Math 1313
Conditional Probability

Basic Information

We have already covered the basic rules of probability, and we have learned the techniques for solving problems with large sample spaces.

Next we will work with conditional probability. In these problems, we will be given information that changes the size of the sample space, and therefore, changes the probabilities in the problem.

We begin with an example:

Example 1: Suppose five universities are selected as finalists for hosting a presidential debate and that each university is equally likely to be chosen. The probability of any one of the universities being chosen is 1/5.

Now suppose one of the campuses decides to withdraw from contention. The probability of any one of the remaining universities being chosen has changed. Now the probability of any one of them being chosen is ¼.

Why? The size of the sample space changed, so the probability changed.

So, the probability of an event is affected by other events or information that changes the size of the underlying sample space.

Example 2: Two cards are drawn from a well-shuffled deck of 52 playing cards.

What is the probability that the first card drawn is a three?

What is the probability that the second card drawn is a three given that the first card drawn was not a three?

What is the probability that the second card drawn is a three given that the first card drawn was a three?

In parts 2 and 3 of this example, we were given more information. In each case that information affected the size of the sample space. We had to make adjustments to find the correct probability. We need to develop a generalized way of doing this.

Look at the Venn diagram of two typical events.

Now suppose we know that event A has occurred. We're no longer interested in the entire sample space.

If we want to know the probability that B occurs given that A has already occurred, we look at the reduced sample space.

All of B that's left is the part of B that is also in A (i.e, $A \cap B$). The remaining sample space is A

So $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$

This leads us to the definition of Conditional Probability:

If A and B are events in an experiment with $P(A) \neq 0$, then the conditional probability that the event B will occur given that event A has already occurred is

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Sometimes we know $P(B \mid A)$ and we are interested in finding $P(B \cap A)$. We can use the formula for conditional probability to develop a formula for $P(B \cap A)$.

Product Rule:

Example 3: Suppose $P(A) = .54$, $P(B) = .61$, and $P(A \cap B) = .25$. Find

$P(A \mid B)$

$P(B \mid A)$
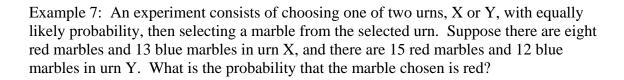
$P(B^c \mid A)$

$P(A^c \mid B)$

$P(B \mid A^c)$

$P(A \mid B^c)$

By now you should be pretty adept at working with the formula. Let's look at some application problems.

Example 4: A pair of fair dice is tossed. What is the probability that the sum of the numbers falling uppermost is 7 if it is known that one of the numbers was 3?

Example 5: A recent survey of 1000 college students, 700 male and 300 female, showed that 435 males were more than 10% over their ideal weight and 78 females were more than 10% over their ideal weight. Given that a surveyed student, selected at random, is male, what is the probability that the surveyed student is more than 10% over his/her ideal weight?

We can use the product rule to solve some problems. We will also use tree diagrams in these problems to help us organize the information.

Example 6: An urn contains 11 blue marbles and 15 green marbles.

Two marbles are drawn in succession and without replacement from the urn. What is the probability that both marbles are green?

Two marbles are drawn in succession and with replacement from the urn. What is the probability that both marbles are green?

Two marbles are drawn in succession and without replacement. What is the probability that the second one drawn is green?

Example 7: An experiment consists of choosing one of two urns, X or Y, with equally likely probability, then selecting a marble from the selected urn. Suppose there are eight red marbles and 13 blue marbles in urn X, and there are 15 red marbles and 12 blue marbles in urn Y. What is the probability that the marble chosen is red?

Example 8: Box #1 contains 9 violet and 7 white marbles. Box #2 contains 5 violet and 8 white marbles. Box #3 contains 7 violet and 10 white marbles. The probabilities that Box #1, Box #2, and Box #3 will be chosen are 3/8, 2/5, and 9/40, respectively. A box is chosen and then a marble is chosen from the box. What is the probability that Box #2 was chosen and a violet marble was chosen?

Example 9: Families in a certain community were surveyed. In this community 98% indicated that they own more than 2 vehicles. Of those that own more than 2 vehicles, 84% are a family of more than 2. Of those that do not own more than 2 vehicles, 31% are not a family of more than 2. If a family is chosen at random, what is the probability that that family is a family of more than 2?