Numbers, Operations and Quantitative Reasoning

This material is not intended to be a self contained construction of the real and complex number systems. Our goal is to give an overview of the basic properties of some important subsets of the real and complex number systems.

You should know how to add, subtract, multiply and divide real numbers.

$$2 + 3 = 5$$

$$13 - 7 = 6$$

$$7 \times (-3) = -21$$

$$6 \div 4 = \frac{3}{2}$$

You should also be comfortable with decimal representations of real numbers, and you should know that real numbers can sometimes be represented in more than one form. For example, $1/2$ and 0.5 represent the same real number.

The algebraic structure of the real number system is described by the **Field Axioms**. For the purposes of our discussion, we will use the notation $\mathbb{R}$ for the set of real numbers, and the symbol $\in$ as shorthand for "is/are an element of." The Field Axioms are given below.

Axiom 0.1: (The Field Axioms) The operations $+$ and $\times$ satisfy the following properties for every $a, b, c \in \mathbb{R}$.

Name	Addition	Multiplication
Commutative Property	$a + b = b + a$	$a \times b = b \times a$
Associative Property	$(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
Distributive Property	$a \times (b + c) = a \times b + a \times c$	
Identity	$a + 0 = 0 + a = a$	$a \times 1 = 1 \times a = a$
Inverse	There exists $-a \in \mathbb{R}$ so that $a + (-a) = (-a) + a = 0$	if $a \neq 0$ there exists $a^{-1} \in \mathbb{R}$ so that $a \times a^{-1} = a^{-1} \times a = 1$

Remark 0.2: For simplicity, we will use **implied multiplication** instead of the symbol $\times$. Consequently, if a and b are real numbers, we will write ab for $a \times b$. As a result, $2(a + 3) - 4b$ is the same as $2 \times (a + 3) - 4 \times b$. **Subtraction** is equivalent to addition of an additive inverse, and **division** is interpreted as multiplication by a multiplicative inverse. That is, $a - b = a + (-b)$, $a/b = a \times b^{-1}$, and $b^{-1} = 1/b$ when $b \neq 0$.

Remark 0.3: We will use **exponential notation** for repeated multiplication. For example,

$$a^1 = a,\ a^2 = aa,\ a^3 = aaa\ ,\ \text{etc.}$$

We will also use the convention that $a^0 = 1$ when $a \neq 0$. The meaning for negative exponents follows from the fact that $a^{-1} = 1/a$ when $a \neq 0$. As a result,

$$a^{-2} = \frac{1}{a^2},\ a^{-3} = \frac{1}{a^3},\ \text{etc.}$$

Finally, you should also be familiar with the basic **order operations** $<, >, =, \leq, \geq$ and their properties. In general, if $a, b, c \in R$ then the following hold:

O1. $a = b$ implies $a + c = b + c$ and $ac = bc$

O2. $a < b$ implies $a + c < b + c$, and $c > 0$ implies $ac < bc$

O3. $a \leq b$ implies $a + c \leq b + c$, and $c > 0$ implies $ac \leq bc$

O4. $c < 0$ and $a < b$ implies $ac > bc$

These field and order properties above will be used to discuss the structure of a variety of subsets of $\mathbb{R}$.