

Math 1313
Bayes' Theorem

Example 1

Bayes' Theorem is an application of conditional probability, so if you aren't familiar with conditional probability or feel as though you need more practice with that topic, you should review the previous lesson before listening to this one.

In the previous section, we looked at several problems that involved the product rule. These required that we multiply together the probabilities we had on our labeled tree diagram. We will continue to use this technique in this section. However, the questions will be somewhat different in this section.

Example 1: A software manufacturer obtains its CDs from two different suppliers, A and B. The manufacturer gets 43% of its CDs from A and 57% of its CDs from B. In the past, 2% of the CDs from A have been defective, and 1.5% of the CDs from B have been defective. Assuming this holds true, if a CD at the manufacturer's is found to be defective what is the probability that it came from supplier A?

Example 2

Example 2: Three factories, A, B and C, produce engine components for a major auto-maker. Factory A produces 27% of the components, factory B produced 39% of the components, and factory C produces the remaining 34% of the components. Of the components produced at factory A, 5% do not meet specifications. Of the components produced at factory B, 3% do not meet specifications. Of the components produced at factory C, 4% do not meet specifications. One component is selected at random and is found to be defective. What is the probability it came from factory C?

Example 3

Example 3: In a recent Senatorial election, 46% of the voters in a certain district were registered Democrats, 38% were registered Republicans and 16% were registered Independents. The incumbent Republican was reelected over his Democratic and Independent challengers. Exit polls showed that 61% of the registered Democrats voted for the incumbent, 88% of Republicans voted for the incumbent and 51% of registered Independents voted for the incumbent. Assuming that the exit poll was accurate, what is the probability that a vote for the incumbent was cast by a registered Independent?

Example 4

Example 4: An experiment consists of drawing two marbles in succession and without replacement from an urn which contains 18 green marbles and 25 yellow marbles. What is the probability that the first marble was green if we know that the second marble was yellow?

Example 5

Example 5: An experiment consists of selecting one of three urns and then selecting a marble from that urn. Urn X contains 6 red marbles and 8 blue marbles. Urn Y contains 9 red marbles and 5 blue marbles. Urn Z contains 8 red marbles and 7 blue marbles. The probability that Urn X, Urn Y and Urn Z will be chosen is $1/6$, $2/5$, and $13/30$, respectively.

What is the probability that a red marble was chosen?

What is the probability that the marble came from Urn X, given that the marble was red?

What is the probability that the marble came from Urn Z if we know that the marble selected was blue?

Example 6

Example 6: An urn contains 6 white marbles and 12 green marbles. Two marbles are selected in succession with replacement.

What is the probability that the second marble drawn was green?

What is the probability that both marbles drawn were white?

What is the probability that the first marble drawn was white if we know that the second marble drawn was green?