Post-Test - Solutions

Numbers, Operations and Quantitative Reasoning

1. Describe the set of natural numbers.

   Solution: The set of natural numbers is given by $\mathbb{N} = \{1, 2, 3, ...\}$.

2. What are prime numbers?

   Solution: The prime numbers are the natural numbers with exactly 2 divisors. An equivalent definition is the set of natural numbers larger than 1 which are only divisible by 1 and themselves.

3. There are exactly 25 prime numbers less than 100. How many more prime numbers do you need to know to determine whether an arbitrary natural number is prime if this number is smaller than 10,000? Explain.

   Solution: You only need to know the 25 prime numbers smaller than 100 since 100 is not prime and $\sqrt{10,000} = 100$.

4. Give the prime factorization of 240.

   Solution: $240 = (2)^4 (3)(5)$

5. The prime factorizations of 33,264 and 1,950 are give by $33,264 = (2)^4 (3)^3 (7)(11)$ and $1,950 = (2)(3)(5)^2(13)$. Give the prime factorizations for both **gcd**(33,264 , 1,950) and **lcm**(33,264 , 1,950).

   Solution: $\gcd(33264, 1950) = (2)(3)$ and $\operatorname{lcm}(33264, 1950) = (2)^4 (3)^3 (5)^2 (7)(11)(13)$.

6. Suppose *a* and *b* are natural numbers. How are **gcd**(*a*,*b*), **lcm**(*a*,*b*) and *ab* related?

   Solution: *ab* is the product of **gcd**(*a*,*b*) and **lcm**(*a*,*b*).

7. Give three methods for computing the **gcd** of two natural numbers.

   Solution: The **gcd** of two natural numbers can be computed by either using their prime factorizations, by using a table of their factors, or by using the Euclidean algorithm.

8. What method is used by many calculators and computers to compute the greatest common divisor of two natural numbers?

   Solution: The Euclidean algorithm.

9. Write the base 10 number 123 in base 6.

   Solution: $123 = 323_6$ since $123 = 3(6)^2 + 2(6) + 3$.

10. Give the base 10 representation of the base 11 number $\Delta 23_{11}$, where Δ is the single digit representation of the number 10 in base 11.

    Solution: $\Delta 23_{11} = 10(11)^2 + 2(11) + 3 = 1210 + 22 + 3 = 1235$.

11. How are rational numbers different from irrational numbers?

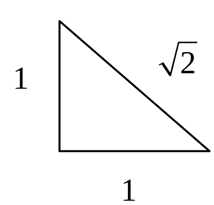
    Solution: A rational number is the quotient of two integers *a* and *b* where *b* is not 0. An irrational number is a real number that is not a rational number.

12. Which of the sets $\mathbb{N}$, $\mathbb{Q}$ and $\mathbb{R}$ have the same number of elements? Which of these sets is smaller than the set of irrational numbers?

    Solution: $\mathbb{N}$ and $\mathbb{Q}$ have the same number of elements. Both of these sets are smaller than the set of irrational numbers.

13. Use the Pythagorean Theorem to give a geometric interpretation of the irrational number $\sqrt{2}$.

    Solution: We can give a geometric interpretation of the number $\sqrt{2}$ by sketching a right triangle whose legs both have length one. The Pythagorean theorem tells us that the hypotenuse will have length $\sqrt{2}$.



14. What is the triangle inequality?

    Solution: The triangle inequality says that $|a+b| \le |a| + |b|$ for all real numbers *a* and *b*.

15. Give a definition of absolute value. Then use this definition to solve the inequality $|2q-1| \geq 5$.

Solution: If a is a real number, then $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$. Consequently, $|2q-1| \geq 5$ if and only if $2q-1 \geq 5$ or $2q-1 \leq -5$. The solution to $2q-1 \geq 5$ is given by $q \geq 3$, and the solution to $2q-1 \leq -5$ is given by $q \leq -2$. Therefore, $|2q-1| \geq 5$ if and only if $q \geq 3$ or $q \leq -2$.

16. Write the quotient $\frac{1-i}{1+i}$ in the form $a+bi$.

Solution: $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1-1-i-i}{1+1} = \frac{-2i}{2} = -i$

17. What is the conjugate of the complex number $a+bi$? How is the conjugate of $a+bi$ related geometrically with $a+bi$ (in the complex plane)?

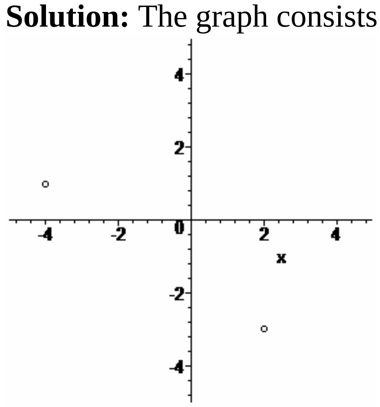
Solution: The conjugate of the complex number $a+bi$ is the complex number $a-bi$. The conjugate of $a+bi$ in the complex plane is the reflection of the point (a,b) across the *real* axis (the horizontal axis).

18. Give the value of $(a+bi)\left(\overline{a+bi}\right)$.

Solution: $(a+bi)\left(\overline{a+bi}\right) = (a+bi)(a-bi) = a^2 + b^2$

19. Graph the complex numbers $2-3i$ and $-4+i$ in the complex plane.

Solution: The graph consists of the points (2,-3) and (-4,1).



20. Graph the set of points $a+bi$ in the complex plane satisfying $|a+bi| \leq 1$.

Solution: The set of points is the solid disk of radius 1 centered at the origin. That is, the set of points that are less than or equal to 1 unit away from the origin.