Exercise Set 2.2: Polynomial Functions

Answer the following.

(a) State whether or not each of the following expressions is a polynomial. (Yes or No.)

(b) If the answer to part (a) is yes, then state the degree of the polynomial.

(c) If the answer to part (a) is yes, then classify the polynomial as a monomial, binomial, trinomial, or none of these. (Polynomials of four or more terms are not generally given specific names.)

1. \(4 + 3x^3\)
2. \(6x^5 + 3x^3 + \frac{8}{x}\)
3. \(3x - 5\)
4. \(2x^3 + 4x^2 - 7x - 4\)
5. \(\frac{5x^3 - 6x^2 + 7}{x^2 - 4x + 5}\)
6. \(8\)
7. \(\frac{7}{2}x^2 - \frac{5}{3}x + 9\)
8. \(\frac{7}{x^3} + \frac{5}{x^2} - \frac{3}{x} - 2\)
9. \(3^{-1}x^4 - 7^{-1}x + 2\)
10. \(-9x^{\frac{3}{2}} + 2x^{\frac{3}{2}} - 4x^{\frac{3}{2}}\)
11. \(|x^2 - 3x + 1|\)
12. \(-\frac{3}{2}x^6\)
13. \(6x^3 + 8x^2\)
14. \(x\)
15. \(-3 + 5x^3 + 6x^4 - 3x^9\)
16. \(3a^3b^4 - 2a^2b^2\)
17. \(-4x^5y^{-2} - 3x^{-4}y^9\)
18. \(4x^5y^3 + \frac{3}{xy^2}\)
19. \(\frac{2}{3}x^3y^9z + 3xy - \frac{1}{2}x^3y^4z^2\)
20. \(-4xyz^3 - \frac{2}{3}y^7 - \frac{3}{7}x^4y^3z^2\)
21. \(-a^7 + 2a^3b^5 + b^6 - 3a^2b^4\)

Answer True or False.

21. (a) \(7x - 2x^3\) is a trinomial.
(b) \(7x - 2x^3\) is a third degree polynomial.
(c) \(7x - 2x^3\) is a binomial.
(d) \(7x - 2x^3\) is a first degree polynomial

22. (a) \(x^2 - 4x + 7x^3\) is a second degree polynomial.
(b) \(x^2 - 4x + 7x^3\) is a binomial.
(c) \(x^2 - 4x + 7x^3\) is a third degree polynomial.
(d) \(x^2 - 4x + 7x^3\) is a trinomial.

23. (a) \(3x^7 - 2x^3y^6 - 3y^8\) is a tenth degree polynomial.
(b) \(3x^7 - 2x^3y^6 - 3y^8\) is a binomial.
(c) \(3x^7 - 2x^3y^6 - 3y^8\) is an eighth degree polynomial.
(d) \(3x^7 - 2x^3y^6 - 3y^8\) is a trinomial.

24. (a) \(-3a^4b^5\) is a fifth degree polynomial.
(b) \(-3a^4b^5\) is a trinomial.
(c) \(-3a^4b^5\) is a ninth degree polynomial.
(d) \(-3a^4b^5\) is a monomial.

Sketch a graph of each of the following functions.

25. \(P(x) = x^3\)
26. \(P(x) = x^4\)
27. \(P(x) = x^6\)
28. \(P(x) = x^5\)
29. \(P(x) = x^n\), where \(n\) is odd and \(n > 0\).
30. \(P(x) = x^n\), where \(n\) is even and \(n > 0\).

Answer the following.

31. The graph of \(P(x) = (x - 1)(x - 2)(x + 4)^2\) has \(x\)-intercepts at \(x = 1\), \(x = 2\), and \(x = -4\).
(a) At and immediately surrounding the point \(x = 2\), the graph resembles the graph of what familiar function? (Choose one.)
\[
\begin{align*}
y & = x \\
y & = x^2 \\
y & = x^3
\end{align*}
\]
Continued on the next page...
Exercise Set 2.2: Polynomial Functions

(b) At and immediately surrounding the point \( x = -4 \), the graph resembles the graph of what familiar function? (Choose one.)

\[
y = x \quad y = x^2 \quad y = x^3
\]

(c) If \( P(x) \) were to be multiplied out completely, the leading term of the polynomial would be: (Choose one; do not actually multiply out the polynomial.)

\[
x^3; \quad -x^3; \quad x^4; \quad -x^4; \quad x^5; \quad -x^5; \quad x^6; \quad -x^6
\]

32. The graph of \( Q(x) = -(x + 3)^2(x - 5)^3 \) has \( x \)-intercepts at \( x = -3 \) and \( x = 5 \).

(a) At and immediately surrounding the point \( x = -3 \), the graph resembles the graph of what familiar function? (Choose one.)

\[
y = x \quad y = x^2 \quad y = x^3
\]

(b) At and immediately surrounding the point \( x = 5 \), the graph resembles the graph of what familiar function? (Choose one.)

\[
y = x \quad y = x^2 \quad y = x^3
\]

(c) If \( P(x) \) were to be multiplied out completely, the leading term of the polynomial would be: (Choose one; do not actually multiply out the polynomial.)

\[
x^3; \quad -x^3; \quad x^4; \quad -x^4; \quad x^5; \quad -x^5; \quad x^6; \quad -x^6
\]

Match each of the polynomial functions below with its graph. (The graphs are shown in the next column.)

33. \( P(x) = (x - 2)(x + 1)(x + 4) \)
34. \( Q(x) = -(x + 2)(x - 1)(x - 4) \)
35. \( R(x) = -(x - 2)^2(x + 1)^2(x + 4)^2 \)
36. \( S(x) = (x - 2)^2(x + 1)(x + 4) \)
37. \( U(x) = (x + 2)^2(x - 1)^3(x - 4) \)
38. \( V(x) = -(x + 2)^3(x - 1)^3(x - 4)^2 \)

Choices for 33-38:

A. \[
\]

B. \[
\]

C. \[
\]

D. \[
\]

E. \[
\]

F. \[
\]

For each of the functions below:

(a) Find the \( x \)- and \( y \)-intercepts.

(b) Sketch the graph of the function. Be sure to show all \( x \)- and \( y \)-intercepts, along with the proper behavior at each \( x \)-intercept, as well as the proper end behavior.

39. \( P(x) = (x - 5)(x + 3) \)
40. \( P(x) = -(x - 3)(x + 1) \)
41. \( P(x) = -(x - 6)^2 \)
42. \( P(x) = (x + 3)^2 \)
43. \( P(x) = (x - 5)(x + 2)(x + 6) \)
44. \( P(x) = 3x(x - 4)(x - 7) \)
45. \( P(x) = -\frac{1}{2}(x - 4)(x - 1)(x + 3) \)
46. \( P(x) = -(x + 6)(x - 2)(x - 5) \)
47. \( P(x) = (x + 2)^2 (x - 4) \)
48. \( P(x) = (5 - x)(x + 3)^2 \)
49. \( P(x) = (3x - 2)(x + 4)(x - 5)(x + 1) \)
50. \( P(x) = -\frac{1}{3}(x + 5)(x + 1)(x + 3)(x - 2) \)
51. \( P(x) = x(x + 2)(4 - x)(x + 6) \)
52. \( P(x) = (x - 1)(x - 3)(x + 2)(x + 5) \)
53. \( P(x) = (x - 3)^2 (x + 4)^2 \)
54. \( P(x) = -x(2x - 5)^3 \)
55. \( P(x) = (x + 5)^3 (x - 4) \)
56. \( P(x) = x^2 (x - 6)^2 \)
57. \( P(x) = (x + 3)^2 (x - 4)^3 \)
58. \( P(x) = -2x(3 - x)^3 (x + 1) \)
59. \( P(x) = -x(x - 2)^2 (x + 3)^2 (x - 4) \)
60. \( P(x) = (x - 5)^3 (x - 2)^2 (x + 1) \)
61. \( P(x) = x^8 (x - 1)^6 (x + 1)^7 \)
62. \( P(x) = -x^3 (x + 1)^4 (x - 1)^7 \)
63. \( P(x) = x^3 - 6x^2 + 8x \)
64. \( P(x) = x^3 - 2x^2 - 15x \)
65. \( P(x) = 25x - x^3 \)
66. \( P(x) = -3x^3 - 5x^2 + 2x \)
67. \( P(x) = -x^4 + x^3 + 12x^2 \)
68. \( P(x) = x^4 - 16x^2 \)
69. \( P(x) = x^5 - 9x^3 \)
70. \( P(x) = -x^5 - 3x^4 + 18x^3 \)
71. \( P(x) = x^3 + 4x^2 - x - 4 \)
72. \( P(x) = x^3 - 5x^2 - 4x + 20 \)
73. \( P(x) = x^4 - 13x^2 + 36 \)
74. \( P(x) = x^4 - 17x^2 + 16 \)

**Polynomial functions can be classified according to their degree, as shown below. (Linear and quadratic functions have been covered in previous sections.)**

<table>
<thead>
<tr>
<th>Degree</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1</td>
<td>Linear</td>
</tr>
<tr>
<td>2</td>
<td>Quadratic</td>
</tr>
<tr>
<td>3</td>
<td>Cubic</td>
</tr>
<tr>
<td>4</td>
<td>Quartic</td>
</tr>
<tr>
<td>5</td>
<td>Quintic</td>
</tr>
<tr>
<td>(n)</td>
<td>(n^{th}) degree polynomial</td>
</tr>
</tbody>
</table>

**Answer the following.**

75. Write the equation of the cubic polynomial \( P(x) \) that satisfies the following conditions:
   \( P(-4) = P(1) = P(3) = 0 \), and \( P(0) = -6 \).
76. Write an equation for a cubic polynomial \( P(x) \) with leading coefficient \(-1\) whose graph passes through the point \((2, 8)\) and is tangent to the \(x\)-axis at the origin.
77. Write the equation of the quartic polynomial with \(y\)-intercept \(12\) whose graph is tangent to the \(x\)-axis at \((-2, 0)\) and \((1, 0)\).
78. Write the equation of the sixth degree polynomial with \(y\)-intercept \(-3\) whose graph is tangent to the \(x\)-axis at \((-2, 0), (-1, 0),\) and \((3, 0)\).

**Use transformations (the concepts of shifting, reflecting, stretching, and shrinking) to sketch each of the following graphs.**

79. \( P(x) = x^3 + 5 \)
80. \( P(x) = -x^3 - 2 \)
81. \( P(x) = -(x - 2)^3 + 4 \)
82. \( P(x) = (x + 5)^3 - 1 \)
83. \( P(x) = 2x^4 - 3 \)
84. \( P(x) = -(x - 2)^4 + 5 \)
85. \( P(x) = -(x + 1)^5 - 4 \)
86. \( P(x) = (x + 3)^5 + 2 \)