Math 1324
Section 2.1
Solving Linear Programming Problems

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can also view the videos within the Math 1324 textbook.

An **objective function** is a linear function in two or more variables that is to be optimized; that is, maximized or minimized.

In linear programming, an objective function is always subjected to a system of constraints. The **constraints** are a system of linear inequalities that represent certain restrictions in the problem.

A **linear programming problem** consists of an objective function to be optimized subject to a system of constraints.

In linear programming problems the solution set of a system of linear inequaltilites is called the **feasible set**, and it represents all possible solutions to the problem. However, we want to find the point(s) that optimizes the objective function. Depending on the question posed, the **optimal solution** is the point and/or value that optimizes the objective function.

Fundamental Theorem of Linear Programming

- Given that an optimal solution to a linear programming problem exists, it must occur at a vertex of the feasible set.
- If the optimal solution occurs at two adjacent vertices of the feasible set, then the linear programming problem has infinitely many solutions. The infinitely many solutions are produced by the infinitely many points on the line segment joining the two vertices.

The Graphical Method for Solving Linear Programming Problems

1. Graph the system of constraints. This will give the feasible set.
2. Find each vertex of the feasible set.
3. Substitute each vertex into the objective function to determine which vertex optimizes the objective function.
4. State the solution to the problem.

Example: Maximize $A = 10x + 6y$

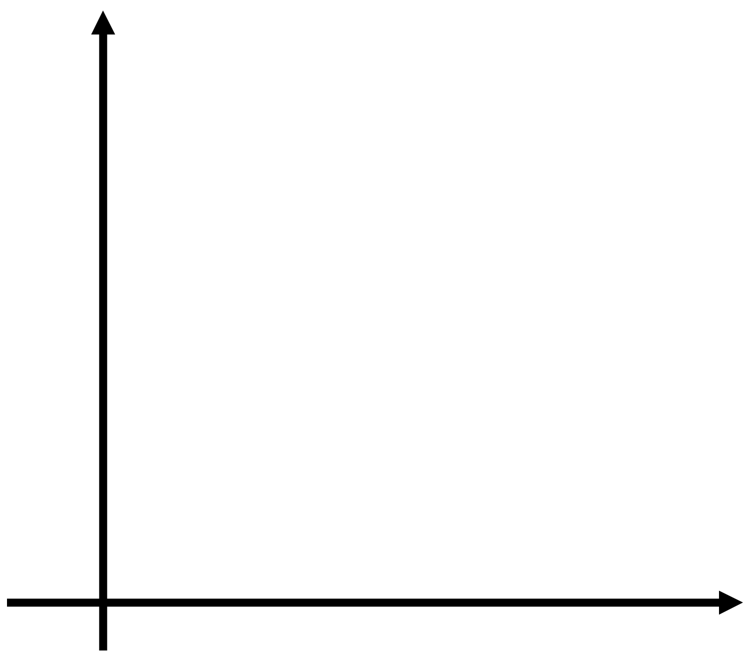
subject to

$$5x + 2y \leq 10$$

$$x + 2y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$



Example: Minimize $B = 3x + 5y$

subject to

$$2x + y \geq 6$$
$$4x + y \geq 8$$
$$x \geq 0$$
$$y \geq 0$$