# Math 1324
# Section 7.4
# The Binomial Distribution

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can also view the videos within the Math 1324 textbook.

**Math 1324**
**The Binomial Distribution**

## Binomial Experiments

The remainder of the course material will be presentation of two very important probability distributions and the types of problems that we can solve using them.

The first one is the binomial distribution.

First, we need to know what we mean by a binomial experiment.

A **binomial trial**, or a Bernoulli trial, is an experiment with exactly two outcomes. For ease, we can describe these outcomes as success and failure.

Suppose you roll a fair die and observe the number on the uppermost face and define success to be "rolling a 3." This is an example of a Bernoulli trial, since we can define failure as rolling anything else.

A **binomial experiment** is a sequence of binomial (or Bernoulli) trials.

A binomial experiment has these properties:

- There are exactly two outcomes, which can be expressed as success and failure
- The number of trials of the experiment is fixed
- The probability of success is the same in each trial.
- The trials are independent

We can revise some of our familiar experiments so that they become binomial experiments.

Example 1: An experiment consists of rolling a fair die seven times and observing the number on the uppermost face. A roll is considered a success if the uppermost number is a 5. Otherwise, it is considered a failure.

Is this a binomial experiment?

**Binomial formula**

Let $X$ be a random variable that denotes the number of successes in a binomial experiment where $p$ is the probability of success and $q$ is the probability of failure. Then the probability of $x$ successes in $n$ trials of the experiment is $P(X = x) = C(n, x)\, p^x q^{n-x}$.

In binomial experiments, we call the random variable X a **binomial random variable**.

Example 2: A fair die is thrown 15 times. Find the probability of obtaining

at least 14 fives in four trials.

at most 2 fives in four trials.

Example 3: Use the formula to determine the probability of each of these:

- Find the probability of exactly three successes in six trials of a binomial experiment in which $p = 0.39$.

- Find the probability of at least 2 successes in 3 trials of a binomial experiment in which the probability of failure is 0.45.

- Find $P(3 \le X \le 5)$ in a binomial experiment with seven trials in which $p = 0.67$.

**Examples 4 – 5**

Example 4: A recent survey shows that 58% of households in a certain city have high speed internet connections. If 15 households in that city are selected at random, what is the probability that exactly five of the households have high speed internet connection.

What is the probability that at least one of the households have high speed internet connection?

Example 5: Historically, one-fifth of all new buildings in a town are in violation of the town's building codes. A building inspector inspects seven new buildings in the town. Find the probability that the first four buildings he inspects will meet building codes and pass inspection and the remaining three will not.

What is the probability that at least two of the buildings he inspects will pass inspection?

**Expected Value, Variance and Standard Deviation**

We can also compute the expected value, variance and standard deviation of binomial experiments. The formulas for these values are very easy to use. But, caution, use these only on binomial random variables! These formulas won't give you correct answers for general random variables!!

Expected Value of a Binomial Random Variable

Variance of a Binomial Random Variable

Standard Deviation of a Binomial Random Variable

Example 6: Find the expected value, variance and standard deviation of the experiment where $n = 6$ and $p = 0.39$.

Example 7: Suppose a binomial experiment consists of rolling a fair die 12 times and observing the number that is uppermost. Rolling a number greater than 4 is called a success. Find the expected value, variance and standard deviation of this experiment.

Example 8: The probability that a marriage will end in divorce within 25 years of its start is 0.65. Eight couples were just married. What is the probability that over the next 25 years all of the couples will be divorced?

What is the probability that over the next 25 years none of the couples will be divorced?

What is the probability that over the next 25 years exactly five of the couples will be divorced?

What is the probability that after 25 years, at least two of the couples will be divorced?

What is the probability that after 25 years, at most four of the couples will be divorced?

Find the mean, variance and standard deviation of this experiment.