Problem 1.

(a) Determine whether the given function $f$ has an inverse. If $f$ has an inverse, find $f^{-1}$.

(i) $f(x) = (x + 1)^2 + 2$

(ii) $f(x) = 2e^{2x}$

(b) The function $G(x) = 2x - 1 + \sin x$ has an inverse. Find $[G^{-1}]'(1)$.

(c) Calculate the derivative of each of the following functions:

(i) $f(x) = \ln(\sec^{-1}x)$

(ii) $F(x) = 5^x \sinh(2e^x)$

(iii) $(\tan^{-1}x)^x$

Problem 2.

(a) The world population in 1960 was 3 billion. In 1980, it was 4.5 billion. Assuming that the population at time $t$ is increasing at a rate proportional to the size of the population at time $t$, in what year will the world population be double the size of the 1980 population?

(b) Cobalt-60 is a radioactive substance that is used extensively in medical technology. It has a half-life of 5.3 years. Suppose an initial sample of cobalt-60 has a mass of 100 grams. How long will it take for 90% of the sample to decay?

Problem 3. Calculate each of the following indefinite integrals:

(a) $\int x^2 e^{2x} \, dx$  
(b) $\int \frac{x^2}{\sqrt{9 - x^2}} \, dx$

(c) $\int \frac{e^x}{4 + e^{2x}} \, dx$  
(d) $\int \sin^{-1}x \, dx$

Problem 4. Calculate each of the following indefinite integrals:

(a) $\int \frac{3x^2 + 2x + 3}{(x^2 - 1)(x^2 + 1)} \, dx$  
(b) $\int x^2 \ln x \, dx$

(c) $\int \frac{1}{\sqrt{4 + x^2}} \, dx$  
(d) $\int \sqrt{\sin x} \cos^3 x \, dx$

Problem 5.

(a) Identify each of the differential equations (separable, linear, or both), and find the general solution.

(i) $y y' + xy^2 - x = 0$

(ii) $x y' - y = x^2 \sin 2x$

(b) Find a solution of the initial value problem: $y' = \frac{x}{y + x^2 y}$, $y(1) = 0$
Problem 6.

(a) Determine whether or not the given limit is an indeterminate form. If it is, give the form (e.g., \(0/0\), \(\infty/\infty\), etc.) and then evaluate the limit.

\[
\begin{align*}
& (i) \lim_{x \to \infty} x^2 e^{-x} \\
& (ii) \lim_{x \to \pi} \frac{\cos x}{x - \pi} \\
& (iii) \lim_{x \to \infty} \left(\frac{\cos 1}{x}\right)^x \\
& (iv) \lim_{x \to -0} \frac{3^x - 2^x}{x} \\
& (v) \lim_{x \to -0} \frac{x + 1 - e^x}{x^2} \\
& (vi) \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}
\end{align*}
\]

(b) State whether or not the given integral is improper. If it is improper, determine whether it converges or diverges; in the case of convergence, give the value.

\[
\begin{align*}
& (i) \int_{-1}^{1} \frac{1}{4 - x^2} \, dx \\
& (ii) \int_{0}^{4} \frac{1}{\sqrt{4 - x}} \, dx \\
& (iii) \int_{0}^{\infty} e^{-2x} \, dx
\end{align*}
\]

Problem 7.

(a) Find all possible polar coordinates for the point with rectangular coordinates \((-3, -3\sqrt{3})\).

(b) Write the equation \(r = 2 \sin \theta\) in rectangular coordinates.

(c) Sketch the graph of \(r = 1 - 2 \cos \theta, \quad 0 \leq \theta \leq \pi\).

(d) Find the area of one petal of the four-petal curve \(r = 4 \cos 2\theta\).

Problem 8.

(a) A particle moves along the curve \(C\) specified by the parametric equations:
\[
x(t) = t^2, \quad y(t) = \frac{4}{3} t^3, \quad 0 \leq t \leq 2.
\]

(i) Represent \(C\) by an equation in \(x\) and \(y\) and give the limits on \(x\).

(ii) Find an equation for the tangent line to \(C\) at the point where \(t = 1\).

(b) Find the points on the polar curve \(r = 1 + \sin \theta\) where the tangent line is vertical.

(c) A curve \(C\) is defined parametrically by:
\[
x = \cos t, \quad y = \frac{1}{2}(\cos 2t - 1), \quad 0 \leq t \leq 2\pi.
\]

Represent \(C\) by an equation in \(x\) and \(y\) and draw the graph.

Problem 9.

(a) Determine whether or not the sequence is bounded and whether it converges or diverges. If it is convergent, give the limit.

\[
\begin{align*}
& (i) -1, \frac{2}{3}, -\frac{4}{5}, \frac{4}{7}, -\frac{5}{9}, \ldots \\
& (ii) \left\{ \frac{n^2 \cos(n\pi)}{2n + 1} \right\} \\
& (iii) a_1 = 1, \quad a_{n+1} = \frac{1}{3} a_n + 1
\end{align*}
\]
(b) Determine whether the series is absolutely convergent, conditionally convergent or divergent.

\[
(i) \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 3^{k+1}}{4^k} \\
(ii) \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k+1)}{\sqrt{4k^2 + 3k + 2}} \\
(iii) \sum_{k=0}^{\infty} \frac{\cos(k\pi)}{\sqrt{k+1}}
\]

**Problem 10.** Determine whether the given (nonnegative) series converges or diverges. State the test being used to determine convergence/divergence and show all work.

(a) \[\sum_{k=1}^{\infty} \frac{3^{k+1}}{k^2 e^k}\]

(b) \[\sum_{k=2}^{\infty} \frac{1}{k (\ln k)^3}\]

(c) \[\sum_{k=0}^{\infty} \frac{2k + 1}{\sqrt{k^3 + 2k^3 + 1}}\]

(d) \[\sum_{k=0}^{\infty} \frac{k!}{(k + 1)^2 2^k}\]

**Problem 11.**

(a) Determine the radius of convergence and the interval of convergence of each of the following power series:

(i) \[\sum_{k=0}^{\infty} \frac{(-1)^k \cdot 5^k}{(k + 1)!} x^k\]

(ii) \[\sum_{k=0}^{\infty} \frac{1}{(k + 1)4^k} x^k\]

(b) The Taylor series expansion (in powers of x) of \(f(x) = \frac{1}{1 + x^2}\) is

\[f(x) = 1 - x^2 + x^4 - \cdots = \sum_{k=0}^{\infty} (-1)^k x^{2k}\].

Determine the Taylor series expansion of \(F(x) = \tan^{-1} x\) and express it in the form \(\sum_{k=0}^{\infty} a_k x^k\).

**Problem 12.**

(a) The Taylor series expansion of \(e^x\) is: \(e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k\). Determine the Taylor series expansion of \(\sinh x = \frac{1}{2} (e^x - e^{-x})\).

(b) Determine the Taylor polynomial \(P_5(x)\) for the function \(g(x) = x^2 + \sin 2x\).

**Problem 13.**

(a) Use the Taylor polynomial \(P_3(x)\) for \(f(x) = e^x\) to approximate \(\sqrt{e}\). Estimate the error in the approximation.

(b) Approximate \(\ln(0.5)\) correct to 4 decimal places.