

Math 1313
Chapter 1 – Section 1.5
The Method of Least Squares

Least-Squares Method

The **least-squares method** is a method for determining a straight line that, in some sense, best fits a set of data points when the points are scattered about a straight line.

The Method of Least-Squares

Suppose we are given n data points $P_1(x_1, y_1), P_2(x_2, y_2), \dots, P_n(x_n, y_n)$ then, the least-squares (regression) line for the data is given by the linear equation $y = mx + b$ where the constants m and b satisfy the **normal equations**

$$nb + (x_1 + x_2 + \dots + x_n)m = (y_1 + y_2 + \dots + y_n)$$

$$(x_1 + x_2 + \dots + x_n)b + (x_1^2 + x_2^2 + \dots + x_n^2)m = (x_1y_1 + x_2y_2 + \dots + x_ny_n)$$

simultaneously.

Example 1: Find the equation of the least-squares line for the given set of points.

$(2, 5)$, $(4, 8)$ and $(3, 2)$

Example 2: The projected number of cable television subscribers (in millions) in a certain country from 1998 through 2002 is summarized in the accompanying table.

Year, x	1998	1999	2000	2001	2002
Cable TV Subscribers, y	14.6	15.4	17.2	18.6	21.2

- a. Find an equation of the least-squares line for these data. (Let $x = 1$ represent 1998.)

- b. Use the result in part (a) to estimate the number of cable television subscribers in 2005, assuming the trend continued.

Example 3: The following equation is the least-squares line for a set of data given by a small individually owned daycare center. The variable x denotes years and y denotes net sales in thousands of dollars. Assume that the first year in the data set, $x = 1$, represents the year 1985. Assuming that the data given was for the years 1985, 1987, 1989, ..., 1995, estimate the net sales in thousands of dollars in the year 1991, assuming the trend continued.

$$y = 2.78x + 4$$