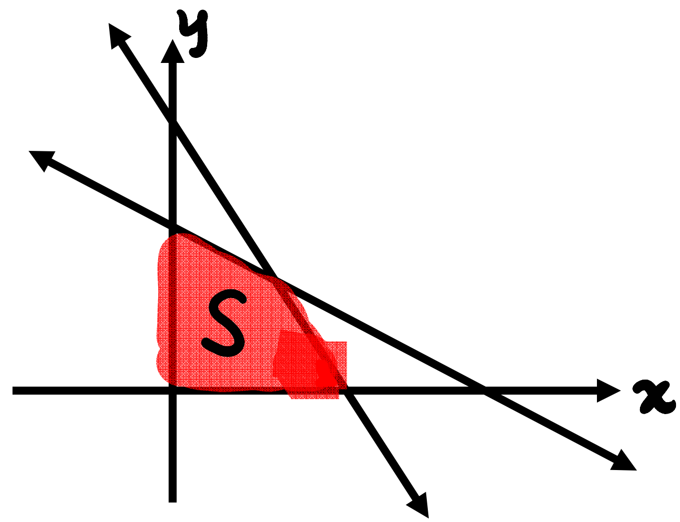
Math 1313
Chapter 3 – Section 3.3
Graphical Solution of Linear Programming Problems

Consider the following figure which is associated with a system of linear inequalities:



The set S is called a **feasible set**. Each point in S is a candidate for the solution of the problem and is called a **feasible solution**. The point(s) in S that optimizes (maximizes or minimizes) the objective function is called the **optimal solution.**

The Method of Corners

1. Graph the feasible set.
2. Find the coordinates of all corner points (vertices) of the feasible set.
3. Evaluate the objective function at each corner point.
4. Find the vertex that renders the objective function a maximum (minimum). If there is only one such vertex, then this vertex constitutes a unique solution to the problem. If the objective function is maximized (minimized) at two adjacent corner points of S, there are infinitely many optimal solutions given by the points on the line segment determined by these two vertices.

Theorem: Linear Programming

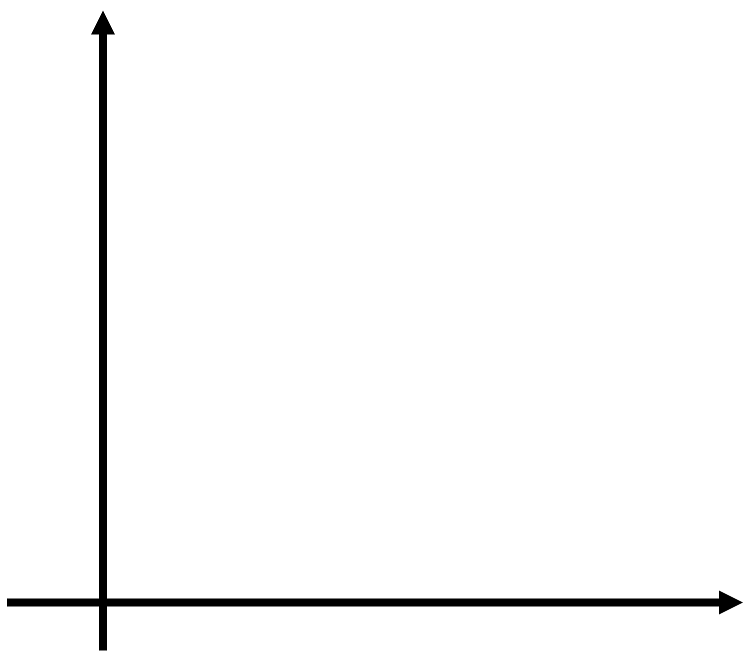
If a linear programming problem has a solution, then it must occur at a vertex, or corner point of the feasible set *S* associated with the problem. Furthermore, if the objective function P is optimized at two adjacent vertices of *S*, then it is optimized at every point on the line segment joining these vertices, in which case there are infinitely many solutions to the problem.

Example 1: The Bax Company produces two models of night lights – Sleep Tight and Rest Well. Each Sleep Tight night light requires 1 hr of work on assembly line I and 3 hr of work on assembly line II. Each Rest Well night light requires 2 hr of work on assembly line I and 4 hr of work on assembly line II. At most 32 hr of assembly time on line I and at most 84 hr of assembly time on line II are available per week. It is anticipated that Bax will realize a profit of $4 on each Sleep Tight night light and $6 on each Rest Well night light. How many night lights of each model should be produced per week in order to maximize Bax's profit?

Recall: x = number of Sleep Tight night lights
y = number of Rest Well night lights

Recall the linear programming problem:

Maximize $P = 4x + 6y$
subject to $x + 2y \leq 32$
$3x + 4y \leq 84$
$x \geq 0$
$y \geq 0$

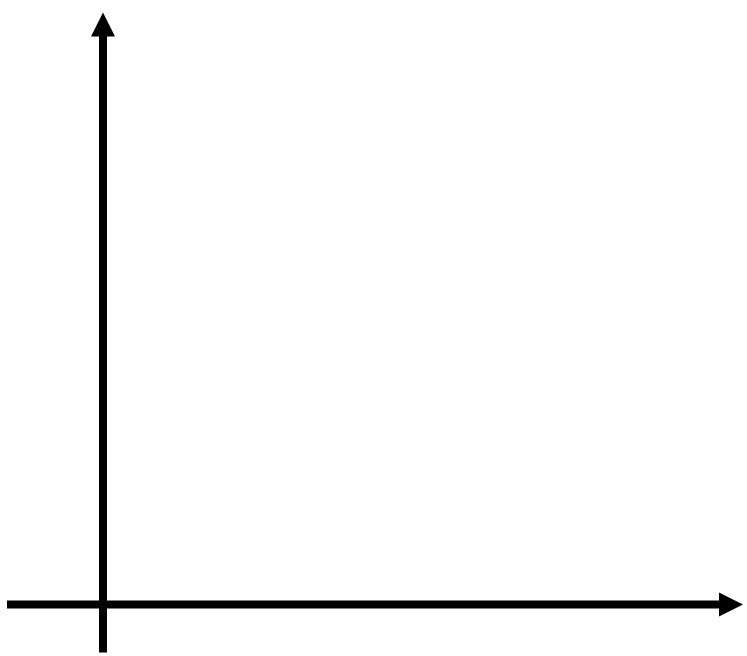


Example 2: A doctor advises a patient to increase his Vitamin A and B intake. Two vitamin pills are suitable: Extra Energy and Healthier You. Each Extra Energy pill contains 40 mg of Vitamin A and 30 mg of Vitamin B. Each Healthier You pill contains 20 mg of Vitamin A and 40 mg of Vitamin B. Each Extra Energy pill cost 7 cents and each Healthier You pill cost 5 cents. The patient must take at least 2,000 mg of Vitamin A and at least 2,400 mg of Vitamin B. How many pills of each brand should the patient purchase in order to meet the minimum requirements at the lowest cost?

Recall: x = number of Extra Energy pills
y = number of Healthier You pills

Recall the linear programming problem:

Minimize $C = 7x + 5y$
subject to $40x + 20y \geq 2000$
$30x + 40y \geq 2400$
$x \geq 0$
$y \geq 0$



Example 3: A farmer can use two types of fertilizer on his crops, Best Crops and Fert Fertilizer. Each bag of Best Crops contains 2 pounds of chlorine, 4 pounds of phosphoric acid and 8 pounds of nitrogen. Each bag of Fert Fertilizer contains 1 pound of chlorine, 4 pounds of phosphoric acid and 3 pounds of nitrogen. Tests indicate that the crops need at most 400 pounds of chlorine and at least 1,000 pounds of phosphoric acid. How many bags of each mix should be used, if the farmer wants to minimize the amount of nitrogen added to his crops?

Recall: x = number of Best Crops bags
y = number of Fert Fertilizer bags

Recall the linear programming problem:

Minimize $N = 8x + 3y$
subject to $2x + y \leq 400$
$4x + 4y \geq 1000$
$x \geq 0$
$y \geq 0$