Math 1313
Chapter 2 – Section 2.4

A **matrix** is an array of numbers.

The real numbers that make up the array are called the **entries** or **elements** of the matrix.

The **size** or **dimension** of a matrix is described in terms of the number of rows and columns of the matrix.

For example, if a matrix has 4 rows and 2 columns, then the dimension of that matrix is 4x2.

A **matrix** is an ordered rectangular array of numbers. A matrix with *m* rows and *n* columns has size *m*x*n*. The entry in the *i*th row and *j*th column is denoted by a_{ij}.

A **row matrix** is a matrix of size 1x*n*, it has one row and *n* columns. A row matrix is also called a row vector.

A **column matrix** is a matrix of size *n*x1, it has *n* rows and one column. A column matrix is also called a column vector.

A matrix with the same number of rows as columns is called a **square matrix**.

Example 1: Given the following matrix:

$$A = \begin{pmatrix} -9 & 1 & 0 & 6 & 99 \\ 4 & -3 & -1 & 0 & 0 \\ 8 & 11 & 12 & 4 & 7 \end{pmatrix}$$

answer the following questions.

a. What is the size of A?
b. Find

a_{24}, a_{15}, a_{35}, and, a_{33}.

Equality of Matrices

Two matrices are equal if they have the same size and their corresponding entries are equal.

Example 2: Solve the following matrix equation for x, y, z, and w.

$$\begin{pmatrix} w+3 & 0 & 8 & -17 \\ 4 & -5z & 19 & -1 \\ 8 & -5 & 6 & -6y+1 \end{pmatrix} = \begin{pmatrix} 13 & 0 & 8 & -17 \\ 4 & 25 & 19 & x+4 \\ 8 & -5 & 6 & 30 \end{pmatrix}$$

Addition and Subtraction of Matrices

If A and B are two matrices of the same size,

1. The sum A + B is the matrix obtained by adding the corresponding entries in the two matrices.

2. The difference A - B is the matrix obtained by subtracting the corresponding entries in B from A.

Laws for Matrix Addition

If A, B, and C are matrices of the same size, then

1. A + B = B + A
2. (A + B) + C = A + (B + C)

The **zero matrix** is one in which all entries are zero. The capital letter O represents the zero matrix.

The zero matrix has the property: A + O = O + A = A

Example 3: Let $A = \begin{pmatrix} 9 & -3 & 1 \\ 0 & 12 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 & -9 \\ 5 & 20 & -1 \end{pmatrix}$.
Find A - B.

Scalar Product

If A is a matrix and c is a real number, then the **scalar product** cA is the matrix obtained by multiplying each entry of A by c.

Example 4: Solve for u, x, y, and z in the matrix equation.

$$\begin{pmatrix} -9+u & -50 \\ 1 & -2z+3 \\ 8 & 4 \\ y-10 & -5 \end{pmatrix} + 4\begin{pmatrix} 8 & 10 \\ 7 & 5 \\ -3 & -11 \\ 2 & 7x \end{pmatrix} = -1\begin{pmatrix} u & 10 \\ -29 & z+3 \\ 4 & 40 \\ 2 & 7x \end{pmatrix}$$

If A is an *m*x*n* matrix with elements A^T , then the **transpose** of A is the *n*x*m* matrix with elements.

Example 5: Refer to the following matrices.

$$A = \begin{pmatrix} 7 & -8 & -3 \\ 11 & 10 & 4 \\ 0 & -9 & 7 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 5 & 2 \\ 4 & 5 & 10 \\ 1 & -6 & 7 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -4 & 8 & 10 \end{pmatrix}$$

a. Find the transpose of A and C.
b. Compute, if possible, -2A + 4B.