Math 1313
Chapter 2 – Section 2.6

Let A be a square matrix of size *n*. A square matrix A^{-1} of size *n* such that $AA^{-1} = A^{-1}A = I$ is called the **inverse of A**.

Note: Not every square matrix has an inverse. A matrix with no inverse is called **singular**.

Finding the Inverse of a Matrix

Given the nxn matrix *A*:

1. Adjoin the nxn identity matrix *I* to obtain the augmented matrix (A|I).

2. Use a sequence of row operations to reduce (A|I) to the form (I|B), if possible.

The matrix *B* is the inverse of *A*.

A matrix B is the inverse of a matrix A if and only if AB = BA = I, where I is the identity matrix.

Matrices That Have No Inverses

If there is a row to the left of the vertical line in the augmented matrix containing all zeros, then the matrix does not have an inverse.

Example 1: Find the inverse of each matrix below, if possible.

a. $A = \begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix}$

b. $B = \begin{pmatrix} -2 & 6 \\ -4 & 12 \end{pmatrix}$

Example 2: Find the inverse of $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{pmatrix}$ if possible.

Formula for the Inverse of a 2X2 Matrix

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Suppose $D = ad - bc$ is not equal to zero. Then A^{-1} exists and is given by $A^{-1} = \frac{1}{D}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Example 3: Find the inverse of $C = \begin{pmatrix} -7 & -8 \\ 10 & 1 \end{pmatrix}$, if possible.

The use of inverses to solve systems of equations is advantageous when we are required to solve more than one system of equations AX = B, involving the same coefficient matrix, A, and different matrices of constants, B.

Example 4: Write a matrix equation that is equivalent to the given system of linear equations and then solve the system using the inverse of the coefficient matrix.

$2x + y = b_1$
$5x + 3y = b_2$

where i. $b_1 = 5,\ b_2 = 13$
and ii. $b_1 = -9,\ b_2 = 1$

Example 5: A movie theatre had a special once a day showing of *Funny Girl*. The theatre in which the movie played has a capacity of 50. The price for an adult ticket was $12 and the price for a child ticket was $7. On Saturday, the movie was sold out and total receipts were $565. On Sunday, the combined number of adults and children attending the movie was 39 and total receipts were $428. How many adults and how many children attended the movie for each showing?