Problem 1.

(a) (i) $\overrightarrow{AB} = (2,4,1)$, $\overrightarrow{AC} = (-3,3,-6)$, $\overrightarrow{BC} = (-5-1-7)$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 \implies \overrightarrow{AB} \perp \overrightarrow{AC}$, A, B, C are the vertices of a right triangle.

(ii) hypotenuse: $\|\overrightarrow{BC}\| = \sqrt{75} = 5\sqrt{3}$

(b) (i) $3(2\mathbf{a} - \mathbf{b} + \mathbf{c}) = 3\,\mathbf{i} - 9\,\mathbf{k}$

(ii) $\text{proj}_{\mathbf{b}}\mathbf{c} = \dfrac{\mathbf{b} \cdot \mathbf{c}}{\|\mathbf{b}\|^2}\,\mathbf{b} = \frac{9}{14}(2\,\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}) = \frac{9}{7}\,\mathbf{i} + \frac{27}{14}\,\mathbf{j} - \frac{9}{14}\,\mathbf{k}$

(c) $\mathbf{b} \times \mathbf{a} = -3\,\mathbf{i} - 6\,\mathbf{k}$; unit vector: $\frac{1}{\sqrt{5}}\,\mathbf{i} - \frac{2}{\sqrt{5}}\,\mathbf{k}$

(d) $V = 15$

Problem 2.

(a) normal vector for plane: $\mathbf{N} = (3, -1, 4)$; direction vector for line: $\mathbf{d} = (-2, 2, 2)$

$\mathbf{N} \cdot \mathbf{d} = 0 \implies \mathbf{N} \perp \mathbf{d}$; P and L are parallel.

(b) $\dfrac{-3-1}{-2} = \dfrac{0+4}{2} \neq \dfrac{5+3}{2}$; A does not line on L; $d(A, L) = \frac{4\sqrt{6}}{3}$

(c) $x = -3 + 3t$, $y = -t$, $z = 5 + 4t$

(d) $P\ (-51/13,\ 4/13,\ 49/13)$

Problem 3.

(a) $(x+2) + 4(y-1) - z = 0$

(b) $3(-2) - 1(1) + 2(0) = -7 \neq 4$; A is not on P; $d\,(A, P_2) = \dfrac{11}{\sqrt{14}}$

(c) $\mathbf{i} + 4\,\mathbf{j} - \mathbf{k} \neq \lambda\,(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k})$; P_1 and P_2 are not parallel.

direction vector for line of intersection: $\mathbf{N}_1 \times \mathbf{N}_2 = 7\,\mathbf{i} - 5\,\mathbf{j} - 13\,\mathbf{k}$;

point on line of intersection: $P\,(0,\ 24/7,\ 26/7)$

symmetric equations for line of intersection: $\dfrac{x}{7} = \dfrac{y - 24/7}{-5} = \dfrac{z - 26/7}{-13}$

(d) $\cos\theta = \dfrac{|\,\mathbf{N}_1 \times \mathbf{N}_2|}{\|\mathbf{N}_1 \times \mathbf{N}_2\|} = \dfrac{3}{\sqrt{252}} \cong 0.1890$; $\theta \cong 1.3$ rad. $\cong 79.11^o$

Problem 4.

(a) $[\mathbf{f}(t)\,\cdot\,\mathbf{g}(t)]' = 6t^2 + 2t + 3\pi\,\sin\,\pi t$

(b) (*i*) $\mathbf{v}(t) = \mathbf{r}'(t) = (-\,e^{-t}\sin t + e^{-t}\cos t)\ \mathbf{i} - (-\,e^{-t}\cos t + e^{-t}\sin t)\ \mathbf{j} + 3\,\mathbf{k}$

(*ii*) speed: $\|\mathbf{v}(t)\| = \sqrt{2e^{-2t} + 9}$

(*iii*) $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = -2\,e^{-t}\cos t\,\mathbf{i} + 2\,e^{-t}\sin t\,\mathbf{j}$

(*iv*) $\lim\limits_{t\to\infty} \mathbf{v}(t) = 3\,\mathbf{k}$

Problem 5.

(a) (*i*) $L(C) = \int_0^3 \sqrt{t^2 + t^4}\,dt = \int_0^3 t\sqrt{1+t^2}\,dt = \frac{1}{3}\Big[\left(1+t^2\right)^{3/2}\Big]_0^3 = \frac{1}{3}\left[(10)^{3/2} - 1\right]$

(*ii*) $\kappa = \dfrac{\sqrt{2}}{4}$

(b) (*i*) $L(C) = \int_0^3 \sqrt{4 + 4t^2 + t^4}\,dt = \int_0^4 \left(2 + t^2\right)\,dt = 15$

(*ii*) $\mathbf{T}(1) = \frac{2}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} + \frac{1}{3}\,\mathbf{k}$; $\quad \mathbf{N}(1) = -\,\frac{2}{3}\,\mathbf{i} + \frac{1}{3}\,\mathbf{j} + \frac{2}{3}\,\mathbf{k}$

(*iii*) normal vector: $\mathbf{T}(1) \times \mathbf{N}(1) = \mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$; point: $\mathbf{r}(1) = 2\,\mathbf{i} + \mathbf{j} + \frac{1}{3}\,\mathbf{k}$, i.e. $\left(2, 1, \frac{1}{3}\right)$;

osculating plane: $(x-2) - 2(y-1) + 2\left(z - \frac{1}{3}\right) = 0$

Problem 6.

(a) $x = -\,\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\,t, \quad y = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\,t, \quad z = 3\pi + 4t$

(b) $\mathbf{T}(t) = -\,\frac{3}{5}\sin t\,\mathbf{i} + \frac{3}{5}\,\cos t\,\mathbf{j} + \frac{4}{5}\,\mathbf{k}$; $\quad \mathbf{N}(t) = -\,\cos t\,\mathbf{i} -\,\sin t\,\mathbf{j}$

(c) $\kappa = \frac{3}{25}$

(d) $\mathbf{a}(t) = 0\,\mathbf{T} + 3\,\mathbf{N}$; $\quad a_{\mathbf{T}} = 0, \quad a_{\mathbf{n}} = 3$