Section 2.4: Applications and Writing Functions

- Setting up Functions to Solve Applied Problems
- Maximum or Minimum Value of a Quadratic Function

Setting up Functions to Solve Applied Problems

Note: For a review of geometric formulas, please refer to Appendix A.3: Geometric Formulas.

Example:
Suppose that the perimeter of a rectangle is 600 ft. If \( x \) represents the width of the rectangle (in ft), then express the area of the rectangle as a function of \( x \).

Solution:
Step 1)
Perimeter = 600 ft, \( x \) = width (in ft), \( L \) = length (in ft)

\[
\begin{array}{c}
\text{} \\
x \\
\text{} \\
L
\end{array}
\]

Step 2)
We want to express the area in terms of the width \( x \). Let \( A \) denote the area of the rectangle. Then \( A = L \cdot x \).

Step 3)
In the equation \( A = L \cdot x \), we need to rewrite \( L \) in terms of \( x \). To do this, recall that the perimeter of a rectangle is equal to \( 2L + 2x \). We are given that the perimeter is 600 ft. We now have
\[ 2L + 2x = 600 \]
\[ 2L = 600 - 2x \]
\[ L = \frac{600 - 2x}{2} \]
\[ L = 300 - x. \]

Step 4)
\[ A = L \cdot x \]
\[ A = (300 - x)x \]
\[ A = 300x - x^2 \]

\[ A = 300x - x^2 \] is the required equation (the area is given in terms of the width \( x \)). To emphasize that \( A \) depends upon \( x \), we can use function notation to rewrite the equation.

\[ A(x) = 300x - x^2 \]

The domain of this function is \((0, 300)\). The width must be positive, so \( x > 0 \). Since the length is also positive, we see from the equation \( L = 300 - x \) that \( x < 300 \).

**Example:**
Suppose that a right circular cylinder has a volume \( V \) of 30 cm\(^3\). Express the surface area \( S \) as a function of \( r \), the radius of the base.

**Solution:**
Step 1)
Volume = 30 cm\(^3\), \( r = \) radius (in cm), \( h = \) height (in cm)

![Diagram of a right circular cylinder with volume \( V = 30 \text{ cm}^3 \).]

Step 2)
We want to express the surface area \( S \) in terms of \( r \). We know that the surface area is given by the formula \( S = 2\pi r^2 + 2\pi rh \).
Step 3)
In the equation $S = 2\pi r^2 + 2\pi rh$, we need to write $h$ in terms of $r$. To do this, we recall that the volume $V$ of a right circular cylinder is equal to $\pi r^2 h$. We are given that the volume is 30 cm$^3$. We now have

\[
\pi r^2 h = 30
\]

\[
h = \frac{30}{\pi r^2}.
\]

Step 4)

\[
S = 2\pi r^2 + 2\pi rh
\]

\[
S = 2\pi r^2 + 2\pi r \left( \frac{30}{\pi r^2} \right)
\]

\[
S = 2\pi r^2 + \frac{60}{r}
\]

$S = 2\pi r^2 + \frac{60}{r}$ is the required equation. To emphasize that $S$ depends upon $r$, we can use the function notation to rewrite the equation.

\[
S(r) = 2\pi r^2 + \frac{60}{r}
\]

The only restriction on $r$ is that it be positive. Thus, the domain of the function is $(0, \infty)$.

Additional Example 1:
Suppose that a rectangle is inscribed in a circle whose diameter is 12 cm. If $x$ represents the width of the rectangle (in cm), then express the perimeter of the rectangle as a function of $x$.

Solution:
Step 1)
Diameter = 12 cm, $x =$ width (in cm), $L =$ length (in cm)
Step 2)
We want to express the perimeter in terms of the width $x$. Let $P$ denote the perimeter of the rectangle. Then $P = 2L + 2x$.

Step 3)
In the equation $P = 2L + 2x$, we need to rewrite $L$ in terms of $x$. To do this, recall from the Pythagorean Theorem that $L^2 + x^2 = (12)^2 = 144$. We now have

$$L^2 + x^2 = 144$$
$$L^2 = 144 - x^2$$
$$L = \sqrt{144 - x^2}.$$

Step 4)

$$P = 2L + 2x$$
$$P = 2\sqrt{144 - x^2} + 2x$$

$P = 2\sqrt{144 - x^2} + 2x$ is the required equation (the perimeter is given in terms of the width $x$). To emphasize that $P$ depends upon $x$, we can use function notation to rewrite the equation.

$$P(x) = 2\sqrt{144 - x^2} + 2x$$

The domain of the function is $(0, 12)$.

Additional Example 2:
Suppose that a rectangle is inscribed in a circle whose diameter is 12 cm. If $x$ represents the width of the rectangle (in cm), then express the area of the rectangle as a function of $x$.

Solution:
Step 1)
Diameter = 12 cm, $x =$ width (in cm), $L =$ length (in cm)
Step 2)
We want to express the area in terms of the width \( x \). Let \( A \) denote the area of the rectangle. Then \( A = L \cdot x \).

Step 3)
In the equation \( A = L \cdot x \), we need to rewrite \( L \) in terms of \( x \). To do this, recall from the Pythagorean Theorem that \( L^2 + x^2 = (12)^2 = 144 \). We now have

\[
L^2 + x^2 = 144
\]
\[
L^2 = 144 - x^2
\]
\[
L = \sqrt{144 - x^2}.
\]

Step 4)
\[
A = L \cdot x
\]
\[
A = x \cdot L
\]
\[
A = x \sqrt{144 - x^2}
\]

\( A = x \sqrt{144 - x^2} \) is the required equation (the area is given in terms of the width \( x \)). To emphasize that \( A \) depends upon \( x \), we can use function notation to rewrite the equation.

\[
A(x) = x \sqrt{144 - x^2}
\]

The domain of the function is \((0, 12)\).

Additional Example 3:
Suppose that \( P(x, y) \) is a point that lies on the parabola \( y = x^2 \). If \( D \) represents the distance from the point \( P \) to the point \((0, 3)\), express \( D \) as a function of \( x \).

Solution:
Step 1)
Step 2)  
Let $D$ denote the distance from the point $P(x, y)$ to the point $(0, 3)$. Recall from the distance formula that 
$$D = \sqrt{(x-0)^2 + (y-3)^2} = \sqrt{x^2 + (y-3)^2}.$$ 

Step 3)  
In the equation $D = \sqrt{x^2 + (y-3)^2}$, we need to rewrite $y$ in terms of $x$. To do this, recall that we are given that $y = x^2$. 

Step 4)  
$$D = \sqrt{x^2 + (y-3)^2}$$ 
$$D = \sqrt{x^2 + (x^2 - 3)^2}$$ 
$$D = \sqrt{x^2 + x^4 - 6x^2 + 9}$$ 
$$D = \sqrt{x^4 - 5x^2 + 9}$$ 

$D = \sqrt{x^4 - 5x^2 + 9}$ is the required equation. To emphasize that $D$ depends upon $x$, we can use function notation to rewrite the equation. 

$$D(x) = \sqrt{x^4 - 5x^2 + 9}$$ 

The domain of the function is $(-\infty, \infty)$ since the $x$-coordinate of a point on the curve $y = x^2$ can be any real number.

**Additional Example 4:** 
Suppose that $P(x, y)$ is a point that lies on the line $y = 2x - 1$. If $D$ represents the distance from the point $P$ to the point $(-1, 2)$, express $D$ as a function of $x$.

**Solution:**  
Step 1] 

![Graph showing the line $y = 2x - 1$ and the point $P(x, y)$ connected to the point $(-1, 2)$ with distance $D$.]}
Step 2)
Let \( D \) denote the distance from the point \( P(x,y) \) to the point \((-1,2)\). Recall from the distance formula that \( D = \sqrt{(x-(-1))^2 + (y-2)^2} = \sqrt{(x+1)^2 + (y-2)^2} \).

Step 3)
In the equation \( D = \sqrt{(x+1)^2 + (y-2)^2} \), we need to rewrite \( y \) in terms of \( x \). To do this, recall that we are given that \( y = 2x-1 \).

Step 4)
\[
\begin{align*}
D &= \sqrt{(x+1)^2 + (y-2)^2} \\
D &= \sqrt{(x+1)^2 + (2x-1-2)^2} \\
D &= \sqrt{(x+1)^2 + (2x-3)^2} \\
D &= \sqrt{x^2 + 2x + 1 + 4x^2 - 12x + 9} \\
D &= \sqrt{5x^2 - 10x + 10}
\end{align*}
\]

\( D = \sqrt{5x^2 - 10x + 10} \) is the required equation. To emphasize that \( D \) depends upon \( x \), we can use function notation to rewrite the equation.

\[
D(x) = \sqrt{5x^2 - 10x + 10}
\]

The domain of the function is \((-\infty, \infty)\) since the \( x \)-coordinate of a point on the curve \( y = 2x-1 \) can be any real number.

**Maximum or Minimum Value of a Quadratic Function**

**Example:**
If the sum of two numbers is 24, find the smallest possible value of the sum of their squares.

**Solution:**
Let \( x \) and \( y \) denote the two numbers. We want to set up a function that represents the sum of the squares of the two numbers. Let \( S \) denote the sum of their squares.

\[
S = x^2 + y^2
\]
In the equation $S = x^2 + y^2$, we need to write $S$ in terms of $x$ only. Thus, we need to rewrite $y$ in terms of $x$. Use the given information that the sum of the two numbers is 24.

\[ x + y = 24 \]
\[ y = 24 - x \]

Substitute $24 - x$ for $y$ in the equation $S = x^2 + y^2$.

\[
S = x^2 + (24 - x)^2 \\
= x^2 + 576 - 48x + x^2 \quad \text{Expand} \ (24 - x)^2. \\
= 2x^2 + 576 - 48x \quad \text{Combine like terms.} \\
= 2x^2 - 48x + 576
\]

The quadratic function $S(x) = 2x^2 - 48x + 576$ denotes the sum of the squares of the two numbers in terms of $x$.

The graph of $S$ is a parabola that opens upward ($a = 2$). So, $S$ has a minimum value. To find this minimum value, we can complete the square to find the vertex of the parabola.

\[
S(x) = 2x^2 - 48x + 576 \\
= 2\left(x^2 - 24x + 144\right) + 576 - 288 \quad \text{Complete the square for} \ x^2 - 24 \text{ by adding} \ \left[\frac{1}{2}(-24)\right]^2 = 144. \\
= 2(x - 12)^2 + 288
\]

The vertex is the point $(12, 288)$.

We now see that $x = 12$ yields the minimum value. Thus, one of the two numbers is 12. The other number is also 12 since $y = 24 - x = 24 - 12 = 12$.

The minimum value of $S$ is $S(12) = 288$.

Therefore, the smallest possible sum of the squares of the two numbers is 288.
Additional Example 1:
If the sum of two numbers is 18, find the largest possible value of their product.

Solution:
Let $x$ and $y$ denote the two numbers. We want to set up a function that represents the product of the two numbers. Let $P$ denote their product.

$$P = xy$$

In the equation $P = xy$, we need to write $P$ in terms of $x$ only. Thus, we need to rewrite $y$ in terms of $x$. Use the given information that the sum of the two numbers is 18.

$$x + y = 18$$
\[y = 18 - x\]

Substitute $18 - x$ for $y$ in the equation $P = xy$.

$$P = x(18-x)$$
\[= 18x - x^2\] Use the distributive property.

The quadratic function $P(x) = 18x - x^2$ denotes the product of the two numbers in terms of $x$.

The graph of $P$ is a parabola that opens downward ($a = -1$). So, $P$ has a maximum value. To find this maximum value, we can complete the square to find the vertex of the parabola.

$$P(x) = 18x - x^2$$
\[= -x^2 + 18x\]
\[= -(x^2 - 18x)\] Factor out a $-1$.

\[= -(x^2 - 18x + 81) + 81\] Complete the square for $x^2 - 18x$ by adding $\left[\frac{1}{2}(-18)\right]^2 = 81$.
\[= -(x - 9)^2 + 81\]
The vertex is the point \((9,81)\).

We now see that \(x = 9\) yields the maximum value. Thus, one of the two numbers is 9. The other number is also 9 since \(y = 18 - x = 18 - 9 = 9\).

The maximum value of \(P\) is \(P(9) = 81\).

Therefore, the largest possible value of the product of the two numbers is 81.

**Additional Example 2:**

If the sum of two numbers is 8, find the smallest possible value of the sum of their squares.

**Solution:**
Let \(x\) and \(y\) denote the two numbers. We want to set up a function that represents the sum of the squares of the two numbers. Let \(S\) denote the sum of their squares.

\[ S = x^2 + y^2 \]

In the equation \(S = x^2 + y^2\), we need to write \(S\) in terms of \(x\) only. Thus, we need to rewrite \(y\) in terms of \(x\). Use the given information that the sum of the two numbers is 8.

\[ x + y = 8 \]
\[ y = 8 - x \]

Substitute \(8 - x\) for \(y\) in the equation \(S = x^2 + y^2\).

\[ S = x^2 + (8 - x)^2 \]
\[ = x^2 + 64 - 16x + x^2 \quad \text{Expand} \ (8 - x)^2. \]
\[ = 2x^2 - 16x + 64 \quad \text{Combine like terms.} \]

The quadratic function \(S(x) = 2x^2 - 16x + 64\) denotes the sum of the squares of the two numbers in terms of \(x\).
The graph of $S$ is a parabola that opens upward ($a = 2$). So, $S$ has a minimum value. To find this minimum value, we can complete the square to find the vertex of the parabola.

\[
S(x) = 2x^2 - 16x + 64
= 2(x^2 - 8x) + 64 \quad \text{Factor out a 2 from the first two terms.}
\]

\[
= 2(x^2 - 8x + 16) + 64 - 32 \quad \text{Complete the square for } x^2 - 8x \text{ by adding } \left[\frac{1}{2}(-8)\right]^2 = 16.
\]

\[
= 2(x - 4)^2 + 32
\]

The vertex is the point $(4,32)$.

We now see that $x = 4$ yields the minimum value. Thus, one of the two numbers is 4. The other number is also 4 since $32 - x = 8 - 4 = 4$.

The minimum value of $S$ is $S(4) = 32$.

Therefore, the smallest possible value of the sum of the squares of the two numbers is 32.

**Additional Example 3:**

If a rectangle has a perimeter of 50 cm, find the largest possible value of its area.

**Solution:**

Let $x$ denote the width of the rectangle (in cm) and let $y$ denote the length of the rectangle (in cm). We want to set up a function that represents the area of the rectangle. Let $A$ denote its area.

\[
A = xy
\]

In the equation $A = xy$, we need to write $A$ in terms of $x$ only. Thus, we need to rewrite $y$ in terms of $x$. Use the given information that the perimeter of the rectangle is 50 cm.

\[
2x + 2y = 50
\]

\[
2y = 50 - 2x
\]

\[
y = 25 - x
\]
Substitute $25 - x$ for $y$ in the equation $A = xy$.

$$A = x(25 - x) = 25x - x^2$$

Use the distributive property.

The quadratic function $A(x) = 25x - x^2$ denotes the area of the rectangle in terms of $x$.

The graph of $A$ is a parabola that opens downward ($a = -1$). So, $A$ has a maximum value. To find this maximum value, we can complete the square to find the vertex of the parabola.

$$A(x) = 25x - x^2$$

$$= -x^2 + 25x$$

$$= -\left(x^2 - 25x\right)$$

Factor out $-1$.

$$= -\left(\frac{x^2 - 25x}{4}\right) + \frac{625}{4}$$

Complete the square for $x^2 - 25x$ by adding $\left(\frac{1}{2}(-25)\right)^2 = \frac{625}{4}$.

$$= -\left(\frac{x - 25}{2}\right)^2 + \frac{625}{4}$$

The vertex is the point $\left(\frac{25}{2}, -\frac{625}{4}\right)$.

We now see that $x = \frac{25}{2}$ yields the maximum value. Thus, the width of the rectangle is $\frac{25}{2}$ cm. The length of the rectangle is also $\frac{25}{2}$ cm since $y = 25 - x = 25 - \frac{25}{2} = \frac{25}{2}$.

The maximum value of $A$ is $A\left(\frac{25}{2}\right) = \frac{625}{4}$.

Therefore, the largest possible value of the area of the rectangle is $\frac{625}{4}$ sq cm.
CHAPTER 2 Polynomial and Rational Functions

Additional Example 4:
If the sum of two numbers is 10, find the smallest possible value of the sum of the first number and square of the second number.

Solution:
Let \( x \) denote the first number and let \( y \) denote second number. We want to set up a function that represents the sum of first number and square of the second. Let \( S \) denote this sum.

\[ S = x + y^2 \]

In the equation \( S = x + y^2 \), we need to write \( S \) in terms of \( x \) only. Thus, we need to rewrite \( y \) in terms of \( x \). Use the given information that the sum of the two numbers is 10.

\[
\begin{align*}
x + y &= 10 \\
y &= 10 - x
\end{align*}
\]

Substitute \( 10 - x \) for \( y \) in the equation \( S = x + y^2 \).

\[
\begin{align*}
S &= x + (10 - x)^2 \\
&= x + 100 - 20x + x^2 & \text{Expand \((10 - x)^2\).} \\
&= -19x + 100 + x^2 & \text{Combine like terms.} \\
&= x^2 - 19x + 100
\end{align*}
\]

The quadratic function \( S(x) = x^2 - 19x + 100 \) denotes the sum of the first number and square of the second number in terms of \( x \).

The graph of \( S \) is a parabola that opens upward (\( a = 1 \)). So, \( S \) has a minimum value. To find this minimum value, we can complete the square to find the vertex of the parabola.

\[
S(x) = x^2 - 19x + 100
\]

\[
= \left( x^2 - 19x + \frac{361}{4} \right) + 100 - \frac{361}{4} \quad \text{Complete the square for } x^2 - 19x \text{ by adding } \left( \frac{1}{2}(-19) \right)^2 = \frac{361}{4}.
\]

\[
= \left( x - \frac{19}{2} \right)^2 + \frac{39}{4}
\]
The vertex is the point \( \left( \frac{19}{2}, \frac{39}{4} \right) \).

We now see that \( x = \frac{19}{2} \) yields the minimum value. Thus, the first number is \( \frac{19}{2} \).

The second number is \( \frac{1}{2} \) since \( y = 10 - x = 10 - \frac{19}{2} = \frac{1}{2} \).

The minimum value of \( S \) is \( S \left( \frac{19}{2} \right) = \frac{39}{4} \).

Therefore, the smallest possible value of the sum of the first number and square of the second number is \( \frac{39}{4} \).
Answer the following. If an example contains units of measurement, assume that any resulting function reflects those units. (Note: Refer to Appendix A.3 if necessary for a list of Geometric Formulas.)

1. The perimeter of a rectangle is 54 feet.
   (a) Express its area, $A$, as a function of its width, $w$.
   (b) For what value of $w$ is $A$ the greatest?
   (c) What is the maximum area of the rectangle?

2. The perimeter of a rectangle is 62 feet.
   (a) Express its area, $A$, as a function of its length, $l$.
   (b) For what value of $l$ is $A$ the greatest?
   (c) What is the maximum area of the rectangle?

3. Two cars leave an intersection at the same time. One is headed south at a constant speed of 50 miles per hour. The other is headed east at a constant speed of 120 miles per hour. Express the distance, $d$, between the two cars as a function of the time, $t$.

4. Two cars leave an intersection at the same time. One is headed north at a constant speed of 30 miles per hour. The other is headed west at a constant speed of 40 miles per hour. Express the distance, $d$, between the two cars as a function of the time, $t$.

5. If the sum of two numbers is 20, find the smallest possible value of the sum of their squares.

6. If the sum of two numbers is 16, find the smallest possible value of the sum of their squares.

7. If the sum of two numbers is 8, find the largest possible value of their product.

8. If the sum of two numbers is 14, find the largest possible value of their product.

9. A farmer has 1500 feet of fencing. He wants to fence off a rectangular field that borders a straight river (needing no fence along the river). What are the dimensions of the field that has the largest area?

10. A farmer has 2400 feet of fencing. He wants to fence off a rectangular field that borders a straight river (needing no fence along the river). What are the dimensions of the field that has the largest area?

11. A farmer with 800 feet of fencing wants to enclose a rectangular area and divide it into 3 pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 3 pens?

12. A farmer with 1800 feet of fencing wants to enclose a rectangular area and divide it into 5 pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 5 pens?

13. The hypotenuse of a right triangle is 6 m. Express the area, $A$, of the triangle as a function of the length $x$ of one of the legs.

14. The hypotenuse of a right triangle is 11 m. Express the area, $A$, of the triangle as a function of the length $x$ of one of the legs.

15. The area of a rectangle is 22 ft$^2$. Express its perimeter, $P$, as a function of its length, $l$.

16. The area of a rectangle is 36 ft$^2$. Express its perimeter, $P$, as a function of its width, $w$.

17. A rectangle has a base on the $x$-axis and its upper two vertices on the parabola $y = 9 - x^2$.
   (a) Express the area, $A$, of the rectangle as a function of $x$.
   (b) Express the perimeter, $P$, of the rectangle as a function of $x$.

18. A rectangle has a base on the $x$-axis and its lower two vertices on the parabola $y = x^2 - 16$.
   (a) Express the area, $A$, of the rectangle as a function of $x$.
   (b) Express the perimeter, $P$, of the rectangle as a function of $x$.

19. In a right circular cylinder of radius $r$, if the height is twice the radius, express the volume, $V$, as a function of $r$. 

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20. In a right circular cylinder of radius \( r \), if the height is half the radius, express the volume, \( V \), as a function of \( r \).

21. A right circular cylinder of radius \( r \) has a volume of 300 cm\(^3\).
   (a) Express the lateral area, \( L \), in terms of \( r \).
   (b) Express the total surface area, \( S \), as a function of \( r \).

22. A right circular cylinder of radius \( r \) has a volume of 750 cm\(^3\).
   (a) Express the lateral area, \( L \), in terms of \( r \).
   (b) Express the total surface area, \( S \), as a function of \( r \).

23. In a right circular cone of radius \( r \), if the height is four times the radius, express the volume, \( V \), as a function of \( r \).

24. In a right circular cone of radius \( r \), if the height is five times the radius, express the volume, \( V \), as a function of \( r \).

25. An open-top box with a square base has a volume of 20 cm\(^3\). Express the surface area, \( S \), of the box as a function of \( x \), where \( x \) is the length of a side of the square base.

26. An open-top box with a square base has a volume of 12 cm\(^3\). Express the surface area, \( S \), of the box as a function of \( x \), where \( x \) is the length of a side of the square base.

27. A piece of wire 120 cm long is cut into several pieces and used to construct the skeleton of a rectangular box with a square base.
   (a) Express the surface area, \( S \), of the box in terms of \( x \), where \( x \) is the length of a side of the square base.
   (b) What are the dimensions of the box with the largest surface area?
   (c) What is the maximum surface area of the box?

28. A piece of wire 96 in long is cut into several pieces and used to construct the skeleton of a rectangular box with a square base.
   (a) Express the surface area, \( S \), of the box in terms of \( x \), where \( x \) is the length of a side of the square base.
   (b) What are the dimensions of the box with the largest surface area?
   (c) What is the maximum surface area of the box?

29. A wire of length \( x \) is bent into the shape of a circle.
   (a) Express the circumference, \( C \), in terms of \( x \).
   (b) Express the area of the circle, \( A \), as a function of \( x \).

30. A wire of length \( x \) is bent into the shape of a square.
   (a) Express the area, \( A \), of the square as a function of \( x \).
   (b) Express the diagonal, \( d \), of the square as a function of \( x \).

31. Let \( P(x, y) \) be a point on the graph of \( y = x^2 - 10 \).
   (a) Express the distance, \( d \), from \( P \) to the origin as a function of \( x \).
   (b) Express the distance, \( d \), from \( P \) to the point \((0, -2)\) as a function of \( x \).

32. Let \( P(x, y) \) be a point on the graph of \( y = x^2 - 7 \).
   (a) Express the distance, \( d \), from \( P \) to the origin as a function of \( x \).
   (b) Express the distance, \( d \), from \( P \) to the point \((0, 5)\) as a function of \( x \).

33. A circle of radius \( r \) is inscribed in a square. Express the area, \( A \), of the square as a function of \( r \).

34. A square is inscribed in a circle of radius \( r \). Express the area, \( A \), of the square as a function of \( r \).

35. A rectangle is inscribed in a circle of radius 4 cm.
   (a) Express the perimeter, \( P \), of the rectangle in terms of its width, \( w \).
   (b) Express the area, \( A \), of the rectangle in terms of its width, \( w \).
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36. A rectangle is inscribed in a circle of diameter 20 cm.
   (a) Express the perimeter, $P$, of the rectangle in terms of its length, $\ell$.
   (b) Express the area, $A$, of the rectangle in terms of its length, $\ell$.

37. An isosceles triangle has fixed perimeter $P$ (so $P$ is a constant).
   (a) If $x$ is the length of one of the two equal sides, express the area, $A$, as a function of $x$.
   (b) What is the domain of $A$?

38. Express the volume, $V$, of a sphere of radius $r$ as a function of its surface area, $S$.

39. Two cars are approaching an intersection. One is 2 miles south of the intersection and is moving at a constant speed of 30 miles per hour. At the same time, the other car is 3 miles east of the intersection and is moving at a constant speed of 40 miles per hour.
   (a) Express the distance, $d$, between the cars as a function of the time, $t$.
   (b) At what time $t$ is the value of $d$ the smallest?

40. Two cars are approaching an intersection. One is 5 miles north of the intersection and is moving at a constant speed of 40 miles per hour. At the same time, the other car is 6 miles west of the intersection and is moving at a constant speed of 30 miles per hour.
   (a) Express the distance, $d$, between the cars as a function of the time, $t$.
   (b) At what time $t$ is the value of $d$ the smallest?

41. A straight wire 40 cm long is bent into an L shape. What is the shortest possible distance between the two ends of the bent wire?

42. A straight wire 24 cm long is bent into an L shape. What is the shortest possible distance between the two ends of the bent wire?

43. An equilateral triangle is inscribed in a circle of radius $r$, as shown below. Express the circumference, $C$, of the circle as a function of the length, $x$, of a side of the triangle.

44. An equilateral triangle is inscribed in a circle of radius $r$, as shown below. Express the area, $A$, within the circle, but outside the triangle, as a function of the length, $x$, of a side of the triangle.

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