

**Math 1313**  
**Distributions of Random Variables**

**Basic Information**

Sometimes we want to assign a numerical value to the outcomes of our experiment.

Example 1: Suppose an experiment consists of tossing a coin three times and observing the sequence of heads and tails that results.

Recall, the sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Suppose we are interested in the number of heads that occur in the sequence. Then for HHH, the number of heads is 3. For THT, the number of heads is 1. For each of the outcomes, we can find such a number. This is an example of what we mean by a random variable.

Definition: Random Variable

Our goal in this section will be to construct a probability distribution of a random variable and to draw a histogram of this type of probability distribution.

Notation:

Example 2: Suppose an experiment consists of tossing a coin three times and observing the sequence of heads and tails that results.

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let the random variable  $X$  denote the number of heads that occurs in the sequence of heads and tails.

Construct a probability distribution for the random variable  $X$ .

Example 3: Use the information and probability distribution from Example 2 to answer the following questions.

Find the event comprising the outcomes to which a value of one has been assigned by the random variable  $X$ .

Find the probability that  $X = 2$  or  $X = 3$ .

Example 4: Let  $X$  denote the random variable that gives the sum of the numbers falling uppermost when two fair dice are cast.

Find the set of all outcomes of the experiment.

Find the value assigned to each outcome of the experiment by the random variable  $X$ .

Find the event comprising the outcomes to which a value of 7 has been assigned by the random variable  $X$ .

Find the probability distribution of  $X$ .

Draw a histogram of the probability distribution of the random variable  $X$ .

Most of the time, you will just need to construct the probability distribution.

Example 5: A survey was conducted by the housing authority among 1000 families to determine the distribution of families by size. Here are the results

2 members	300 families
3 members	250 families
4 members	225 families
5 members	141 families
6 members	70 families
7 members	11 families
8 members	3 families

Construct a probability distribution of the random variable  $X$ , where  $X$  is the number of persons in a randomly selected family.

Find  $P(X \geq 6)$

Find  $P(X \leq 4)$

Find  $P(2 \leq X \leq 5)$

Draw a histogram of the probability distribution of the random variable  $X$ .

### **Types of Random Variables**

We need to be familiar with three types of random variables. We'll look at an example of each:

Let  $X$  be a random variable denoting the number of heads observed in 6 flips of a coin.

Suppose an experiment consists of flipping a coin until we observe a heads. Let  $Y$  be a random variable denoting the number of tosses it takes before we observe a heads.

Suppose an experiment consists of observing the amount of time it takes for a battery-operated computer game to die. Let  $Z$  denote the amount of time before the game dies.