Math 1324
Section 3.3
Matrix Operations

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can alternatively view the videos within the Math 1324 textbook.

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Addition and Subtraction of Matrices

Let A and B be two matrices of the same size, mxn.

1. $A + B$ is the matrix of size mxn that is obtained by adding the corresponding elements in A to B.

2. $A - B$ is the matrix of size mxn that is obtained by subtracting the corresponding elements in B from A.

Properties for Matrix Addition

Let A, B, and C be matrices of the same size.

1. Commutative Property: $A + B = B + A$
2. Associative Property: $(A + B) + C = A + (B + C)$

Transpose of a Matrix

The transpose of a matrix A of size mxn, is the matrix A^T of size nxm that results from interchanging the rows and columns of the matrix A.

Properties of the Transpose

Let A and B be two matrices of the same size, and s be a scalar.

1. $\left(A^T\right)^T = A$
2. $\left(A+B\right)^T = A^T + B^T$
3. $\left(sA\right)^T = sA^T$

Scalar Multiplication

A **scalar** is a real number.

Scalar multiplication is the product of a scalar and a matrix. To perform scalar multiplication, each element in the matrix is multiplied by the scalar; hence, it "scales" the elements in the matrix.

Properties for Scalar Multiplication

Let A and B be matrices of the same size, and r and s be scalars.

1. Distributive Property: $r(A + B) = rA + rB$
   $(r + s)A = rA + sA$
2. Associative Property: $r(sA) = (rs)A$

Example 1: Let $A = \begin{pmatrix} 9 & -3 & 1 \\ 0 & 12 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 & -9 \\ 5 & 20 & -1 \end{pmatrix}$. Find A - B.

Example 2 Refer to the following matrices.

$$A = \begin{pmatrix} 7 & -8 & -3 \\ 11 & 10 & 4 \\ 0 & -9 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 5 & 2 \\ 4 & 5 & 10 \\ 1 & -6 & 7 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & -4 & 8 & 10 \end{pmatrix}$$

a. Find the transpose of A and C.
b. Compute, if possible, -2A + 4B.

Equality of Matrices

Two matrices are equal if and only if their corresponding elements are equal.

Example 3: Solve the following matrix equation for x, y, z, and w.

$$\begin{pmatrix} w+3 & 0 & 8 & -17 \\ 4 & -5z & 19 & -1 \\ 8 & -5 & 6 & -6y+1 \end{pmatrix} = \begin{pmatrix} 13 & 0 & 8 & -17 \\ 4 & 25 & 19 & x+4 \\ 8 & -5 & 6 & 30 \end{pmatrix}$$

Example 4: Solve for u, x, y, and z in the matrix equation.

$$\begin{pmatrix} -9+u & -50 \\ 1 & -2z+3 \\ 8 & 4 \\ y-10 & -5 \end{pmatrix} + 4\begin{pmatrix} 8 & 10 \\ 7 & 5 \\ -3 & -11 \\ 2 & 7x \end{pmatrix} = -1\begin{pmatrix} u & 10 \\ -29 & z+3 \\ 4 & 40 \\ 2 & 7x \end{pmatrix}$$