Math 1313
Rules of Probability

Five Basic Rules

At this point, you should be familiar with experiments, events and sample spaces, and with the definition, notation and properties of probability.

In this lesson, we will learn five basic properties of probability and we will use them to solve problems.

Suppose S is a sample space of an experiment, and suppose E and F are events of the experiment.

Rule 1: $P(E) \geq 0$ for any E

Rule 2: $P(S) = 1$

Rule 3: If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$

Note, Rule 3 can be extended to any finite number of events.

Rule 4: For any events E and F, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Rule 5: If E is an event of an experiment, and E^c denotes the complement of event E, then $P(E)+P(E^c)=1$.

Example 1: The SAT-II Math scores of the senior class at Central High are as follows:

Score, x	Probability
$x > 700$	.03
$600 < x \le 700$	.09
$500 < x \le 600$	.23
$400 < x \le 500$	.31
$300 < x \le 400$	.28
$x \le 300$	.06

If a student is selected at random, what is the probability that his score was more than 400?

If a student is selected at random, what is the probability that his score was less than or equal to 500?

If a student is selected at random, what is the probability that his score was greater than 400 but less than or equal to 600?

Example 2: Suppose E and F are events of an experiment. $P(E) = .27$, $P(F) = .19$, and $P(E \cap F) = .06$. Find $P(E \cup F)$.

Example 3: Suppose E and F are events of an experiment. $P(E) = .62$, $P(F) = .53$, and $P(E \cup F) = .77$. Find $P(E \cap F)$.

Example 4: Suppose E and F are events of an experiment. $P(E) = .29$, $P(F^c) = .34$, and $P(E \cap F) = .11$. Find $P(E \cup F)$.

Example 5: Suppose E and F are events of an experiment. $P(E) = .43$, $P(F) = .22$, and $P(E \cap F) = .09$. Find

$P(E \cup F)$

$P(E^c \cap F^c)$

$P(E^c \cap F)$

$P(E \cup F^c)$

Example 6: Suppose E and F are events of an experiment, $P(E^c) = .61$, $P(F^c) = .44$ and $P(E \cup F)^c = .3$. Find $P(E \cap F)$.

Example 7: Suppose E and F are events of an experiment, $P(E) = .79$, $P(F^c) = .24$ and $P(E \cup F^c) = .82$. Find $P(E \cap F^c)$.

Example 8: 500 students were surveyed regarding their enrollment in History and Sociology. The survey revealed that 274 are enrolled in exactly one of the two courses, 118 are not enrolled in History, and 181 are enrolled in History only. A student surveyed is chosen at random, what is the probability that he/she is enrolled in both History and Sociology?

Example 9: Among 500 college freshmen in the engineering school, 322 are enrolled in a math class, 99 are enrolled in a foreign language class, 261 are enrolled in physics, 148 are enrolled in both math and physics, 51 are enrolled in both math and foreign language, 39 are enrolled in both physics and foreign language, and 27 are enrolled in all three courses. What is the probability that an engineering student selected at random is enrolled in math and/or physics?

What is the probability that an engineering student selected at random is enrolled in exactly one of these three courses?

What is the probability that an engineering student selected at random is enrolled in at least two of these three courses?

What is the probability that an engineering student selected at random is enrolled in none of these three courses?

Example 10: A survey of people at a school yielded the following results:
15 speak Spanish, Russian and French;
25 speak Spanish and French;
42 speak Spanish;
20 speak Russian and Spanish;
6 speak Russian and French but not Spanish;
4 speak French, but neither Spanish nor Russian;
10 speak none of these languages mentioned here;
18 speak Russian but not French.

A person surveyed is chosen at random, what is the probability that that person speaks at most 1 of these languages mentioned here?

Example 11: University officials wanted to find out how many hours per week students spend studying, so they commissioned a survey of 500 students. The results showed the following:

Number of hours, x	**Number of students**
$0 \le x \le 1$	84
$1 < x \le 3$	129
$3 < x \le 6$	216
$x > 6$	71

Find the probability distribution associated with this experiment.

What is the probability that a surveyed student selected at random studied more than 3 hours per week?

What is the probability that a surveyed student selected at random studied no more than 6 hours per week?