Math 1324
Section 3.2
Solving Systems of Linear Equations Using Matrices

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can alternatively view the videos within the Math 1324 textbook.

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In order to solve systems of linear equation using matrices, we'll only need the augmented matrix.

The following row operations, that are a result of the elimination method in Section 1.3, will allow us to write a linear system in a simplified and equivalent form. Equivalent systems have the same solution sets.

Row Operations

If any of the following row operations are performed on an augmented matrix, the resulting matrix is an equivalent matrix.

- Swap two rows.
  Notation: $R_1 \leftrightarrow R_2$ means Row 1 was swapped with Row 2.
- A row is multiplied by a nonzero constant.
  Notation: $-5R_1$ means -5 is multiplied to Row 1.
- A row is multiplied by a nonzero constant then added to another row.
  Notation: $2R_1 + R_2$ means 2 is multiplied to Row 1 then added to Row 2

We'll use row operations to write the augmented matrix in a specific form called the **row reduced form**, which will allow us to read off the solution to the system quite easily.

Row Reduced Form

A matrix is in row reduced form if the following conditions are satisfied.

1. If a row contains all zeros, it must lie at the bottom of the matrix.

2. The first nonzero element in each row must be a one, called a leading one. Applying any row operations to obtain a leading one is called **pivoting the matrix** about that element that becomes a one.

3. All other elements in each column containing a leading one are zeros. This defines a **unit column**.
4. In any two successive rows, the leading one in the row below lies to the right of the leading one in the row above.

Example 1: Determine which of the following matrices are in row-reduced from. If a matrix is not in row-reduced form, state which condition is violated.

a. $\left(\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & -5 \end{array}\right)$

b. $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 \end{array}\right)$

c. $\left(\begin{array}{ccc|c} 1 & 8 & 0 & -20 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$

d. $\left(\begin{array}{ccc|c} 1 & -8 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & -4 \end{array}\right)$

Gauss-Jordan Elimination Method

Basically, you will apply row operations to write the augmented matrix in row reduced form and read off the solution(s) easily.

1. Write the augmented matrix associated with the given system.

2. Use row operations to write the augmented matrix in row reduced form. If at any point a row in the matrix contains zeros to the left of the vertical line and a nonzero number to its right, stop the process the problem has no solution.

3. Read off the solution(s).

Example 2: Solve the system of linear equations using the Gauss-Jordan elimination method.

a. $\begin{aligned} 2x - 4y &= -14 \\ 3x + 2y &= 3 \end{aligned}$

b. $\begin{aligned} 3x + y + 2z &= 31 \\ x + y + 2z &= 19 \\ x + 3y + 2z &= 25 \end{aligned}$

Example 3: You invested a total of $38,000 in two municipal bonds – Bond A and Bond B, that have a yield of 4% and 6% interest per year, respectively. The interest you earned from the bonds was $1,930. How much did you invest in each bond?

Example 4: A popular play at a certain performance hall sold 1,000 tickets on opening night. The seats in section A, the best section, sold for $80 each, each seat in the middle section, Section B, sold for $60, and each seat in Section C, the farthest section, sold for $50. The combined number of tickets sold for Sections A and B exceeded twice the number of Section C tickets sold by 400. The total receipts for the performance were $62,800. Determine how many tickets of each type were sold.

Example 5: Given that the augmented matrix in row-reduced form is equivalent to the augmented matrix of a system of linear equations, determine whether the system has a solution and find the solution or solutions to the system, if they exists.

$$\left(\begin{array}{ccc|c} 1 & 0 & -10 & 5 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 9 \end{array}\right)$$

Example 6: Given that the augmented matrix in row-reduced form is equivalent to the augmented matrix of a system of linear equations, determine whether the system has a solution and find the solution or solutions to the system, if they exists.

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 9 \\ 0 & 1 & 8 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Example 7: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x + y - z &= -20 \\ -3y + 3z &= 51 \end{aligned}$$

Example 8: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x - y &= 3 \\ 2x + y &= 7 \\ x + 3y &= 4 \end{aligned}$$

Example 9: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x - 3y &= 3 \\ 8x - 24y &= 24 \\ 10x - 30y &= 30 \end{aligned}$$

Example 10: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$
\begin{aligned}
x - 2y + z &= 5 \\
2x + y - z &= 2 \\
-2x + 4y - 2z &= 2
\end{aligned}
$$