7. The Real Number System

At the end of the last section we learned that the set of irrational numbers is uncountable, and consequently it is MUCH LARGER than $\mathbb{Q}$. One of the difficulties that you might have thinking about this could arise from your lack of experience with irrational numbers. It turns out that they are everywhere. Theorem 5.13 and definition 6.16 tell us that the irrational numbers are the numbers which can be written in decimal form without any repeated pattern in the decimal expansion. We can use this to devise strategies to create irrational numbers. For example, we could create a decimal expansion that starts with a 0, followed by a 1, followed by a 0, followed by two 1's, followed by a 0, followed by three 1's, etc...

$$.0\underset{1}{\underbrace{1}}0\underset{2}{\underbrace{11}}0\underset{3}{\underbrace{111}}0\underset{4}{\underbrace{1111}}0\underset{5}{\underbrace{11111}}0\underset{6}{\underbrace{111111}}0\underset{7}{\underbrace{1111111}}0\cdots$$

This decimal expansion does not eventually repeat itself indefinitely. Consequently, this number is irrational. You can use other ideas to create many other irrational numbers.

There are also very well known irrational numbers.

Theorem 7.1: $\sqrt{2}$ is an irrational number.

Proof: Suppose $\sqrt{2}$ is a rational number. Let $\dfrac{a}{b}$ be the reduced form of the rational number $\sqrt{2}$ ($a,b \in \mathbb{Z}$, $b \neq 0$).

So, $a = \sqrt{2}\,b$. This implies that $a^2 = 2b^2$. Hence, a^2 is an even integer. That is, a is also even (the square of an odd number is odd, the square of an even number is even).
Let $a = 2k$ where $k \in \mathbb{Z}$.
$$a^2 = 2b^2 \quad \Rightarrow \quad 4k^2 = 2b^2 \quad \Rightarrow \quad b^2 = 2k^2 .$$
Hence, b^2 is even. This implies that b is also even which contradicts to our assumption that $\dfrac{a}{b}$ is in the reduced form. Therefore, $\sqrt{2}$ is irrational. ∎

Theorem 7.2: If $n \in \mathbb{N}$ and $\sqrt{n}$ is not a natural number, then $\sqrt{n}$ is an irrational number.

Proof: Let $n \in \mathbb{N}$ and $\sqrt{n}$ is not a natural number. That is, n is not a perfect square.

Suppose $\sqrt{n}$ is a rational number. Let $\dfrac{a}{b}$ be the reduced form of the rational number $\sqrt{n}$ ($a,b \in \mathbb{Z}$, $b \neq 0$).

$\sqrt{n} = \dfrac{a}{b} \implies n = \dfrac{a^2}{b^2}$. We know that n is a natural number; i.e., $\dfrac{a^2}{b^2}$ is a natural number.

This means that b^2 divides a^2. And since a and b are integers, this implies that b divides a, which is a contradiction to our assumption.

Hence, if n is a natural number which is not a perfect square, then $\sqrt{n}$ is irrational. ∎

There are other well known irrational numbers. For example, Johann Lambert showed in 1760 that π is an irrational number. This particular irrational number can be constructed geometrically. For example, a circle with radius 1 has area π, and a circle with radius ½ has circumference π.
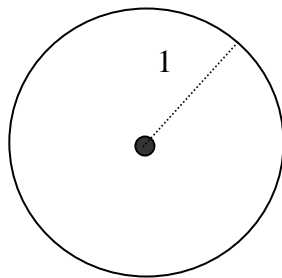

The area of this
circle is π.

We show in the example below that it is also possible to use geometry to construct physical representations of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$. The main idea stems from the Pythagorean theorem.
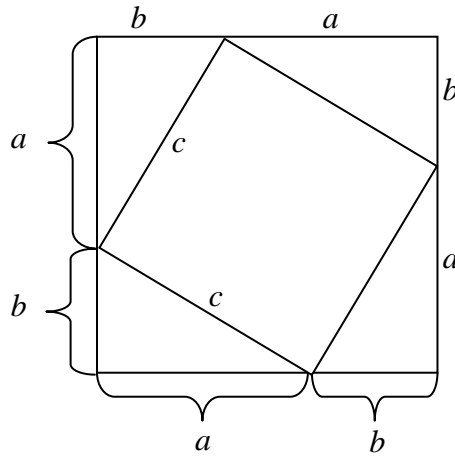
Theorem 7.3: (Pythagorean theorem) Suppose a right triangle has side lengths a, b and c as shown in the figure below. Then $a^2 + b^2 = c^2$.
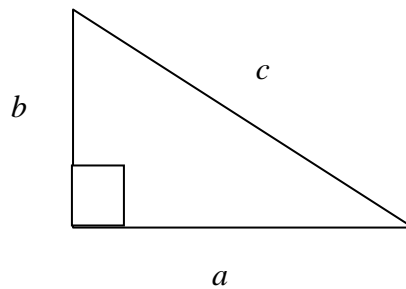
Proof: Let's start with a square whose side has length $a + b$ (see next page).
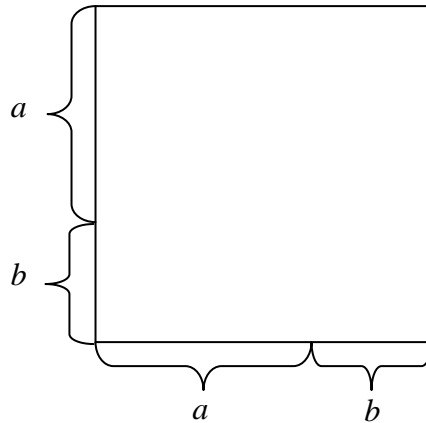
We get a new figure by joining the points separating the sides into two parts, a part with length a and another part with length b. Here is the figure;


The figure formed is a square since all the triangles in the figure are congruent. Let's say its one side is c units. Then, the area of the first square (the one in the first figure) must be equal to the sum of the areas of the triangles and the new square (the ones in the second figure). Hence, $(a+b)^2 = 4\left(\dfrac{ab}{2}\right) + c^2$ ($\dfrac{ab}{2}$ is the area of one triangle).

$$(a+b)^2 = a^2 + 2ab + b^2 = 2ab + c^2 \implies a^2 + b^2 = c^2. \; \blacksquare$$

The Pythagorean theorem can be used to construct $\sqrt{2}$ and $\sqrt{3}$, as well as many other irrational numbers.

Example 7.4: Give geometric constructions of $\sqrt{2}$ and $\sqrt{3}$.

Solution: We start with a right triangle whose legs have length 1 as shown in the figure below.
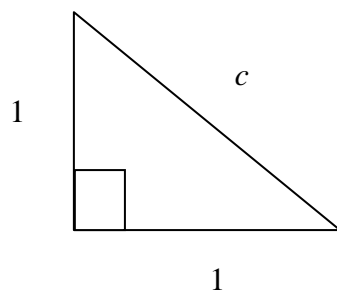
We can use the Pythagorean theorem to conclude that $c^2 = 1^2 + 1^2 = 1 + 1 = 2$.
Consequently, $c = \sqrt{2}$. So we have constructed a right triangle that has a hypotenuse with length $\sqrt{2}$. We can build on this picture as shown below by placing another right triangle on the hypotenuse of this triangle as shown.
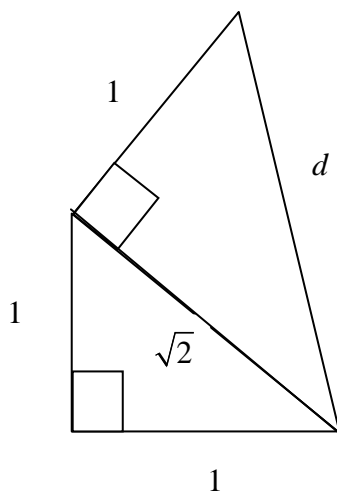
From the Pythagorean theorem, the value of d satisfies $d^2 = 1^2 + \left(\sqrt{2}\right)^2 = 1 + 2 = 3$.
Consequently, $d = \sqrt{3}$.

As stated above, there are many more irrational numbers, and in fact, there are more irrational numbers than rational numbers. Also, just like the rational numbers, the irrational numbers are nearly everywhere on the real line.

Theorem 7.5: The irrational numbers are a dense subset of $\mathbb{R}$.

Proof: Omitted.

Approximating Irrational Numbers With Rational Numbers

We learned earlier that the rational numbers are dense in $\mathbb{R}$. This is important, because these are the numbers that calculators and other computing devises display when we ask for an irrational number such as $\sqrt{3}$. One natural question is "How does a calculator get a number when we ask for it?" First, let's dispel the widely help belief that calculators have all of the numbers that they need stored in memory. This is IMPOSSIBLE, since there are infinitely many numbers. Actually, calculators store algorithms (recipes) for creating numbers on demand. Since a calculator only displays numbers to 11-16 decimal places (depending upon the calculator), the recipe only needs to give an approximation that is accurate within this number of decimal places. One such recipe is the one given below for approximating $\sqrt{n}$ whenever n is a natural number. The process comes from a technique from calculus known as Newton's method, but it is not necessary to know calculus to use the method.

Approximating $\sqrt{n}$
Let $x_0 = n$.
Define $x_1 = \dfrac{1}{2}\left(x_0 + \dfrac{n}{x_0} \right)$.
Define $x_2 = \dfrac{1}{2}\left(x_1 + \dfrac{n}{x_1} \right)$.
Define $x_3 = \dfrac{1}{2}\left(x_2 + \dfrac{n}{x_2} \right)$.
Define $x_4 = \dfrac{1}{2}\left(x_3 + \dfrac{n}{x_3} \right)$.
Continue until $-10^{-11} < x_k - x_{k-1} < 10^{-11}$

The number 10^{-11} is simply an example of what might be used in practice. The point is that the process is stopped when successive values are very close to one another. The algorithm above typically converges very quickly! We see this in the example below.

Example 7.6: Use the algorithm to approximate $\sqrt{2}$.

Solution: In this case, $n = 2$. So, $x_0 = 2$. Substituting this into the formula above for x_1 gives

$$x_1 = \frac{1}{2}\left(2 + \frac{2}{2} \right) = \frac{3}{2}$$

Substituting this into the formula for x_2 gives

$$x_2 = \frac{1}{2}\left(x_1 + \frac{n}{x_1}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{4}{3}\right) = \frac{17}{12} = 1.41\overline{6}$$

We can see that x_2 is already a reasonable approximation to the calculator value for $\sqrt{2} \approx 1.41421356237310$. Continuing,

$$x_3 = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{17/12}\right) = \frac{577}{408} \approx 1.41421568627451$$

$$x_4 = \frac{1}{2}\left(x_3 + \frac{2}{x_3}\right) \approx 1.41421356237469$$

$$x_5 = \frac{1}{2}\left(x_4 + \frac{2}{x_4}\right) \approx 1.41421356237310$$

That's amazing! We can see that after 3 steps we already have a very good approximation, and in 5 steps, we have the same value as our calculator approximation for $\sqrt{2}$. Of course, your calculator can do this series of calculations in a fraction of a second, so it almost seems like your calculator already knows the value when you ask for it. That's why some people believe (incorrectly) that calculators keep a table of all of these types of values in memory.

It is also possible to give an extension of this algorithm that gives roots of numbers other than square roots. We give one of these below for computing $\sqrt[p]{n}$ whenever $n, p \in \mathbb{N}$ and $p \geq 2$. In the case of odd roots, this algorithm only returns the approximation to the positive root.

Approximating $\sqrt[p]{n}$
Let $x_0 = n$.
Define $x_1 = \frac{1}{p}\left((p-1)x_0 + \frac{n}{x_0^{p-1}}\right)$.
Define $x_2 = \frac{1}{p}\left((p-1)x_1 + \frac{n}{x_1^{p-1}}\right)$.
Define $x_3 = \frac{1}{p}\left((p-1)x_2 + \frac{n}{x_2^{p-1}}\right)$.
Define $x_4 = \frac{1}{p}\left((p-1)x_3 + \frac{n}{x_3^{p-1}}\right)$.
Continue until $-10^{-11} < x_k - x_{k-1} < 10^{-11}$

Notice how this algorithm reduces to the earlier one when $p = 2$.

Example 7.7: Use the algorithm above to approximate $\sqrt[3]{5}$.

Solution: In this case $p = 3$ and $n = 5$. So we start by setting $x_0 = 5$. Then

$$x_1 = \frac{1}{3}\left((3-1)5 + \frac{5}{5^{3-1}}\right) = \frac{17}{5} = 3.4$$

$$x_2 = \frac{1}{3}\left((3-1)x_1 + \frac{5}{x_1^{3-1}}\right) \approx 2.41084198385236$$

$$x_3 = \frac{1}{3}\left((3-1)x_2 + \frac{5}{x_2^{3-1}}\right) \approx 1.89398315995149$$

$$x_4 = \frac{1}{3}\left((3-1)x_3 + \frac{5}{x_3^{3-1}}\right) \approx 1.72727396648750$$

$$x_5 = \frac{1}{3}\left((3-1)x_4 + \frac{5}{x_4^{3-1}}\right) \approx 1.71014860175656$$

$$x_6 = \frac{1}{3}\left((3-1)x_5 + \frac{5}{x_5^{3-1}}\right) \approx 1.70997596410721$$

$$x_7 = \frac{1}{3}\left((3-1)x_6 + \frac{5}{x_6^{3-1}}\right) \approx 1.70997594667670$$

$$x_8 = \frac{1}{3}\left((3-1)x_7 + \frac{5}{x_7^{3-1}}\right) \approx 1.70997594667670$$

Notice that we do not see any improvement from the seventh to the eighth value. Also, checking the calculator, we get the approximation for $\sqrt[3]{5}$ given by 1.70997594667670, so the agreement is excellent!

The following TI-83 program can be used to observe the convergence of this process.

ROOT
:   Disp "APPROX THE P-TH"
:   Disp "ROOT OF N"
:   Prompt P
:   Prompt N
:   Disp   "HOW MANY STEPS? "
:   Prompt M
:   N → X
:   For(I,1,M)
:   ((P-1)X+N/X^(P-1))/P → X
:   Disp X
:   End

The program ROOT approximates $\sqrt[p]{n}$ using the algorithm above. It prompts the user for the values of p and n, and then it asks for the number of steps to be used in the algorithm. The program displays the output from the steps to the screen. Notice that only the steps

that will fit on the screen will be visible. The screen shots below show the result of requesting an approximation of $\sqrt[3]{5}$ using 8 steps and then requesting the TI-83 to give the its approximation for $\sqrt[3]{5}$. The values for the first four steps do not appear on the screen. Only the values for x_5, x_6, x_7, x_8 appear. The agreement is very good, but the TI-83 is not capable of the type of accuracy that we have displayed in the previous example.

prgmROOT	1.710148602
APPROX THE P-TH	1.709975964
ROOT OF N	1.709975947
P=?3	1.709975947
N=?5	Done
HOW MANY STEPS	5^(1/3)
M=?8	1.709975947

Absolute Value

So far, we have focused on algebraic properties of various subsets of the real line. In order to give more structure to $\mathbb{R}$, we need a measuring stick for computing distances between numbers. This is the role of absolute value.

Definition 7.8: Let $a \in \mathbb{R}$ The **absolute value** of a is given by
$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

The use of the expression $-a$ in the definition of $|a|$ is sometimes a point of confusion. However, notice that $|a| = -a$ only when $a < 0$, and the negative of a negative is positive!

The absolute value function has some very important properties.

Theorem 7.9: Let $a, b \in \mathbb{R}$.
- $|a| \geq 0$, and $|a| = 0$ if and only if $a = 0$.
- $|ab| = |a||b|$
- $|a| = |-a|$
- $|a + b| \leq |a| + |b|$ (triangle inequality)
- If $b > 0$, then $|a| < b$ if and only if $-b < a < b$.

Proof: The bulleted items are proved below.

- We know that $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$. Hence, $|a|$ is defined to be always non-negative. If a is negative then $-a$ is positive, so $|a|$ is always non-negative; $|a| \geq 0$. The facts

  - if $a = 0$ then $|a| = a = 0$
  - if $|a| = 0$ then $a = 0$

  follow from the definition.

- We need to deal with some cases here.

  Case-1: $a, b \geq 0$. Then $ab \geq 0$. So, $|a| = a$, $|b| = b$, and $|ab| = ab$. Hence $|ab| = ab = |a||b|$.

  Case-2: $a, b \leq 0$. Then $ab \geq 0$. So, $|a| = -a$, $|b| = -b$, and $|ab| = ab$. Hence, $|ab| = ab = (-a)(-b) = |a||b|$.

  Case-3: $a < 0, b \geq 0$. Then, $ab \leq 0$. So, $|a| = -a$, $|b| = b$, and $|ab| = -ab$. Thus, $|ab| = -ab = (-a)(b) = |a||b|$.

  Therefore, in any case, $|ab| = |a||b|$.

- We have two cases here.

  Case-1: $a \geq 0$. Then $-a \leq 0$. So, we have $|a| = a$ and $|-a| = -(-a) = a$; i.e., $|a| = |-a|$.

  Case-2: $a < 0$. Then $-a > 0$. So, $|a| = -a$ and $|-a| = -a$; i.e., $|a| = |-a|$.

  Hence, $|a| = |-a|$.

- It is important to observe that if a and b are non-negative real numbers, then $a \leq b$ if and only if $a^2 \leq b^2$. We know that $|a+b|$ and $|a|+|b|$ are non-negative numbers. Moreover,

$$|a+b|^2 = (a+b)^2$$
$$= a^2 + 2ab + b^2$$
$$= |a|^2 + 2ab + |b^2|$$
$$\leq |a|^2 + 2|a||b| + |b|^2 = \left(|a| + |b|\right)^2.$$

  Hence, by the observation, we can conclude that $|a+b| \leq |a| + |b|$

- Since $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$, $|a| < b$ implies that either $a < b$ or $-a < b$. But $-a < b$ means $a > -b$ (just multiply both sides by -1). We also know that $b > 0$, which means $-b < b$. So, $-b$ is less than a, and a is less than b. That is, $-b < a < b$. ∎

Remark 7.10: The term "triangle inequality" used above comes from the extension of this idea to 2 and 3 dimensions. In the 2 dimensional setting, distances can be interpreted using vectors. The diagram below shows the vectors u, v and $u+v$.
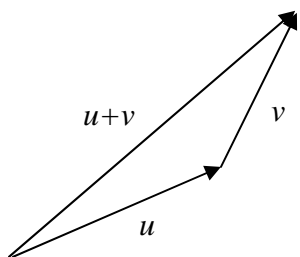

Notice that these vectors form a triangle and that the length of $u+v$ will certainly be smaller than the length of u plus the length of v. The length of vectors is typically denoted using the same absolute value symbol as above. Consequently, this information can be written in the form

$$|u+v| \le |u| + |v|.$$

Example 7.11: $|\pi| = \pi$, $|-3.24| = 3.14$, $|\sqrt{2}| = \sqrt{2}$, $\left|-\dfrac{3}{7}\right| = \dfrac{3}{7}$, and $|0| = 0$.

Example 7.12: Illustrate the points on the real number line which satisfy $|x| = 4$.

Solution: There are only two values of x which satisfy $|x| = 4$; namely, $x = -4$ and $x = 4$. We show these on the number line below.

-4   4
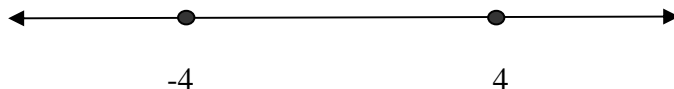
Example 7.13: Illustrate the points on the real number line which lie less than 3 units away from 4.

Solution: The numbers which are less than 3 units away from 4 satisfy $|x-4| < 3$. This implies $-3 < x - 4 < 3$. Adding 4 across this inequality gives $1 < x < 7$. These values are shown on the number line below. Notice the open circles at 1 and 7 denoting that these values are not included.

1   7

Exercises

1. Use the algorithm in this section to approximate $\sqrt[5]{7}$. How many steps do you need to use to obtain the accuracy that your calculator gives?
2. Create a table with 4 columns. The first column should have the header n and it should include the values 10, 20, ..., 200. The second column should have the

header $\sqrt{n}$, and it should include the calculator values for $n = 10, 20, ..., 200$. The third column should include the results of applying the algorithm above to approximate $\sqrt{n}$ for each value of n using 5 steps. The fourth column should include the results of applying the algorithm using 10 steps. Do you see any noticeable difference in the performance for the different values of n?

3. Repeat the exercise above using the values $n = 1000, 2000, ..., 20000$. This time include a 5th column which gives the result of using 15 steps. Do you see any noticeable difference in the performance for the different values of n?

4. Illustrate the points on the real number line which are less than 3 units away from 2 or less than 4 units away from -7. Also, write each of these requirements using absolute values and inequalities.

5. Illustrate the points on the real number line which are less than 1 units from -7 or less than 5 units from 6. Also, write each of these requirements using absolute values and inequalities.

6. Illustrate the points on the real number line which satisfy the inequality $|2x+5| \le 7$.

7. Illustrate the points on the real number line which satisfy the inequality $|x^2 -1| \le 3$. Hint: $|a| \le 3$ if and only if $-3 \le a \le 3$.

8. Illustrate the points on the real number line which satisfy the inequality $\dfrac{2}{|2x-1|} > 3$. Be careful!

Solutions:

1. In this case $p = 5$ and $n = 7$. So we start by setting $x_0 = 7$. Then

$$x_1 = \frac{1}{5}\left((5-1)7 + \frac{7}{7^{5-1}}\right) \approx 5.6005830903790$$

$$x_2 = \frac{1}{5}\left((5-1)x_1 + \frac{7}{x_1^{5-1}}\right) \approx 4.48222296774677$$

$$x_3 = \frac{1}{5}\left((5-1)x_2 + \frac{7}{x_2^{5-1}}\right) \approx 3.58924697411137$$

$$x_4 = \frac{1}{5}\left((5-1)x_3 + \frac{7}{x_3^{5-1}}\right) \approx 2.87983315376313$$

$$x_5 = \frac{1}{5}\left((5-1)x_4 + \frac{7}{x_4^{5-1}}\right) \approx 2.32422094279246$$

$$x_6 = \frac{1}{5}\left((5-1)x_5 + \frac{7}{x_5^{5-1}}\right) \approx 1.90735213110943$$

$$x_7 = \frac{1}{5}\left((5-1)x_6 + \frac{7}{x_6^{5-1}}\right) \approx 1.63166193804920$$

$$x_8 = \frac{1}{5}\left((5-1)x_7 + \frac{7}{x_7^{5-1}}\right) \approx 1.50287834677902$$

$$x_9 = \frac{1}{5}\left((5-1)x_8 + \frac{7}{x_8^{5-1}}\right) \approx 1.47673339845297$$

$$x_{10} = \frac{1}{5}\left((5-1)x_9 + \frac{7}{x_9^{5-1}}\right) \approx 1.47577440955909$$

$$x_{11} = \frac{1}{5}\left((5-1)x_{10} + \frac{7}{x_{10}^{5-1}}\right) \approx 1.47577316159347$$

$$x_{12} = \frac{1}{5}\left((5-1)x_{11} + \frac{7}{x_{11}^{5-1}}\right) \approx 1.47577316159455$$

$$x_{13} = \frac{1}{5}\left((5-1)x_{12} + \frac{7}{x_{12}^{5-1}}\right) \approx 1.47577316159455$$

Notice that we do not see any improvement from the 12th to the 13th value. Also, checking the calculator, we get the approximation for $\sqrt[5]{7}$ given by 1.47577316159455.

2. Here is the table that shows the calculator values of square roots of 10,20,…,200, together with the values obtained by using the algorithm for 5 and 10 steps.

N	$\sqrt{n}$	after 5 steps	after 10 steps
10	3.16227766	3.162277665	3.16227766
20	4.472135955	4.472140217	4.472135955
30	5.477225575	5.477306379	5.477225575
40	6.32455532	6.325023209	6.32455532
50	7.071067812	7.072628276	7.071067812
60	7.745966692	7.749784171	7.745966692
70	8.366600265	8.374286298	8.366600265
80	8.94427191	8.957833156	8.94427191
90	9.486832981	9.508604615	9.486832981
100	10	10.03257851	10
110	10.48808848	10.5342704	10.48808848
120	10.95445115	11.0171801	10.95445115
130	11.40175425	11.48407658	11.40175425
140	11.83215957	11.93718797	11.83215957
150	12.24744871	12.37833279	12.24744871
160	12.64911064	12.80901331	12.64911064
170	13.03840481	13.23048364	13.03840481
180	13.41640786	13.64380042	13.41640786

| 190 | 13.78404875 | 14.04986116 | 13.78404875 |
| 200 | 14.14213562 | 14.44943382 | 14.14213562 |

3. Here is the table showing also the values found by using the algorithm for 5,10 and 15 steps.

n	$\sqrt{n}$	after 5 steps	after 10 steps	after 15 steps
1000	31.6227766	41.24542607	31.6227766	31.6227766
2000	44.72135955	72.81065909	44.72135955	44.72135955
3000	54.77225575	104.172262	54.77225575	54.77225575
4000	63.2455532	135.4793778	63.2455532	63.2455532
5000	70.71067812	166.7640805	70.71067812	70.71067812
6000	77.45966692	198.0373991	77.45966692	77.45966692
7000	83.66600265	229.304146	83.66600266	83.66600265
8000	89.4427191	260.5667563	89.44271912	89.4427191
9000	94.86832981	291.8265942	94.86832988	94.86832981
10000	100	323.0844833	100.0000003	100
11000	104.8808848	354.3409506	104.8808855	104.8808848
12000	109.5445115	385.5963486	109.5445132	109.5445115
13000	114.0175425	416.8509223	114.0175461	114.0175425
14000	118.3215957	448.1048471	118.3216029	118.3215957
15000	122.4744871	479.358252	122.4745005	122.4744871
16000	126.4911064	510.6112338	126.4911299	126.4911064
17000	130.3840481	541.8638668	130.3840874	130.3840481
18000	134.1640786	573.1162088	134.1641416	134.1640786
19000	137.8404875	604.3683055	137.8405847	137.8404875
20000	141.4213562	635.6201936	141.4215015	141.4213562

4. The points less that 3 units away from 2 satisfy the inequality: $|x - 2| < 3$. So, $-3 < x - 2 < 3$. If we add 2 to all sides of inequality, we get : $-1 < x < 5$.

-1   5

The points less than 4 units away from -7 satisfy the inequality: $|x-(-7)| < 4$, i.e. $|x+7| < 4$. Hence, $-4 < x+7 < 4$. If we subtract 7 from all sides of the inequality, we get: $-11 < x < -3$.
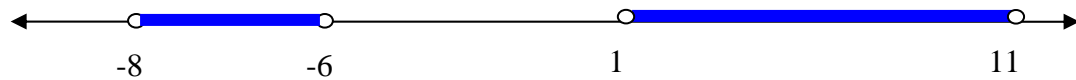

-11   -3

5. We're looking for real numbers satisfying $|x+7| < 1$ or $|x-6| < 5$.
$|x+7| < 1 \Rightarrow -1 < x+7 < 1 \Rightarrow -8 < x < -6$,
$|x-6| < 5 \Rightarrow -5 < x-6 < 5 \Rightarrow 1 < x < 11$.
So we need real numbers between -8 and -6, or between 1 and 11.

-8   -6   1   11

6. $|2x+5| \le 7 \Rightarrow -7 \le 2x+5 \le 7 \Rightarrow -12 \le 2x \le 2 \Rightarrow -6 \le x \le 1$.
Hence, the real numbers satisfying this inequality are:

-6   1

7. $|x^2-1| \le 3 \Rightarrow -3 \le x^2-1 \le 3 \Rightarrow -2 \le x^2 \le 4$. We know that $x^2 \ge 0$, so the
inequality becomes; $0 \le x^2 \le 4$, i.e. $0 \le x \le 2$.

0   2

8. Since $|2x-1| \geq 0$ (actually in this case $|2x-1| > 0$), we can multiply the inequality

by $|2x-1|$; $\dfrac{2}{|2x-1|} > 3 \Rightarrow 2 > 3|2x-1|$. If we divide both side by 3(we can do

this, 3>0), we get:

$|2x-1| < \dfrac{2}{3} \Rightarrow -\dfrac{2}{3} < 2x-1 < \dfrac{2}{3} \Rightarrow \dfrac{1}{3} < 2x < \dfrac{5}{3} \Rightarrow \dfrac{1}{6} < x < \dfrac{5}{6}$. Hence, the

real numbers satisfying this inequality are:

1/6   5/6