

### Exercises 2.3

Find the general solution of the Bernoulli equation.

1.  $y' + \frac{1}{x}y = 3x^2y^2$ .

2.  $y' + xy = xy^3$ .

3.  $y' - 4y = 2e^x\sqrt{y}$ .

4.  $2xyy' = 1 + y^2$

5.  $3y' + 3x^{-1}y = 2x^2y^4$ .

6.  $(x - 2)y' + y = 5(x - 2)^2y^{1/2}$ .

Show that each of the following differential equations is homogeneous and find the general solution of the equation.

7.  $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ .

8.  $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$ .

9.  $\frac{dy}{dx} = \frac{x^2e^{y/x} + y^2}{xy}$ .

10.  $y' = \frac{x^4 + 2y^4}{xy^3}$ .

11.  $y' = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$ .

12.  $y' = \frac{y + \sqrt{x^2 - y^2}}{x}$ .

Find the general solution. (These equations are a mixture of linear, separable, Bernoulli and homogeneous equations.)

13.  $x(1 + y^2) + y(1 + x^2)y' = 0$ .

14.  $xy' = y + x^2e^x$ .

15.  $xy' + y - \sec x = 0$ .

16.  $y' = \frac{2xy}{x^2 - y^2}$ .

17.  $(xy + y)y' = x - xy$ .

18.  $\frac{dy}{dx} = 2x - 2xy$ .

$$19. \frac{dy}{dx} = \frac{xe^{y/x} + y}{x}.$$

$$20. y' = (y/x) + \sin(y/x).$$

$$21. (3x^2 + 1)y' - 2xy = 6x.$$

$$22. x(1 - y) + y(1 + x^2) \frac{dy}{dx} = 0.$$

$$23. xy' + y = y^2 \ln x.$$

$$24. y' = -\frac{3y}{x} + x^4 y^{1/3}.$$

$$25. \frac{dy}{dx} = \frac{x^3 + y^3}{3xy^2}.$$

26. (a) Show that the change of variable  $u = \ln y$  transforms

$$y' + yp(x) \ln y = q(x) y$$

into a linear equation.

(b) Find the general solution of

$$y' - \frac{y}{x} \ln y = xy$$

using the change of variable indicated in Exercise .

27. (a) Determine a change of variable that will transform

$$y' \cos y + p(x) \sin y = q(x)$$

into a linear equation.

(b) Find the general solution of

$$y' \cos y + 2x \sin y = 4e^{-x^2}.$$