Section 6.2: More Systems and Applications

- Solving Other Systems

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The methods of elimination and substitution can be used to solve systems of linear equations with more than two variables and also systems in which nonlinear equations appear.

Example Problem 1: Solving a 3x3 Linear System by the Elimination Method

Solve the following system by the elimination method.

$$\begin{aligned} x + y + z &= 4 \\ 2x + 3y + 3z &= 11 \\ x - 5y + 2z &= -7 \end{aligned}$$

Solution:

Eliminate x from the second and third equations.

$$\begin{array}{rl} -2x - 2y - 2z = -8 & \text{First equation times } -2 \\ \underline{2x + 3y + 3z = 11} & \text{Second equation} \\ y + z = 3 & \end{array}$$

$$\begin{array}{rl} -x - y - z = -4 & \text{First equation times } -1 \\ \underline{x - 5y + 2z = -7} & \text{Third equation} \\ -6y + z = -11 & \end{array}$$

We obtain a new system that is equivalent to the original system.

$$\begin{aligned} x + y + z &= 4 \\ y + z &= 3 \\ -6y + z &= -11 \end{aligned}$$

Eliminate y from the third equation.

$$\begin{aligned} 6y + 6z &= 18 \quad &\text{Second equation times 6} \\ -6y + z &= -11 \quad &\text{Third equation} \\ \hline 7z &= 7 \end{aligned}$$

Write the equivalent system.

$$\begin{aligned} x + y + z &= 4 \\ y + z &= 3 \\ 7z &= 7 \end{aligned}$$

Now multiply the third equation by $\frac{1}{7}$.

$$\begin{aligned} x + y + z &= 4 \\ y + z &= 3 \\ z &= 1 \end{aligned}$$

Substitute $z = 1$ into the second equation.

$$\begin{aligned} y + 1 &= 3 \\ y + 1 - 1 &= 3 - 1 \\ y &= 2 \end{aligned}$$

Substitute $z = 1$ and $y = 2$ into the first equation.

$$\begin{aligned} x + 2 + 1 &= 4 \\ x + 3 &= 4 \\ x + 3 - 3 &= 4 - 3 \\ x &= 1 \end{aligned}$$

The solution to the system is $x = 1$, $y = 2$, and $z = 1$ which can be written as $(1, 2, 1)$.

Example Problem 2: Solving a System Which Contains a Nonlinear Equation by the Method of Substitution

Solve the following system by the substitution method.

$$\begin{aligned} x^2 - x - y &= 3 \\ -x + y &= -3 \end{aligned}$$

Solution:

Solve the second equation for y.

$$\begin{aligned} -x + y &= -3 \\ -x + y + x &= -3 + x \\ y &= -3 + x \end{aligned}$$

Now substitute $-3+x$ for y in the first equation.

$$\begin{aligned} x^2 - x - y &= 3 \\ x^2 - x - (-3 + x) &= 3 \\ x^2 - x + 3 - x &= 3 \\ x^2 - 2x + 3 &= 3 \\ x^2 - 2x + 3 - 3 &= 3 - 3 \\ x^2 - 2x &= 0 \\ x(x - 2) &= 0 \\ x = 0 \quad \text{or} \quad x &= 2 \end{aligned}$$

Substitute $x = 0$ into the equation $y = -3 + x$.

$$y = -3 + 0 = -3$$

Substitute $x = 2$ into the equation $y = -3 + x$.

$$y = -3 + 2 = -1$$

The system has two solutions:

$(0, -3)$ $\quad [\, x = 0 \text{ and } y = -3 \,]$

and

$(2, -1)$ $\quad [\, x = 2 \text{ and } y = -1 \,]$

Additional Example 1:

Solve the following system of equations by the elimination method.

$$\begin{aligned} x+2y-z&=-2\\ -x-y+4z&=5\\ 2x+5y-z&=-3 \end{aligned}$$

Solution:

Eliminate x from the second and third equations.

Add the first and second equations together.

$$\begin{aligned} x+2y-z&=-2\\ -x-y+4z&=5\\ \hline y+3z&=3 \end{aligned}$$

Multiply the first equation by -2 and add to the third equation.

$$\begin{aligned} -2x-4y+2z&=4\\ 2x+5y-z&=-3\\ \hline y+z&=1 \end{aligned}$$

The system that appears below is equivalent to the original system.

$$\begin{aligned} x+2y-z&=-2\\ y+3z&=3\\ y+z&=1 \end{aligned}$$

Multiply the second equation by -1 and add to the third equation. This will eliminate y from the third equation.

$$\begin{aligned} -y-3z&=-3\\ y+z&=1\\ \hline -2z&=-2 \end{aligned}$$

Write the equivalent system.

$$\begin{aligned} x+2y-z&=-2\\ y+3z&=3\\ -2z&=-2 \end{aligned}$$

Multiply the third equation by $-\frac{1}{2}$.

$$\begin{aligned} x+2y-z&=-2 \\ y+3z&=3 \\ z&=1 \end{aligned}$$

Substitute $z=1$ in the equation $y+3z=3$ and solve for y.

$$\begin{aligned} y+3(1)&=3 \\ y+3&=3 \\ y+3-3&=3-3 \\ y&=0 \end{aligned}$$

Substitute $z=1$ and $y=0$ in the equation $x+2y-z=-2$ and solve for x.

$$\begin{aligned} x+2(0)-1&=-2 \\ x-1&=-2 \\ x-1+1&=-2+1 \\ x&=-1 \end{aligned}$$

The solution is $x=-1$, $y=0$, and $z=1$ which can be written as $(-1,0,1)$.

Additional Example 2:

Solve the following system of equations by the substitution method.

$$\begin{aligned} x^2+y^2&=4 \\ -x+y&=2 \end{aligned}$$

Solution:

Solve the second equation for y.

$$\begin{aligned} -x+y&=2 \\ -x+y+x&=2+x \\ y&=2+x \end{aligned}$$

Substitute $2+x$ for y in the first equation.

$$x^2+y^2=4$$
$$x^2+(2+x)^2=4$$
$$x^2+4+4x+x^2=4$$
$$2x^2+4x+4=4$$
$$2x^2+4x+4-4=4-4$$
$$2x^2+4x=0$$
$$2x(x+2)=0$$

$2x=0$ or $x+2=0$

$x=0$ $\quad$ $x+2-2=0-2$

$x=-2$

Substitute $x=0$ and then $x=-2$ into the equation $y=2+x$.

For $x=0$, $y=2+0=2$.

For $x=-2$, $y=2+(-2)=0$.

The solutions are $(0,2)$ and $(-2,0)$.

Additional Example 3:

Solve the following system of equations by the elimination method.

$$x^2+y=-1$$
$$2x^2-y=4$$

Solution:

Eliminate y by adding the equations together.

$$\begin{array}{r} x^2+y=-1 \\ \underline{2x^2-y=\ 4} \\ 3x^2 \quad\ =\ 3 \end{array}$$

Solve the equation $3x^2 = 3$ by dividing both sides by 3.

$$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{3}{3}$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$
$$x = \pm 1$$

Substitute $x = 1$ into the first equation $x^2 + y = -1$.

$$x^2 + y = -1$$
$$1^2 + y = -1$$
$$1 + y = -1$$
$$1 + y - 1 = -1 - 1$$
$$y = -2$$

Substitute $x = -1$ into the first equation $x^2 + y = -1$.

$$x^2 + y = -1$$
$$(-1)^2 + y = -1$$
$$1 + y = -1$$
$$1 + y - 1 = -1 - 1$$
$$y = -2$$

For $x = 1$, $y = -2$ and for $x = -1$, $y = -2$. The solutions are $(1, -2)$ and $(-1, -2)$.

Additional Example 4:

For the following problem, write a system of equations with two variables to model the problem and then solve the system of equations.

A rectangle's perimeter is 30 feet and its length is 4 times its width. Find the width and length of the rectangle.

Solution:

Let x = width (in feet).

Let y = length (in feet).

The perimeter is given as 30 feet.

$$2x + 2y = 30$$

The length is 4 times the width.

$$y = 4x$$

Set up the system of equations.

$$\begin{aligned} 2x + 2y &= 30 \\ y &= 4x \end{aligned}$$

Substitute $4x$ for y in the first equation.

$$\begin{aligned} 2x + (2)(4x) &= 30 \\ 2x + 8x &= 30 \\ 10x &= 30 \\ \frac{\cancel{10}x}{\cancel{10}} &= \frac{30}{10} \\ x &= 3 \end{aligned}$$

Substitute 3 for x in the equation $y = 4x$

$$y = 4(3) = 12$$

The width is 3 feet and the length is 12 feet.

Exercise Set 6.2: More Systems and Applications

Solve the following systems of equations by using substitution and/or elimination.

1. $$\begin{aligned} 3x-5y+z&=22\\ 2x+y&=1\\ x-3y-4z&=7 \end{aligned}$$

2. $$\begin{aligned} x+3z&=5\\ 3x-2y-z&=-13\\ 5x-7y+4z&=-1 \end{aligned}$$

3. $$\begin{aligned} x+y+z&=1\\ -2x+3y-5z&=20\\ 3x-y+2z&=-1 \end{aligned}$$

4. $$\begin{aligned} x-y+z&=2\\ -4x+2y-3z&=-5\\ 2x+3y+z&=4 \end{aligned}$$

5. $$\begin{aligned} 2x+3y-4z&=-9\\ 3x-5y-2z&=4\\ -2x+4y+3z&=0 \end{aligned}$$

6. $$\begin{aligned} 4x-5y+2z&=7\\ 3x+2y-4z&=10\\ -2x-3y+3z&=-3 \end{aligned}$$

Solve the following equations by using the substitution method.

7. $$\begin{aligned} y&=x^2\\ x+y&=12 \end{aligned}$$

8. $$\begin{aligned} x&=y^2\\ x-y&=2 \end{aligned}$$

9. $$\begin{aligned} x^2+y^2&=10\\ x+3y&=0 \end{aligned}$$

10. $$\begin{aligned} x+y^2&=5\\ x+y&=3 \end{aligned}$$

Solve the following equations by using the elimination method.

11. $$\begin{aligned} 2x^2+3y&=-7\\ 3x^2-4y&=32 \end{aligned}$$

12. $$\begin{aligned} x^2-2y^3&=7\\ -2x^2+5y^3&=-13 \end{aligned}$$

13. $$\begin{aligned} \frac{6}{x}-\frac{8}{y}&=11\\ \frac{4}{x}+\frac{3}{y}&=-1 \end{aligned}$$

14. $$\begin{aligned} \frac{12}{x}+\frac{9}{y}&=0\\ \frac{8}{x}-\frac{6}{y}&=-4 \end{aligned}$$

For each of the following problems:

(a) **Write a system of equations involving two variables to model the problem.**

(b) **Solve your system of equations and answer the question.**

15. Dillan is at a baseball game and is buying hot dogs and sodas for his family. Hot dogs cost \$3 each and sodas cost \$1.75 each. He purchases nine items and spends a total of \$22.00. How many hot dogs did he buy? How many sodas did he buy?

16. Gabrielle is buying notebooks at the bookstore. Red notebooks cost \$3.50 each, and black notebooks cost \$2.20 each. She buys fourteen notebooks and spends a total of \$42.50. How many notebooks of each color did she buy?

17. Two numbers have a sum of 77 and a difference of 13. Find the two numbers.

18. Two numbers have a sum of 130 and a difference of 78. Find the two numbers.

19. A rectangle has a perimeter of 26 centimeters and an area of 36 square centimeters. Find the dimensions of the rectangle.

20. A rectangle has a perimeter of 44 inches and an area of 72 square inches. Find the dimensions of the rectangle.

21. A rectangular garden has a perimeter of 200 feet, and its width is 56 feet less than its length. Find the length and width of the garden.

22. A rectangular picture frame has a perimeter of 50 inches, and its width is $\frac{2}{3}$ of its length. Find the length and width of the picture frame.

23. Paul has 16 coins in his pocket, consisting entirely of dimes and quarters. If he has a total of $3.40 in coins, how many coins of each type are in his pocket?

24. Michael has 105 coins in his piggy bank, consisting entirely of dimes and nickels. If he has a total of $9.10 in coins, how many coins of each type are in his piggy bank?

25. Kathy has $2,500 to invest and she decides to invest it in two different accounts which both yield simple interest $(I = PRT)$. The first account yields 5% interest per year, and the second account yields 6% interest per year. At the end of one year, she earns a total of $139 in interest. How much money was invested in each account?

26. Mark has $12,000 to invest and he decides to invest it in two different accounts which both yield simple interest $(I = PRT)$. The first account yields 4% interest per year, and the second account yields 4.5% interest per year. At the end of one year, he earns a total of $527.50 in interest. How much money was invested in each account?

27. Jen and Anthony have received a total of 64 emails in the past week. If Jen received 5 less than twice the amount of emails that Anthony received, how many emails did they each receive?

28. Brian and Teri have changed a total of 73 diapers this week. If Teri has changed 2 less than four times the amount of diapers that Brian has changed, how many diapers did each of them change?