

Math 1313
Chapter 2 –Section 2.3

In the previous section we studied systems of linear equations that had unique solutions.

In this section we will study systems of linear equations that have infinitely many solutions and those that have no solution.

Theorem

1. If the number of equations is greater than or equal to the number of variables in a linear system, then one of the following is true:
 - a. The system has no solution.
 - b. The system has exactly one solution.
 - c. The system has infinitely many solutions.

2. If there are fewer equations than variables in a linear system, then the system either has no solution or it has infinitely many solutions.

Systems with No Solution

If there is a row in the augmented matrix containing all zeros to the left of the vertical line and a nonzero entry to the right of the line, then the system of equations has no solution.

Example 1: Given that the augmented matrix in row-reduced form is equivalent to the augmented matrix of a system of linear equations, determine whether the system has a solution and find the solution or solutions to the system, if they exist.

$$\left(\begin{array}{ccc|c} 1 & 0 & -10 & 5 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 9 \end{array} \right)$$

A System of Equations with an Infinite Number of Solutions

Example 2: Given that the augmented matrix in row-reduced form is equivalent to the augmented matrix of a system of linear equations, determine whether the system has a solution and find the solution or solutions to the system, if they exist.

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 9 \\ 0 & 1 & 8 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Example 3: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x + y - z &= -20 \\ -3y + 3z &= 51 \end{aligned}$$

Example 4: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x - y &= 3 \\ 2x + y &= 7 \\ x + 3y &= 4 \end{aligned}$$

Example 5: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x - 3y &= 3 \\ 8x - 24y &= 24 \\ 10x - 30y &= 30 \end{aligned}$$

Example 6: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$\begin{aligned} x - 2y + z &= 5 \\ 2x + y - z &= 2 \\ -2x + 4y - 2z &= 2 \end{aligned}$$