Section 1.2
Alphabetic and Multiplicative Systems of Numeration

Alphabetic Systems

The second basic type of system of numeration developed was based on the written alphabet of the culture. Writing words with an alphabet, unlike the Egyptian pictograms, involved using a symbol for each phonetic sound. This allows the culture to write down a huge number of different words with just a few dozen symbols. The earliest written alphabet was that of the Phoenicia, dating from about the fifteenth century BCE. The peoples in the area surrounding Phoenicia each adapted the Phoenician alphabet to their needs, but most of them retained the basic order of the original Phoenician letters. Since the order of the letters remained fairly constant, it was a logical step to use the alphabet as a counting tool, just like using pebbles or sticks as counting tools. Start with "A" for one, "B" for two and continue in this fashion.

The disadvantage to this simple numbering system is that a person can only count to about twenty or thirty using letters. Later versions of alphabetic systems assigned numerical values to the letters by first assigning 1-9 to the first nine letters, then 10-90 in multiples of 10 to the next nine letters, then 100-900 in multiples of 100 to the last nine letters. This allows an alphabetic system to represent numbers from 1-999 easily using only 27 letters and an additive scheme for combining the values of the symbols.

Alphabetic systems were developed by many peoples, including the Armenians, Hebrews and Greeks. We will illustrate this type of system by looking at the Ionic Greek system of numeration.

Greek Numerals

The Ionic Greek system of numeration dates from the fourth century BCE. It used the 24 letters of the Greek alphabet plus 3 older Phoenician letters (digamma, koppa and sampi) to represent numbers. The value of each letter is given in the following table.

A number between 1 and 999 was written by combining the symbols for the appropriate symbols and writing them in descending order. The number 354 would be written as $\tau\nu\delta$ for 300 plus 50 plus 4. Numbers were also written with a small stroke over them to distinguish them from letters. The Greeks also used a small stroke to the left of the letters representing 1-9 to indicate multiplication by 1000. This gave a simple system for extending the system to represent numbers from 1-9,999. The number 10,000 was called a myriad and had a special symbol, M (capital mu).

GREEK NUMERALS

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
β beta	2	κ kappa	20	σ sigma	200
γ gamma	3	λ lambda	30	τ tau	300
δ delta	4	μ mu	40	υ upsilon	400
ε epsilon	5	ν nu	50	φ phi	500
ς digamma	6	ξ xi	60	χ chi	600
ζ zeta	7	o omicron	70	ψ psi	700
η eta	8	π pi	80	ω omega	800
θ theta	9	**Q** koppa	90	ϡ sampi	900

Since we do not need to be concerned about distinguishing numbers from letters, in this text we will not place the stroke over all numbers. For typesetting ease, we will indicate multiplication by 1000 as an apostrophe after the number. Thus α represents one and α' represents 1000.

Example 1: Write the number 3885 in Greek numerals.

Solution:

$$
\begin{aligned}
3885 &= 3000 + 800 + 80 + 5 \\
&= \gamma' \omega \pi \varepsilon
\end{aligned}
$$

Example 2: Write the number 737 in Greek numerals.

Solution:

$$
\begin{aligned}
737 &= 700 + 30 + 7 \\
&= \psi \lambda \zeta
\end{aligned}
$$

Exaample 3: Write the Greek number $\delta'\tau\kappa\eta$ in Hindu-Arabic numerals.

Solution:

Look the Greek letters up on the table. We find $\delta = 4$, $\tau = 300$, $\kappa = 20$ and $\eta = 8$. Thus, $\delta'\tau\kappa\eta = 4000 + 300 + 20 + 8 = 4328$.

Example 4: Write the Greek number $\chi\mu\varsigma$ in Hindu-Arabic numerals.

Solution:

Look the Greek letters up on the table. The values are $\chi = 600$, $\mu = 40$ and $\varsigma = 6$. Adding the values, $\chi\mu\varsigma = 600 + 40 + 6 = 646$.

The advantage of the Greek system over the earlier Egyptian system is that the number of symbols needed to express a large number in written form was considerably less. It only takes four symbols to write 9,999 in Greek, and 36 symbols in Egyptian! Further, there were no “extra” symbols to learn once a student had learned his letters.

The disadvantages of the Greek system compared with our modern Hindu-Arabic system are fairly obvious. It is not easy to perform operations on numbers in the Greek system. Further, multiplying or dividing a Greek number by 10 changes most of the symbols, versus our system in which only the places shift. It is also fairly easy to confuse letters and numbers in an alphabetic system. This provides wonderful material for secret code enthusiasts, but can become very impractical in daily transactions.

Multiplicative Systems

The third type of system of numeration developed by ancient cultures is called a multiplicative system. In a multiplicative system, symbols are needed for the numbers 1-9 and different powers of ten. A number is broken into groups of powers of ten, just like in our Hindu-Arabic system. The difference is that in the multiplicative system, the multiplication is explicitly stated. So, instead of 534 in Hindu-Arabic, a multiplicative system would express this number as 5 times 100 plus 3 times ten plus 4.

TRADITIONAL CHINESE NUMERALS

The traditional system of numeration in China, dating from before 1000 BCE, is a multiplicative system. The numerals were traditionally written vertically, from top to bottom. The symbols are listed in the following table. Notice that the system included a symbol for zero, which is not the case of the Egyptian, Roman or Greek systems.

CHINESE NUMERALS

Number	Symbol
0	零
1	一
2	二
3	三
4	四
5	五
6	六
7	七
8	八
9	九
10	十
100	百
1000	千

The traditional Chinese system was a base ten system, with symbols for the digits 0-9 and symbols for the powers of ten. A number beyond ten was written as a sum of multiples of powers of ten, with the multiplication explicitly shown in the symbols for the various powers of ten. Eleven was “ten one” in traditional Chinese, twelve “ten two” and so on. Twenty is “two ten”, numbers up to 99 were “multiple of ten unit”. One hundred and one was “one hundred zero one”. Three hundred twenty-two would be “three hundred two ten two”. When a number contained a zero, except at the end, the zero would be written, but only once for two or more consecutive zeroes.

Example 5: Write the number 6,592 as a traditional Chinese number.

Solution:

The number 6,592 is 6 times 1000 plus 5 times 100 plus 9 times 10 plus 2.

Therefore, the number 6592 is the following Chinese numeral:

六
千
五
百
九
十
二

Example 6: Write the number 2083 as a traditional Chinese number.

Solution:

The number 2083 is 2 times 1000 plus 0 times 100 plus 8 times ten plus 3.

二
千
零
八
十
三

Example 7: Write the number 4,007 as a traditional Chinese number.

Solution:

The number 4,007 is 4 times 1000 plus 0 times 100 plus 0 times 10 plus 7. The zero hundreds and zero tens is indicated by just one symbol for zero in the middle.

Therefore, the number 4,007 is the following Chinese numeral:

四
千
零
七

Example 8: Write the following traditional Chinese numbers as Hindu-Arabic numbers.

(a)	二 千 零 三 十 八	(b)	六 百 五 十 二	(c)	九 千 八 百 零 一

Solution:

The numbers above can be read by translating the symbols. The following table does this in an organized manner. Notice that the zero is not followed by a power of ten for multiplication, it is merely a place holder.

	Part (a)	**Part (b)**	**Part (c)**
Thousands	2		9
	1000		1000
Hundreds	0	6	8
		100	100
Tens	3	5	0
	10	10	
Ones	8	2	1

Part (a) – The number is 2*1000 + 0*100 + 3*10 + 8 = 2038
Part (b) – The number is 6*100 + 5*10 + 2 = 652
Part (c) – The number is 9*1000 + 8*100 + 0*10 + 1 = 9801

The traditional Chinese system has many advantages over the additive and alphabetic systems we have examined. The base ten system is easy for us to understand, and multiplication and addition can be performed with comparative ease. It also requires less symbols than the additive systems of Egypt and Rome. However, compared to our modern Hindu-Arabic system is requires twice as many symbols to write the same numbers.