

Math 1313
Chapter 5 – Section 5.2
Annuities – Present Value and Future Value

An **Ordinary Annuity** is a sequence of equal periodic payments made at the end of each payment period.

Examples of annuities:

1. Regular deposits into a savings account.
2. Monthly home mortgage payments.
3. Payments into a retirement account.

We will study annuities that are subject to the following conditions:

1. The terms are given by fixed time intervals.
2. The periodic payments are equal in size.
3. The payments are made at the end of the payment periods.
4. The payment periods coincide with the interest conversion periods.

The **Future Value of an Annuity** is the sum of all payments made and interest earned on an account.

Future Value of an Annuity Formula:

The future value S of an annuity of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$S = R \left[\frac{(1+i)^n - 1}{i} \right]$$

Present Value of an Annuity

Why would we want to know the present value of an annuity?

Well in some cases we would like to know how much money we should put away now (at the present time), so that at a later time we could withdraw a certain amount of money for a certain length of time. In other cases, given that we are making a sequence of payments on an investment we may wish to know the purchase price of that investment.

Present Value of an Annuity Formula:

The present value P of an annuity of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$P = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

Example 1: Felipe loves jet skiing, but every time he goes jet skiing he has to rent a jet ski. He finally decides to buy a jet ski and finds one he likes. He decides to save up for a down payment on the jet ski. His bank will pay 3% interest compounded quarterly for 2 years. Felipe decides to deposit \$370 into this account each quarter. How much will he have towards a down payment in 2 years?

Example 2: Trang would like to help his grandmother financially. He'd like for her to have \$300 per month for the next 5 years. An account at his credit union will pay 6% per year compounded monthly for 5 years. How much does he need to invest today in this account, so that he can set up this account for his grandmother?

Example 3: Stacy will invest \$3,000 at the end of each semiannual period for 5 years in an account that earns 4.5% interest per year compounded semiannually. How much will she have in this account at the end of 5 years?

Example 4: Newlyweds, Kate and Leo, have just purchased a new home. They made a down payment of \$5,000 and financed the rest through their credit union. Their monthly payments are \$1,200 over 30 years. The interest charged is 5.25% per year compounded monthly. How much was the purchase price of the house?