Math 1324
Section 3.4
Matrix Multiplication

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can alternatively view the videos within the Math 1324 textbook.

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Matrix Multiplication

Let A be a matrix of size mxn and B be a matrix of size nxp. The product AB is the matrix of size mxp whose entries are obtained by the sum of the products of the i^{th} row of A and the j^{th} column of B.

Note: Recall that for the product of two matrices to be defined, the number of columns in the first matrix must be equal to the number of row in the second matrix. If not, then the product is not defined.

An illustration follows.

Let $C = AB$.

$$C = AB$$

$$C = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}$$

where each entry in the mxp matrix C is determined by calculating

$$\begin{aligned} c_{11} &= a_{11}b_{11} + a_{12}b_{21} + \ldots + a_{1n}b_{n1} \\ c_{12} &= a_{11}b_{12} + a_{12}b_{22} + \ldots + a_{1n}b_{n2} \\ \vdots &= \qquad\qquad \vdots \\ c_{mp} &= a_{m1}b_{1p} + a_{n2}b_{2p} + \ldots + a_{mn}b_{np} \end{aligned}$$

Example 1: A student is part of an organization that sold candy bars for a fundraiser. The kinds of candy bars the student sold were: chocolate, chocolate almond, and chocolate crisp. The following matrix represents the number of each kind of candy bar

the student sold, respectively.

$$A = \begin{pmatrix} 44 & 67 & 59 \end{pmatrix}$$

Each kind of candy bar sold for different prices. The chocolate sold for $1, the chocolate almond for $2 and the chocolate crisp for $1.50. The following matrix represents this information, respectively.

$$B = \begin{pmatrix} 1.00 \\ 2.00 \\ 1.50 \end{pmatrix}$$

Find the total amount of money the student made for the fundraiser.

Example 2: Let

$$A = \begin{pmatrix} -4 & 7 & 9 \\ 10 & -2 & 2 \end{pmatrix}, \ B = \begin{pmatrix} 8 & 1 \\ 4 & -3 \\ -1 & 5 \end{pmatrix}, \text{ and } C = \begin{pmatrix} -5 & 7 & 0 & -10 \\ 7 & -6 & 1 & 1 \end{pmatrix}.$$

Find, if possible, BA, BC, and CB.

A **square matrix** is a matrix that has the same number of rows as columns.

The **identity matrix** is a square matrix that has 1's along its main diagonal (from the upper left corner to the lower right corner) and 0's elsewhere. Since an identity matrix the same number of rows as columns, we simply say an identity matrix is of size n.

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \vdots & 1 & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Some examples: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix of size 2. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix of size 3.

Multiplication Properties of Matrices

Let A, B and C be matrices whose products and sums are defined. Also let k be a scalar.

1. Associative Property: $A(BC) = (AB)C$
2. Associative Property: $k(AB) = (kA)B$
3. Distributive Property: $A(B + C) = AB + AC$

In general matrix multiplication is not commutative. However, if A is a square matrix of size n the identity matrix of size n has the following property:

$$AI_n = I_nA = A$$

Example 3: A company manufactures two kinds of calculators, Basic and Scientific. The labor hours needed for each kind of calculator in the Assembly Department and Packaging Department are given by the following matrix.

	Assembly Dept	Packaging Dept.
Basic	3	2
Scientific	4	3

= A

The company manufactures the calculators at two plants, one is located in the East Coast and the other in the West Coast. The following matrix gives hourly rates, in dollars, for workers in each department at each location.

	East Coast	West Coast
Assembly Dept.	10	12
Packaging Dept.	13	14

= B

Find AB and explain the meaning of each entry.