Math 1313
Chebychev's Inequality

Basic Information

Chebychev's inequality is a nice application of topics covered in the two previous lessons. This formula will give us the proportion of values of the random variable that are within a given number of standard deviations of the mean.

First, let's look at some number lines to see what it means to be within a certain number of standard deviations of the mean.

Chebychev's inequality tells us how much of our data falls within that interval.

Chebychev's Inequality: Let X be a random variable with expected value μ and standard deviation σ. Then the probability that a randomly chosen outcome of the experiment lies between $\mu - k\sigma$ and $\mu + k\sigma$ is at least $1-1/k^2$, that is

So, suppose k = 2. Then $P(\mu - 2\sigma \le X \le \mu + 2\sigma) \ge 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} = .75$ This means that at least 75% of the data lies within 2 standard deviations of the mean.

Examples 1 – 2

Example 1: The probability distribution of a random variable X has mean 12 and standard deviation 2. Use Chebychev's inequality to estimate the probability that an outcome of the experiment lies between 6 and 18.

Example 2: The probability distribution of a random variable X has mean 45 and standard deviation 3.5. Use Chebychev's inequality to estimate $P(31 \le X \le 59)$.

Examples 3 - 4

Example 3: A standard lightbulb has an expected life of 6000 hours and a standard deviation of 100 hours. Estimate the probability that the lightbulb will last between 5500 and 6500 hours.

Example 4: A probability distribution has mean 50 and standard deviation 1.4. Use Chebychev's inequality to find the value of c that guarantees that the probability is at least 96% that an outcome of the experiment lies between $50 - c$ and $50 + c$.