A matrix is an array of numbers.

The real numbers that make up the array are called the entries or elements of the matrix.

The size or dimension of a matrix is described in terms of the number of rows and columns of the matrix.

For example, if a matrix has 4 rows and 2 columns, then the dimension of that matrix is 4x2.

A row matrix is a matrix of size 1xn, it has one row and n columns. A row matrix is also called a row vector.

A column matrix is a matrix of size nx1, it has n rows and one column. A column matrix is also called a column vector.

A matrix with the same number of rows as columns is called a square matrix.

Example 1: Given the following matrix:

\[
A = \begin{pmatrix}
-9 & 1 & 0 & 6 & 99 \\
4 & -3 & -1 & 0 & 0 \\
8 & 11 & 12 & 4 & 7
\end{pmatrix}
\]

answer the following questions.

a. What is the size of A?

b. Find 

\[a_{24}, a_{15}, a_{35}, \text{ and } a_{33}\].

Equality of Matrices

Two matrices are equal if they have the same size and their corresponding entries are equal.
Example 2: Solve the following matrix equation for \( x, y, z, \) and \( w. \)

\[
\begin{pmatrix}
  w+3 & 0 & 8 & -17 \\
  4 & -5z & 19 & -1 \\
  8 & -5 & 6 & -6y+1 \\
\end{pmatrix} =
\begin{pmatrix}
  13 & 0 & 8 & -17 \\
  4 & 25 & 19 & x+4 \\
  8 & -5 & 6 & 30 \\
\end{pmatrix}
\]

**Addition and Subtraction of Matrices**

If \( A \) and \( B \) are two matrices of the same size,

1. The sum \( A + B \) is the matrix obtained by adding the corresponding entries in the two matrices.

2. The difference \( A - B \) is the matrix obtained by subtracting the corresponding entries in \( B \) from \( A. \)

**Laws for Matrix Addition**

If \( A, B, \) and \( C \) are matrices of the same size, then

1. \( A + B = B + A \)

2. \( (A + B) + C = A + (B + C) \)

The **zero matrix** is one in which all entries are zero. The capital letter \( O \) represents the zero matrix.

The zero matrix has the property: \( A + O = O + A = A \)

Example 3: Let \( A = \begin{pmatrix} 9 & -3 & 1 \\ 0 & 12 & 8 \end{pmatrix} \) and \( B = \begin{pmatrix} -4 & 3 & -9 \\ 5 & 20 & -1 \end{pmatrix} \).

Find \( A - B. \)
Scalar Product

If $A$ is a matrix and $c$ is a real number, then the **scalar product** $cA$ is the matrix obtained by multiplying each entry of $A$ by $c$.

Example 4: Solve for $u$, $x$, $y$, and $z$ in the matrix equation.

\[
\begin{pmatrix}
-9 + u & -50 \\
1 & -2z + 3 \\
8 & 4 \\
y - 10 & -5
\end{pmatrix}
+ 4
\begin{pmatrix}
8 & 10 \\
7 & 5 \\
-3 & -11 \\
2 & 7x
\end{pmatrix}
= -1
\begin{pmatrix}
u & 10 \\
-29 & z + 3 \\
4 & 40 \\
2 & 7x
\end{pmatrix}
\]

If $A$ is an $m \times n$ matrix with elements $A^T$, then the **transpose** of $A$ is the $n \times m$ matrix with elements.

Example 5: Refer to the following matrices.

\[
A = \begin{pmatrix}
7 & -8 & -3 \\
11 & 10 & 4 \\
0 & -9 & 7
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
-1 & 5 & 2 \\
4 & 5 & 10 \\
1 & -6 & 7
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
1 & -4 & 8 & 10
\end{pmatrix}
\]

a. Find the transpose of $A$ and $C$.

b. Compute, if possible, $-2A + 4B$. 