Example 2:

Sketch the graph of the function $f(x) = -|x-3|-2$. Do not plot points, but instead apply transformations to the graph of a standard function.

ANALYSIS OF THE ORDER OF TRANSFORMATIONS:

In order to graph this problem correctly, we need to choose a correct order of transformations. First, decide on the transformations that need to be performed on $f(x) = -|x-3|-2$ (without consideration of correct order). Make a note of whether each transformation has a horizontal or vertical effect on the graph.

$f(x) = -|x-3|-2$
Reflect in the *x*-axis
(Vertical Effect)

$f(x) = -|x-3|-2$
Shift right 3
(Horizontal Effect)

$f(x) = -|x-3|-2$
Shift down 2
(Vertical Effect)

Remember that any vertically-oriented transformations have no effect on horizontally-oriented transformations, and vice versa. However, the order in which you perform vertically-oriented transformations may make a difference in the graph, and the order in which you perform horizontally-oriented transformations may make a difference in the graph.

Notice that the shift to the right is the only transformation that has a horizontal effect on the graph. Since it has no effect on the vertically-oriented transformations, the right shift can be performed at any point in the graphing process.

We need to be more careful about the order in which we perform the reflection in the *x*-axis and the downward shift, since they both have a vertical effect on the graph.

Remember that you do NOT need to evaluate all the possible combinations in which the above transformations can be performed, but <u>you need to find one</u> which algebraically yields the correct function (and therefore also yields the correct graph).

Perform the following transformations algebraically on $g(x) = |x|$ to see which one gives the desired function, $f(x) = -|x-3|-2$. (In the choices below, the shift to the right is written first, but we could put that transformation at any point in the process and get the same result.)

Choice 1: Shift right 3, reflect in the *x*-axis, shift down 2

$g(x)=|x| \rightarrow h(x)=|x-3| \rightarrow k(x)=-|x-3| \rightarrow f(x)-|x-3|-2$

Choice 1 yields the desired function, $f(x)-|x-3|-2$.

Choice 2: Shift right 3, shift down 2, reflect in the *x*-axis.

$g(x)=|x| \rightarrow h(x)=|x-3| \rightarrow k(x)=|x-3|-2 \rightarrow f(x)=-(|x-3|-2)$

Notice that in Choice 2, $f(x)=-(|x-3|-2)=-|x-3|+2$, and does NOT yield the desired function, $f(x)=-|x-3|-2$.

We can see from the choices above that the reflection in the *x*-axis needs to be performed before the downward shift. Since the right shift is the only horizontally-oriented transformation, it can be performed at any point in the process. Therefore, any of the following combinations of transformations would yield the correct graph:

Shift right 3, reflect in the *x*-axis, shift down 2
Reflect in the *x*-axis, shift right 3, shift down 2
Reflect in the *x*-axis, shift down 2, shift right 3

In the detailed solution in the text, the order of transformations shown in Choice 1 is used to graph the function. Click on one of the links below to return to that solution and to see the graph of $f(x)=-|x-3|-2$.

Example 2: Complete Solution, Step-by-step, Flash

Sketch the graph of the function $f(x)=-|x-3|-2$. Do not plot points, but instead apply transformations to the graph of a standard function.