THE FUNCTION CONCEPT

Exercises:

1. $f(x) = 2x^2 - 3x + 4$; to find $f(1),\ f(-1),\ f(0)$ and $f(2)$, we need to substitute the values $x = 1, -1, 0, 2$ in the given expression.

$f(1) = 2(1)^2 - 3(1) + 4 = 2 - 3 + 4 = 3$,

$f(-1) = 2(-1)^2 - 3(-1) + 4 = 2 + 3 + 4 = 9$,

$f(0) = 2(0)^2 - 3(0) + 4 = 0 - 0 + 4 = 4$,

$f(2) = 2(2)^2 - 3(2) + 4 = 8 - 6 + 4 = 6$.

2. $f(x) = x^3 + 5x^2 - 1$;

$f(2) = (2)^3 + 5(2)^2 - 1 = 8 + 20 - 1 = 27$,

$f(-2) = (-2)^3 + 5(-2)^2 - 1 = -8 + 20 - 1 = 11$,

$f(0) = (0)^3 + 5(0)^2 - 1 = 0 + 0 - 1 = -1$,

$f(-1) = (-1)^3 + 5(-1)^2 - 1 = -1 + 5 - 1 = 3$.

3. $f(x) = \sqrt{x-1} + 2x$;

$f(1) = \sqrt{1-1} + 2(1) = 0 + 2 = 2$,

$f(3) = \sqrt{3-1} + 2(3) = \sqrt{2} + 6$,

$f(5) = \sqrt{5-1} + 2(5) = \sqrt{4} + 10 = 2 + 10 = 12$,

$f(10) = \sqrt{10-1} + 2(10) = \sqrt{9} + 20 = 3 + 20 = 23$.

4. $f(x) = 2x^2 - x + 3$;

a. $f(c) = 2c^2 - c + 3$

b. $f(-c) = 2(-c)^2 - (-c) + 3 = 2c^2 + c + 3$

c. $-f(c) = -\left(2c^2 - c + 3\right) = -2c^2 + c - 3$

d. $f(c+h) = 2(c+h)^2 - (c+h) + 3 = 2c^2 + 4ch + 2h^2 - c - h + 3$

e. $f(c) + f(h) = \left(2c^2 - c + 3\right) + \left(2h^2 - h + 3\right) = 2c^2 + 2h^2 - c - h + 6$

5. $g(x) = 3x - 8$;

a. $g\left(\frac{1}{a}\right) = 3\left(\frac{1}{a}\right) - 8 = \frac{3}{a} - 8$

b. $\frac{1}{g(a)} = \frac{1}{3a - 8}$

c. $g(a^2) = 3a^2 - 8$

d. $\left[g(a)\right]^2 = \left[3a-8\right]^2 = 9a^2 - 48a + 64$

6. If 4 is in the range of $f(x) = 6x - 5$, then there must exist some *a* such that $f(a) = 6a - 5 = 4$.

$6a - 5 = 4 \;\Rightarrow\; 6a = 9 \;\Rightarrow\; a = \frac{9}{6} = \frac{3}{2}$.

So, for $a = \frac{3}{2}$, $f(a) = 4$ (note that this is true only for $a = \frac{3}{2}$). Hence, 4 is in the range of $f(x) = 6x - 5$.

7. 4 is in the range of $f(x) = 3 - 2x$ if there exists *a* such that $f(a) = 3 - 2a = 4$.

$3 - 2a = 4 \;\Rightarrow\; -2a = 1 \;\Rightarrow\; a = -\frac{1}{2}$.

That is, $f\left(-\frac{1}{2}\right) = 4$. Hence, 4 is in the range of $f(x) = 3 - 2x$.

8. $f(x) = \sqrt{x-3}$; we need *a* such that $f(a) = \sqrt{a-3} = 4$.

$\sqrt{a-3} = 4 \;\Rightarrow\; a - 3 = 16 \;\Rightarrow\; a = 19$.

So, $f(19) = 4$, which means that 4 is in the range of $f(x) = \sqrt{x-3}$.

9. $f(x) = \frac{1}{x}$; we need *a* satisfying $f(a) = \frac{1}{a} = 4$.

$\frac{1}{a} = 4 \;\Rightarrow\; 1 = 4a \;\Rightarrow\; a = \frac{1}{4}$.

So, $f\left(\frac{1}{4}\right) = 4$; 4 is in the range of $f(x) = \frac{1}{x}$.

10. $f(x) = x^2 + 5$; we're looking for some *a* where $f(a) = a^2 + 5 = 4$.

However, $a^2 + 5 = 4 \;\Rightarrow\; a^2 = -1$.

We know that there are no real numbers whose squares are negative. So, we can conclude that there is no *a* such that $f(a) = 4$; that is, 4 is not in the range of $f(x) = x^2 + 5$.

11. a. The set $\{(4,2),(4,3),(3,4),(2,5),(1,4)\}$ can not be a function since 2 different values are assigned to 4; 4 is related with both 2 and 3.

b. The set $\{(1,7),(2,8),(3,9),(4,10),(5,12)\}$ determines a function since every element in the domain (the elements in the first coordinate of each pair forms the domain) is related with just one number (we can say every element in the domain is used once).

c. The set $\{(1,3),(2,3),(3,3),(4,3),(5,3)\}$ determines a function with the reasoning we used in part (b). It is important to note that if every element in the domain is related with the same number; this is not a problem about being a function. This is a function where there is just one element in the range. Notice the difference between part (a) and part (c), an element in the domain can be used just once, but an element in the range can be used lots of times.

12. a. The set $\{(1,2),(2,2),(3,4),(4,4),(4,5)\}$ does not determine a function since 4 is related with both 4 and 5.

b. The set $\{(1,3),(5,1),(5,2),(3,1),(4,0)\}$ is not a function since there are 2 ordered pairs with the same first coordinate and different second coordinates(namely (5,1) and (5,2)).

c. The set $\{(95,15),(72,37),(10,30),(15,45)\}$ is a function since every ordered pair has different first coordinates.

13. Suppose $f(x)=3x+2$ is not one-to-one. Then there must exist real numbers x_1 and x_2, $x_1 \neq x_2$, such that

$$3x_1+2=3x_2+2.$$

However, this implies that $x_1 = x_2$. This is a contradiction to our assumption. Hence, *f* is one-to-one.

14. Assume $f(x)=5-3x$ is not one-to-one. So, there must exist real numbers x_1 and x_2, $x_1 \neq x_2$, such that

$$5-3x_1=5-3x_2.$$

Solving this equation gives $x_1 = x_2$, which is a contradiction to our assumption. Hence, $f(x)=5-3x$ is one-to-one.

15. Assume that $f(x)=\frac{1}{2x+1}$ is not one-to-one. Then, there are real numbers x_1 and x_2, $x_1 \neq x_2$, such that

$$\frac{1}{2x_1+1}=\frac{1}{2x_2+1}.$$

If you solve this equation, you'll get $x_1 = x_2$ which contradicts our assumption. Hence, $f(x) = \frac{1}{2x+1}$ is one-to-one.

16. The function $f(x) = x^2 - 2x + 1$ is not one-to-one since $f(2) = 2^2 - 2(2) + 1 = 1$ and $f(0) = 0 - 0 + 1 = 1$. That is, the elements 2 and 0 are assigned to the same value 1.We can find lots of such numbers for this function, but giving one example is enough to show that a function is not one-to-one.

17. We know the relation between the radius and the circumference of a circle; $C = 2\pi r$. That is, $r = f(C) = \frac{2\pi}{C}$.

If the circumference is increased by 6 cm, then the new radius is $r_2 = f(C+6) = \frac{2\pi}{C+6}$.

And the increase in the radius is $\frac{2\pi}{C+6} - \frac{2\pi}{C}$.

18. If one side of a square is *x* units, then the area of the square is $A = x^2$ and the perimeter is $P = 4x$.

$A = x^2 \Rightarrow x = \sqrt{A}$, so $P = 4x = 4\sqrt{A}$.

Hence, $P = f(A) = 4\sqrt{A}$.

19. If one side of a cube is *a* inches, then its volume is $V = a^3$ and the surface area is $S = 6a^2$.

$S = 6a^2 \Rightarrow a^2 = \frac{S}{6} \Rightarrow a = \sqrt{\frac{S}{6}}$, which means $V = a^3 = \left(\sqrt{\frac{S}{6}}\right)^3$.

Thus, $V = f(S) = \left(\sqrt{\frac{S}{6}}\right)^3$.

If the surface area is 36 sq.in., then $V = f(36) = \left(\sqrt{\frac{36}{6}}\right)^3 = \left(\sqrt{6}\right)^3 = 6\sqrt{6}$.

20. Since the car gets 30 miles per gallon, we can say that it uses $\frac{1}{30}$ gallons for a mile. After m miles, $m \cdot \frac{1}{30} = \frac{m}{30}$ gallons will be used. Since we start with 15 gallons, the number of gallons left is found by the function $g = f(m) = 15 - \frac{m}{30}$.