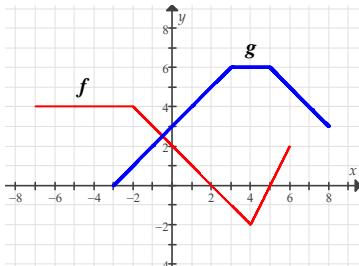


Exercise Set 1.4: Operations on Functions

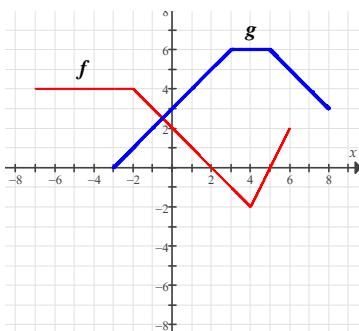
Answer the following.

1.



- (a) Find $f(-3) + g(-3)$.
- (b) Find $f(0) + g(0)$.
- (c) Find $f(-6) + g(-6)$.
- (d) Find $f(5) + g(5)$.
- (e) Find $f(7) + g(7)$.
- (f) Sketch the graph of $f + g$. (Hint: For any x value, add the y values of f and g .)
- (g) What is the domain of $f + g$? Explain how you obtained your answer.

2.



- (a) Find $f(-2) - g(-2)$.
- (b) Find $f(0) - g(0)$.
- (c) Find $f(-4) - g(-4)$.
- (d) Find $f(2) - g(2)$.
- (e) Find $f(4) - g(4)$.
- (f) Sketch the graph of $f - g$. (Hint: For any x value, subtract the y values of f and g .)
- (g) What is the domain of $f - g$? Explain how you obtained your answer.

For each of the following problems:

- (a) Find $f + g$ and its domain.
- (b) Find $f - g$ and its domain.
- (c) Find fg and its domain.
- (d) Find $\frac{f}{g}$ and its domain.

Note for (a)-(d): Do not sketch any graphs.

3. $f(x) = 2x + 3; g(x) = x^2 - 4x - 12$

4. $f(x) = 2x^3 - 5x; g(x) = x^2 + 8x + 15$

5. $f(x) = \frac{3}{x-1}; g(x) = \frac{x}{x+2}$

6. $f(x) = \frac{4}{x-5}; g(x) = \frac{2x}{x-5}$

7. $f(x) = \sqrt{x-6}; g(x) = \sqrt{10-x}$

8. $f(x) = \sqrt{2x-3}; g(x) = \sqrt{x+4}$

9. $f(x) = \sqrt{x^2 - 9}; g(x) = \sqrt{x^2 + 4}$

10. $f(x) = \sqrt{49-x^2}; g(x) = x-3$

Find the domain of each of the following functions.

11. $f(x) = \frac{2}{x-3} + \sqrt{x-1}$

12. $h(x) = \sqrt{x+2} - \frac{3}{x}$

13. $g(x) = \frac{3}{x-7} - \frac{x+1}{x-2}$

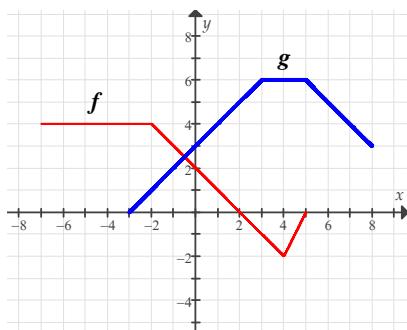
14. $f(x) = \frac{x-2}{x+6} + \frac{5}{x-1} + 7$

15. $f(x) = \frac{\sqrt{x+2}}{x-5}$

16. $g(x) = \frac{x-3}{\sqrt{x-1}}$

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Answer the following, using the graph below.



17. (a) $g(2)$ (b) $f(g(2))$
 (c) $f(2)$ (d) $g(f(2))$
18. (a) $g(0)$ (b) $f(g(0))$
 (c) $f(0)$ (d) $g(f(0))$
19. (a) $(f \circ g)(-3)$ (b) $(g \circ f)(-3)$
20. (a) $(f \circ g)(-1)$ (b) $(g \circ f)(-1)$
21. (a) $(f \circ f)(3)$ (b) $(g \circ g)(-2)$
22. (a) $(f \circ f)(5)$ (b) $(g \circ g)(-3)$
23. (a) $(f \circ g)(4)$ (b) $(g \circ f)(4)$
24. (a) $(f \circ g)(-5)$ (b) $(f \circ g)(2)$

Use the functions f and g given below to evaluate the following expressions:

$$f(x) = 3 - 2x \text{ and } g(x) = x^2 - 5x + 4$$

25. (a) $g(0)$ (b) $f(g(0))$
 (c) $f(0)$ (d) $g(f(0))$
26. (a) $g(-1)$ (b) $f(g(-1))$
 (c) $f(-1)$ (d) $g(f(-1))$
27. (a) $(f \circ g)(-2)$ (b) $(g \circ f)(-2)$
28. (a) $(f \circ g)(4)$ (b) $(g \circ f)(4)$
29. (a) $(f \circ f)(6)$ (b) $(g \circ g)(6)$
30. (a) $(f \circ f)(-4)$ (b) $(g \circ g)(-4)$
31. (a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$
32. (a) $(f \circ f)(x)$ (b) $(g \circ g)(x)$

The following method can be used to find the domain of $f \circ g$:

- (a) Find the domain of g .
- (b) Find $f \circ g$.
- (c) Look at the answer from part (b) as a stand-alone function (ignoring the fact that it is a composition of functions) and find its domain.
- (d) Take the intersection of the domains found in steps (a) and (c). This is the domain of $f \circ g$.

Note:

We check the domain of g because it is the inner function of $f \circ g$, i.e. $f(g(x))$. If an x -value is not in the domain of g , then it also can not be an input value for $f \circ g$.

Use the above steps to find the domain of $f \circ g$ for the following problems:

33. $f(x) = \frac{1}{x^2}; g(x) = \sqrt{x-5}$

34. $f(x) = \frac{1}{x^2}; g(x) = \sqrt{x+2}$

35. $f(x) = \frac{3}{x^2-4}; g(x) = \sqrt{x-6}$

36. $f(x) = \frac{5}{x^2-2}; g(x) = \sqrt{3-x}$

For each of the following problems:

- (a) Find $f \circ g$ and its domain.
- (b) Find $g \circ f$ and its domain.

37. $f(x) = x^2 + 3x; g(x) = 2x - 7$

38. $f(x) = 6x + 2; g(x) = 7 - x^2$

39. $f(x) = x^2; g(x) = \frac{1}{\sqrt{x-4}}$

40. $f(x) = \frac{3}{\sqrt{x+5}}; g(x) = x^2$

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41. $f(x) = \sqrt{x+7}$; $g(x) = -5 - x$

42. $f(x) = \sqrt{3-x}$; $g(x) = 9 - 2x$

Answer the following.

43. Given the functions $f(x) = x^2 + 2$ and $g(x) = 5x - 8$, find:

- | | |
|---------------|---------------|
| (a) $f(g(1))$ | (b) $g(f(1))$ |
| (c) $f(g(x))$ | (d) $g(f(x))$ |
| (e) $f(f(1))$ | (f) $g(g(1))$ |
| (g) $f(f(x))$ | (h) $g(g(x))$ |

44. Given the functions $f(x) = x + 1$ and $g(x) = 3x - 2x^2$, find:

- | | |
|----------------|----------------|
| (a) $f(g(-3))$ | (b) $g(f(-3))$ |
| (c) $f(g(x))$ | (d) $g(f(x))$ |
| (e) $f(f(-3))$ | (f) $g(g(-3))$ |
| (g) $f(f(x))$ | (h) $g(g(x))$ |

45. Given the functions $f(x) = \frac{x+1}{x-2}$ and $g(x) = \frac{3}{x-5}$, find:

- | | |
|----------------|----------------|
| (a) $f(g(-2))$ | (b) $g(f(-2))$ |
| (c) $f(g(x))$ | (d) $g(f(x))$ |

46. Given the functions $f(x) = \frac{2x}{x+5}$ and $g(x) = \frac{7-x}{x-1}$, find:

- | | |
|---------------|---------------|
| (a) $f(g(3))$ | (b) $g(f(3))$ |
| (c) $f(g(x))$ | (d) $g(f(x))$ |

47. Given the functions

$$f(x) = x^2 - 1, \quad g(x) = 3x - 5, \quad \text{and} \quad h(x) = 1 - 2x, \quad \text{find:}$$

- | | |
|-------------------------|-------------------------|
| (a) $f(g(h(2)))$ | (b) $g(h(f(3)))$ |
| (c) $f \circ g \circ h$ | (d) $g \circ h \circ f$ |

48. Given the functions

$$f(x) = 2x^2 + 3, \quad g(x) = x + 4, \quad \text{and} \quad h(x) = 3x - 2, \quad \text{find:}$$

- | | |
|-------------------------|-------------------------|
| (a) $f(g(h(1)))$ | (b) $g(h(f(1)))$ |
| (c) $f \circ g \circ h$ | (d) $g \circ h \circ f$ |

49. Given the functions

$$f(x) = x^2 + 4, \quad g(x) = \sqrt{x+3}, \quad \text{and} \quad h(x) = 2x + 1, \quad \text{find:}$$

- | | |
|-------------------------|-------------------------|
| (a) $h(f(g(4)))$ | (b) $f(g(h(0)))$ |
| (c) $f \circ g \circ h$ | (d) $h \circ f \circ g$ |

50. Given the functions

$$f(x) = \frac{1}{x^2}, \quad g(x) = \sqrt{x-2}, \quad \text{and} \quad h(x) = 3 - 4x, \quad \text{find:}$$

- | | |
|-------------------------|-------------------------|
| (a) $h(f(g(5)))$ | (b) $f(g(h(-2)))$ |
| (c) $f \circ g \circ h$ | (d) $h \circ f \circ g$ |

Functions f and g are defined as shown in the table below.

x	0	1	2	4
$f(x)$	2	4	4	7
$g(x)$	4	5	0	1

Use the information above to complete the following tables. (Some answers may be undefined.)

51.

x	0	1	2	4
$f(g(x))$				

52.

x	0	1	2	4
$g(f(x))$				

53.

x	0	1	2	4
$f(f(x))$				

54.

x	0	1	2	4
$g(g(x))$				