Math 1313
Chapter 2 – Section 2.5

If A is a matrix of size mxn and B is a matrix of size nxp then the product AB is defined and is a matrix of size mxp.

So two matrices can be multiplied if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

How to Multiply Two Matrices

The element in the *i*th row and *j*th column of AB is found by multiplying each element in the *i*th row of A by the corresponding element in the *j*th column of B and adding the products.

Example 1: A student is part of an organization that sold candy bars for a fundraiser. The kinds of candy bars the student sold were: chocolate, chocolate almond, and chocolate crisp. The following matrix represents the number of each kind of candy bar the student sold, respectively.

$$A = \begin{pmatrix} 44 & 67 & 59 \end{pmatrix}$$

Each kind of candy bar sold for different prices. The chocolate sold for \$1, the chocolate almond for \$2 and the chocolate crisp for \$1.50. The following matrix represents this information, respectively.

$$B = \begin{pmatrix} 1.00 \\ 2.00 \\ 1.50 \end{pmatrix}$$

Find the total amount of money the student made for the fundraiser.

Example 2: Let

$$A = \begin{pmatrix} -4 & 7 & 9 \\ 10 & -2 & 2 \end{pmatrix},$$

$$B = \begin{pmatrix} 8 & 1 \\ 4 & -3 \\ -1 & 5 \end{pmatrix}, \text{ and}$$

$$C = \begin{pmatrix} -5 & 7 & 0 & -10 \\ 7 & -6 & 1 & 1 \end{pmatrix}.$$

Find, if possible, BA, BC, and CB.

Laws for Matrix Multiplication

If the products and sums are defined for the matrices A, B, and C, then

1. (AB)C = A(BC)
2. A(B + C) = AB + AC

Note: In general matrix multiplication is not commutative, AB $\neq$ BA.

The square matrix of size *n* having 1s along the main diagonal and zeros elsewhere is called the **identity matrix of size *n***.

$$I_n = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{pmatrix}$$

Matrix multiplication, in general, is not commutative, but the identity matrix has the property that $I_n A = AI_n = A$ for any square matrix *A* of size *n*x*n*.

Matrix Representation

A system of linear equations can be written in matrix form.

Example 3: Write the following system of linear equations in matrix form.

$$x + 5y - 3z = 18$$
$$-6y + 8z = -10$$
$$-4x + 9y = 8$$

Example 4: A company manufactures two kinds of calculators, Basic and Scientific. The labor hours needed for each kind of calculator in the Assembly Department and Packaging Department are given by the following matrix.

	Assembly Dept	Packaging Dept.
Basic	3	2
Scientific	4	3

= A

The company manufactures the calculators at two plants, one is located in the East Coast and the other in the West Coast. The following matrix gives hourly rates, in dollars, for workers in each department at each location.

	East Coast	West Coast
Assembly Dept.	10	12
Packaging Dept.	13	14

= B

Find AB and explain the meaning of each entry.