Math 1324
Section 5.4
Permutations and Combinations

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can also view the videos within the Math 1324 textbook.

Definition: **n-Factorial**

For any natural number n,

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$0! = 1$

Definition: A **permutation** is an arrangement of a specific set where the order in which the objects are arranged is important.

Formula: $P(n,r) = \dfrac{n!}{(n-r)!}$,

n is the number of distinct objects and *r* is the number of distinct objects taken *r* at a time.

Example 1: Let A={1, 2, 3}.

a. Find the number of permutations of A taking 3 elements at a time.

b. List all the permutations of A for part a.

Example 2: Rachel has a math book, a science book, an history book, a psychology book, and an art book that she needs to arrange on a shelf from left to right. In how many ways can she arrange these books on the shelf?

Example 3: Sue's daughter brought home 6 different drawings she had done in art class at school. She asked her mother to put them up on their refrigerator. Since her mother has room for only 3 of them, in how many ways can her mother arrange 3 of the 6 drawings from left to right on her refrigerator if she likes them all equally?

Example 4: The Art History Club at a certain high school has 25 members. A president, vice president, secretary, and treasurer need to be chosen. If a person can hold only one office, in how many ways can the officers be chosen?

Permutations of n objects, not all distinct

Formula : Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind, ..., and, finally, n_r objects are alike and of yet another kind so that $n_1 + n_2 + ... + n_r = n$, then the number of permutations of these n objects taken n at a time is given by $\frac{n!}{n_1! n_2! \cdots n_r!}$.

Example 5: How many permutations can be formed from all the letters in the word OSMOSIS?

Definition: A **combination** is an arrangement of a specific set where the order in which the objects are arranged is not important.

Formula: $C(n,r) = \dfrac{n!}{r!(n-r)!}$,

n is the number of distinct objects and r is the number of distinct objects taken r at a time.

Example 6: Let A={1, 2, 3}.

a. Find the number of combinations of A taking 2 elements at a time.

b. List all the combinations of A for part a.

Example 7: The principal of a certain high school needs to send 5 of his math teachers to a math conference. The principal has 20 math teachers. Assuming all are qualified to attend the conference, in how many ways can he select 5 teachers from the 20?

Example 8: A certain company has come up with a team of people to work together on a project. The manager needs to select two team leaders from the group of 15 workers. In how many ways can he choose the two team leaders?

Example 9: The English Club at a certain high school needs to select a fundraising committee. The club has 20 members. In how many ways can a committee of 10 members be chosen?

Example 10: Carol's mother has taken her on a shopping spree for getting good grades. Her mother will allow her to pick out 6 shirts and 5 pants to purchase. Carol has picked out 11 shirts and 13 pants that she likes. In how many ways can Carol choose 6 shirts and 5 pants to purchase from the ones she likes? (Assume all shirts and pants are different.)

Example 11: A committee of 22 men and 18 women is to form a subcommittee. The subcommittee will contain 10 committee members. How many ways can a subcommittee of 10 members be chosen, if 4 must be men and 6 must be women?

Example 12: Suppose you toss a coin 4 times. How many outcomes are possible if exactly two heads occur?

Example 13: Suppose you toss a coin 11 times. How many outcomes are possible if exactly 8 heads occur?

Example 14: A high school cafeteria has the following items on its menu: 4 appetizers, 5 entrees, 3 salads, 5 desserts, and 2 flavor drinks. In how many ways can a student put together a meal consisting of 2 appetizers, an entrée, a salad, 3 desserts, and a drink?

Example 15: An urn contains 10 blue and 8 green marbles. Five marbles are chosen at random from the urn. In how many ways can 2 blue and 3 green or 1 blue and 4 green marbles be chosen?

Example 16: A coin is tossed 15 times. In how many outcomes do at least 13 tails occur?

Example 17: A committee consisting of 7 senior members and 6 junior members is to make up a 5-member fundraising subcommittee. In how many ways can the subcommittee consist of at most 3 junior members?

Example 18: A coin is tossed 25 times. In how many outcomes do at least 1 heads occur?

Example 19: A box at Fruity Fruit Stand contains 30 apples of which 10 are rotten. A customer chooses 8 at random from the box. In how many ways can the customer choose at most 1 rotten apple?

Example 20: A box contains 25 batteries of which 7 are defective. You choose 10 batteries at random from the box. In how many ways can at least 5 defective batteries be chosen?