Math 1313
Expected Value

Mean of a Data Set

From the last lesson, you should be familiar with random variables, and you should be able to construct a probability distribution for a random variable.

In this lesson, you will learn how to compute the expected value of a probability distribution of a random variable.

We begin with a familiar definition.

Definition: Mean

The mean is one measure of central tendency, or the center of a group of data. Other measures of central tendency are the median and the mode, which you may have encountered in another class. We will not consider them in this course.

Example 1: On the 12th hole of a recent golf tournament, the players listed these scores:

Score	1	2	3	4	5	6	7
Frequency	0	0	1	25	123	35	16

Find the mean score on the 12th hole.

Definition of Expected Value

Example 2: This table gives the data for the number of cars waiting in line at specified times at the drive-through teller's lane at a bank.

Number	0	1	2	3	4	5	6
Frequency	2	7	13	11	8	4	5

Construct the probability distribution of the random variable X, where X denotes the number of cars waiting in line.

Find the average number of cars waiting in line.

So, we can also find the mean by multiplying each value the random variable takes by its respective probability and then adding them up.

This will give us a method to use when we have only the probability distribution of the random variable, and not the raw data.

Definition: Expected Value

Example 3: Find the expected value of X.

x	P(X=x)
0	1/8
1	1/4
2	3/16
3	1/4
4	1/16
5	1/8

We can use expected value to solve some business problems.

Example 4: A group of private investors wants to buy one of two hotels currently being offered for sale in a certain city. The terms of sale for the two hotels are similar, but the Harbor Lights Hotel, with 248 rooms, has a better location than the Gulf View Hotel, which has 275 rooms and is part of a national chain. Records obtained from the current owners of the two hotels indicate the estimated occupancy rates of the hotels, along with corresponding probabilities during the peak tourist season, and are summarized in these two tables:

Harbor Lights Hotel

Occupancy Rate	80%	85%	90%	95%	100%
Probability	.13	.14	.38	.26	.09

Gulf View Hotel

Occupancy Rate	80%	85%	90%	95%	100%
Probability	.35	.13	.21	.19	.12

Suppose the random variable X denotes the occupancy rate at the Harbor Lights Hotel and the random variable Y denotes the occupancy rate at the Gulf View Hotel.

Also, suppose the average profit per occupied room at the Harbor Lights is $55, while the average profit per occupied room at the Gulf View is $48.

Find the average number of rooms occupied per day at each hotel.

Assume that the investors' objective is to achieve the highest possible profit on the hotel they purchase. If that is the case, which hotel should they buy?

Recall that expected value and average mean exactly the same thing. We usually use the term expected value when we are working with a random variable. With the technique presented in this section, finding expected value from the probability distribution of a random variable should now be quite easy.