$, if we choose c to be 0, d to be 1, and e to be any constant number, then we get the equation $y = e$ which is a horizontal line.

Vertical lines have equations like $x = \frac{1}{5}$ or $x = -2$. So, if we choose c to be 1, d to be 0, and e to be any constant we get the equation of a vertical line; $x = e$.

We know that $m = -\frac{c}{d} = 1$. So, we need to have $c = -d$ to get a slope 1. The equation becomes $cx - cy = e$.

If we want to have a slope -1, we need to have $m = -\frac{c}{d} = -1$. That is, $c = d$. Hence, the equation becomes $cx + cy = e$.

Exercise 12:

(a) Let's firstly use the method of substitution to solve the system, and by doing this we'll know whether the system has one solution, infinitely many solutions or no solutions.

From the equation $-x+4y=0$ we get $x=4y$. Now, substitute this in the first equation:

$$2x-2y=1 \Rightarrow 2(4y)-2y=1 \Rightarrow 8y-2y=1 \Rightarrow 6y=1 \Rightarrow y=\frac{1}{6}.$$

So, $x=4y=4\cdot\frac{1}{6}=\frac{2}{3}$.

Hence, there is only one solution to this system, which is $\left(\frac{2}{3},\frac{1}{6}\right)$.

Now, let's use the Cramer's Rule. The matrix of the coefficients is : $\begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}$.The determinant of this matrix is: $\begin{vmatrix} 2 & -2 \\ -1 & 4 \end{vmatrix} = (2)(4)-(-2)(-1)=6$.

Hence, $x=\dfrac{\begin{vmatrix} 1 & -2 \\ 0 & 4 \end{vmatrix}}{6}=\frac{4}{6}=\frac{2}{3}$ and $y=\dfrac{\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix}}{6}=\frac{1}{6}$.

(b) The first equation tells us that $r=-3s-1$. If we substitute this information in the second equation, we get:

$$-(-3s-1)+2s=4 \Rightarrow 3s+1+2s=4 \Rightarrow 5s=3 \Rightarrow s=\frac{3}{5}.$$

Then, $r=-3s-1=-3\left(\frac{3}{5}\right)-1=-\frac{14}{5}$.

Hence, the only solution of the equation is: $\left(-\frac{14}{5},\frac{3}{5}\right)$.

Let's calculate the determinant of the coefficient matrix to use the Cramer's Rule.

$\begin{vmatrix} -1 & -3 \\ -1 & 2 \end{vmatrix} = (-1)(2)-(-3)(-1)=-5$.

Therefore, $x=\dfrac{\begin{vmatrix} 1 & -3 \\ 4 & 2 \end{vmatrix}}{-5}=\dfrac{(1)(2)-(-3)(4)}{-5}=-\frac{14}{5}$ and $y=\dfrac{\begin{vmatrix} -1 & 1 \\ -1 & 4 \end{vmatrix}}{6}=\frac{-3}{-5}=\frac{3}{5}$.

Exercise 13:

i) Let's solve for C in the equation $F = \frac{9}{5}C + 32$.

$$F = \frac{9}{5}C + 32 \;\Rightarrow\; \frac{9}{5}C = F - 32 \;\Rightarrow\; C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$$

Hence, the inverse of the function $f(C) = \frac{9}{5}C + 32$ is $f^{-1}(F) = \frac{5}{9}F - \frac{160}{9}$.

ii) $v = f(t) = gt + v_0$. We need to solve for *t*.

$$v = gt + v_0 \;\Rightarrow\; gt = v - v_0 \;\Rightarrow\; t = \frac{1}{g}(v - v_0).$$

Thus, $f^{-1}(v) = \frac{1}{g}(v - v_0)$.

iii) Since $v = f(t) = P(1 + rt) = P + Prt$, we have $Prt = v - P$, and this implies that $t = \frac{v - P}{Pr}$.

That is, $f^{-1}(v) = \frac{v - P}{Pr}$.

iv) We have $M = f(d) = 500d + 50000$. Let's solve for *d*,

$$M = 500d + 50000 \;\Rightarrow\; 500d = M - 50000 \;\Rightarrow\; d = \frac{M - 50000}{500} = \frac{M}{500} - 100.$$

Hence, $f^{-1}(M) = \frac{M}{500} - 100$.