Math 1324
Section 1.5
Linear Models

The videos corresponding to this worksheet can be found at
https://online.math.uh.edu/Math1324/.
UH students can alternatively view the videos within the Math 1324 textbook.

Linear Depreciation refers to the amount of decrease in the book value of an asset.

Example 1: In 2002, the Boyer Company installed a new machine in one of its factories at a cost of \$300,000. The machine is depreciated linearly over 10 years with a scrap value of \$ 10,000.

a. Find the rate of depreciation for this machine.

b. Find an expression for the machine's book value in the *t*-th year of use ($0 \leq t \leq 10$).

c. Find the machine's book value at the end of 2006.

Example 2: A certain department of a local university bought a copier that had an original value of \$36,000 and will be depreciated linearly over 5 years with scrap value of \$4,000.

a. Find an expression for the copier's book value in the *t*-th year of use ($0 \le t \le 5$).

b. Find the copier's book value at the end of the second year.

The **cost function** is the total amount of money a company spends to produce goods or services.

The cost function involves fixed costs and variable costs. **Fixed Costs** are costs that are independent of the company's production and sales level (i.e. rent, insurance). **Variable Costs** are costs that are dependent and proportional to the company's production and sales level (i.e. utilities, delivery costs).

The **revenue function** is the total amount of money received from the sale of goods or services.

The **profit function** is the total amount of money earned after all costs have been removed.

Formulas

- $C(x) = cx + F$
- $R(x) = sx$
- $P(x) = R(x) - C(x) = (s - c)x - F$

where c is the production cost per unit, F is the fixed costs, s is the selling price per unit, and x is the number of units produced and sold.

Example 3: A manufacturer has a monthly fixed cost of \$150,000 and a production cost of \$12 for each unit produced. The product sells for \$23 per unit.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

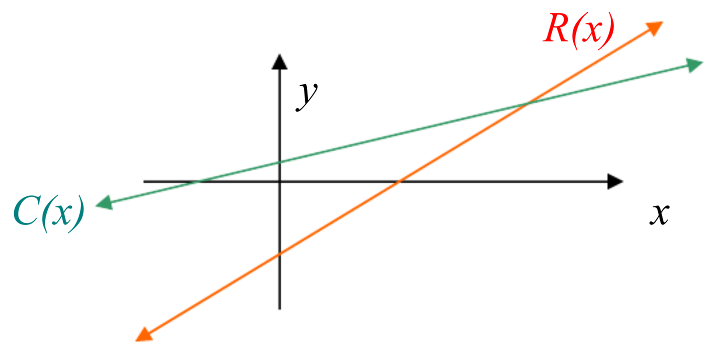
d. Compute the profit (loss) corresponding to production levels of 8,000 units and 25,000 units.

e. How many units must the manufacturer produce and sell to make a profit of \$70,000?

When a company breaks even, this means that the company neither makes a profit nor sustains a loss. So the profit is zero. Hence, the **break-even point** refers the point of intersection of the revenue function and the cost function.

$P(x) = R(x) - C(x) = 0 \Rightarrow R(x) = C(x).$

Consider the following graph:



The x value of the point of intersection is called the **break-even quantity**. The y value of the point of intersection is called the **break-even revenue**.

Note: If an x value is larger than the break-even quantity, then it would result in a profit. If an x value is smaller than the break-even quantity, then it would result in a loss.

Example 4: The Toyco Company has a monthly fixed cost of $84,000 and a production cost of $8 for each unit produced. The product sells for $20 per unit.

a. Find the break-even point for the company.

b. If the company produces and sells 2,000 units, would they have a profit or a loss?

c. If the company produces and sells 10,000 units, what would be the profit or loss?

Example 5: A company's profit function is $P(x) = 16x - 200{,}000$. Find the break-even quantity.

Example 6: A company has a cost function given by $C(x) = 10x + 100{,}000$ and each unit sells for $30. Find the break-even revenue.

Example 7: A company has a cost function given by $C(x) = 15x + 175{,}000$ and a revenue function given by $R(x) = 25x$. How many units need to be produced and sold in order to obtain a profit of $25,000?

Least-Squares Method

Suppose we are given a set of data points, that when plotted do not all lie on a single straight line, we can find a linear equation that "best fits" the set of data. The procedure for finding this best-fit line is called the **Least-Squares Method**. The best-fit line is the line that produces the minimum sum of the squares of the deviations of the points to the line. The best-fit line is called the **least-squares line** or **regression line** and is of the form .

The Least Squares Method

Given a set of *n* data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

1. Calculate the sum of the *x* values.
2. Calculate the sum of the *y* values.
3. Square each *x* value, then find the sum.
4. Multiply the corresponding *x* and *y* values, then find the sum.
5. Substitute into the following normal equations, then use either the substitution method of the elimination method to solve for the variables *m* and *b*.
6. Substitute *m* and *b* into $y = mx + b$ to obtain the least-squares line.

$$nb + (x_1 + \ldots + x_n)m = (y_1 + \ldots + y_n)$$

$$(x_1 + \ldots + x_n)b + (x_1^2 + \ldots + x_n^2)m = (x_1y_1 + \ldots + x_ny_n)$$

Example 8: Find the equation of the least-squares line for the given set of points.
(2, 5), (4, 8) and (3, 2)

Example 9: The projected number of cable television subscribers (in millions) in a certain country from 1998 through 2002 is summarized in the accompanying table.

Year, x	1998	1999	2000	2001	2002
Cable TV Subscribers, y	14.6	15.4	17.2	18.6	21.2

a. Find an equation of the least-squares line for these data. (Let $x = 1$ represent 1998.)

b. Use the result in part (a) to estimate the number of cable television subscribers in 2005, assuming the trend continued.

Example 10: The following equation is the least-squares line for a set of data given by a small individually owned daycare center. The variable x denotes years and y denotes net sales in thousands of dollars. Assume that the first year in the data set, $x = 1$, represents the year 1985. Assuming that the data given was for the years 1985, 1987, 1989,…, 1995, estimate the net sales in thousands of dollars in the year 1991, assuming the trend continued.

$$y = 2.78x + 4$$