Math 1324
Section 7.6
Applications of the Normal Distribution

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can also view the videos within the Math 1324 textbook.

Math 1324
Applications of the Normal Distribution

Example 1

Example 1: The average lifetime for a car battery is 148 weeks with a standard deviation of 8 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution.

Example 2

Example 2: According to the data released by the Chamber of Commerce of a certain city, the weekly wages of office workers are normally distributed with a mean of $700 and a standard deviation of $50. What is the probability that a worker who lives in the city, selected at random, makes a weekly wage of less than $600?

What is the probability that a worker who lives in the city, selected at random, makes a weekly wage of more than $810?

What is the probability that a worker who lives in the city, selected at random, makes a weekly wage of between $620 and $770?

Example 3

Example 3: The serum cholesterol levels in milligrams/dekaliter (mg/dL) in a current Mediterranean population are found to be normally distributed with a mean of 160 and a standard deviation of 50. Scientists of the National Heart, Lung and Blood Institute consider this pattern ideal for a minimal risk of heart attacks. Find the percentage of the population having blood cholesterol levels between 150 and 200 mg/dL.

Example 4

Example 4: The grade point average of the senior class at Central High School is normally distributed with a mean of 3.4 and a standard deviation of 0.52. If a senior in the top 10% of his/her class is eligible to be admitted to any state university, what is the minimum GPA that a senior should have to ensure eligibility for admission?

Approximating the Binomial Distribution

We can also use the normal distribution to approximate the binomial distribution in cases where *n* is large. Recall that we can use the binomial distribution to compute probabilities in binomial experiments using the formula $P(X = x) = C(n,x)p^x q^{n-x}$.

This is easy to use when *n* is small, but as *n* increases it becomes difficult to use. For example, suppose $n = 50$, $p = 0.45$ and $q = 0.55$, and we are interested in finding $P(X \geq 25)$. This involves too much computation. We need to find a better way of computing this!

We can represent a binomial distribution using a histogram.

If we draw a normal distribution with the same mean and standard deviation, it will approximate the shape of the histogram of the binomial distribution.

So we can use the normal distribution to approximate the binomial distribution. Let's go back to the binomial experiment stated earlier where $n = 50$, $p = 0.45$ and $q = 0.55$. Recall we are finding $P(X \geq 25)$.

The rectangle for 25 successes will extend from 24.5 to 25.5, so since we are interested in finding $P(X \geq 25)$, we'll need to compute $P(Y \geq 24.5)$, where Y is a normal random variable. That is, we'll use the left endpoint of the rectangle with 25 as its midpoint. By convention, this is the same as computing $P(Y > 24.5)$.

Next we'll find the mean and standard deviation for our problem:

Next we'll use our formula to convert the normal random variable into a standard normal random variable:

Finally, we compute the probability using the z table.

Example 5

Example 5: Suppose a coin is weighted so that the probability that it lands heads is 60%. The coin is tossed 25 times. Find the probability that it lands heads fewer than 20 times.

Find the probability that it lands heads more than 17 times.

Example 6

Example 6: The probability that a worker will have an on-the-job accident at any time during the year is 0.11. If a company has 950 employees, what is the probability that more the 100 workers will have an on-the-job accident during the year?