Chapter 6
Systems of Equations

Section 6.1: 2x2 Linear Systems

- Solving 2x2 Linear Systems

Solving 2x2 Linear Systems

Solving 2X2 Linear Systems

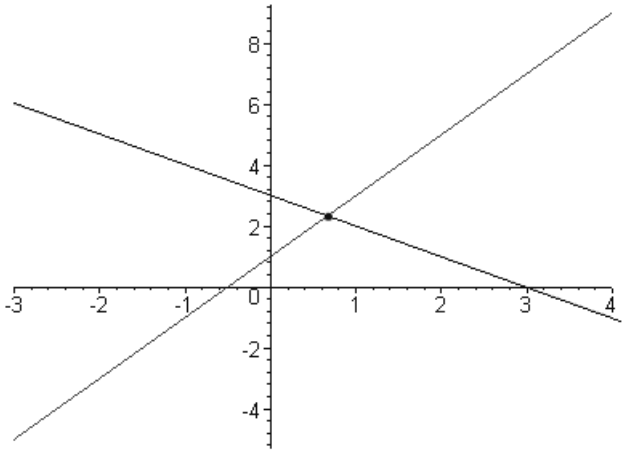
To solve a system of two linear equations

$$\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$$

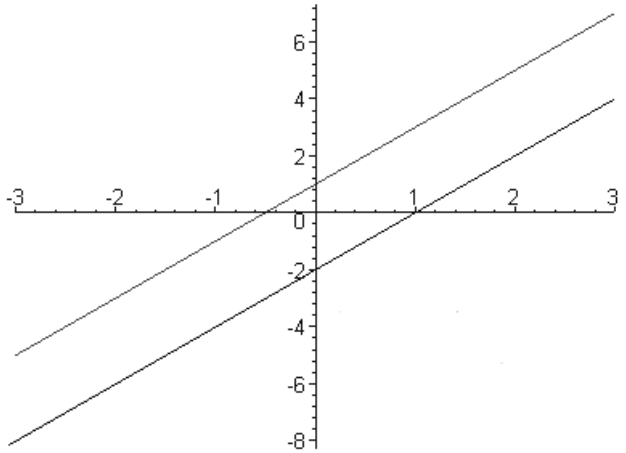
means to find values for x and y that satisfy both equations.

The system will have exactly one solution, no solution, or infinitely many solutions

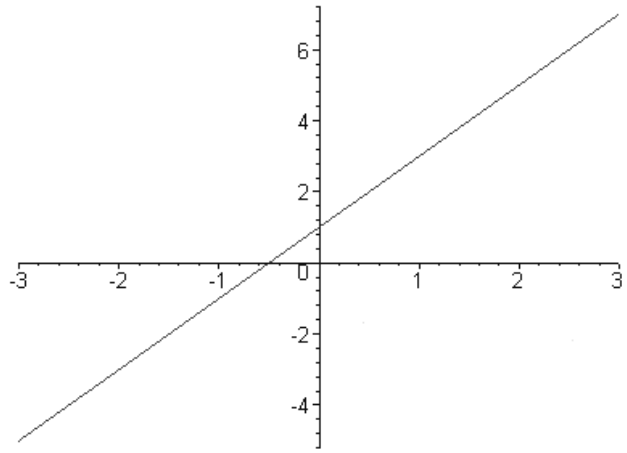
In a 2X2 linear system with one solution the two lines $ax+by=c$ and $dx+ey=f$ intersect at a single point.



In a 2X2 linear system with no solution the two lines $ax+by=c$ and $dx+ey=f$ are parallel.



In a 2X2 linear system with infinitely many solutions the two lines $ax+by=c$ and $dx+ey=f$ coincide.



Example Problem 1: Solving by Graphing

Solve the following system by graphing.

$$3x-y=-2$$
$$6x-2y=3$$

Solution:

Graph the line $3x-y=-2$ by finding the x-intercept and the y-intercept.

Find the x-intercept by substituting $y = 0$ in the equation and solving for x.

$$\begin{aligned} 3x - y &= -2 \\ 3x - 0 &= -2 \\ 3x &= -2 \\ \frac{\cancel{3}x}{\cancel{3}} &= \frac{-2}{3} \\ x &= -\frac{2}{3} \end{aligned}$$

Find the y-intercept by substituting $x = 0$ in the equation and solving for y.

$$\begin{aligned} 3x - y &= -2 \\ 3(0) - y &= -2 \\ 0 - y &= -2 \\ -y &= -2 \\ (-1)(-y) &= (-1)(-2) \\ y &= 2 \end{aligned}$$

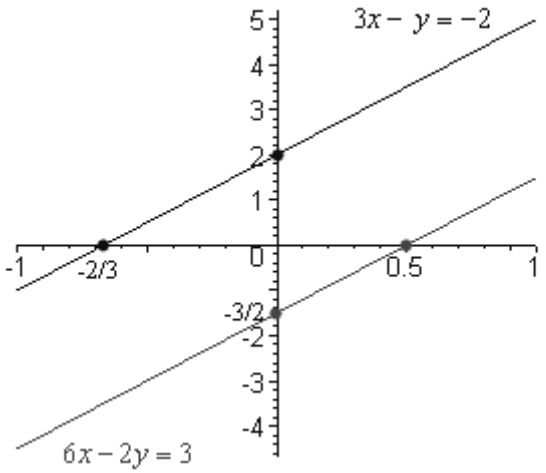
Graph the line $6x - 2y = 3$ by finding the x-intercept and the y-intercept.

Find the x-intercept by substituting $y = 0$ in the equation and solving for x.

$$\begin{aligned} 6x - 2y &= 3 \\ 6x - 2(0) &= 3 \\ 6x - 0 &= 3 \\ 6x &= 3 \\ \frac{\cancel{6}x}{\cancel{6}} &= \frac{3}{6} \\ x &= \frac{1}{2} \end{aligned}$$

Find the y-intercept by substituting $x = 0$ in the equation and solving for y.

$$\begin{aligned} 6x - 2y &= 3 \\ 6(0) - 2y &= 3 \\ 0 - 2y &= 3 \\ -2y &= 3 \\ \frac{\cancel{-2}y}{\cancel{-2}} &= \frac{3}{-2} \\ y &= -\frac{3}{2} \end{aligned}$$



The lines are parallel. The system has no solution.

Example Problem 2: Solving by Elimination

Solve the following system by the elimination method.

$$3x - y = -1$$
$$2x + 5y = 22$$

Solution:

We will eliminate the variable y and solve for x. To eliminate the variable y, multiply the first equation by 5.

$$\begin{array}{r} 15x - 5y = -5 \\ 2x + 5y = 22 \\ \hline 17x \quad = 17 \end{array}$$ Add the equations together.

$$\frac{17x}{17} = \frac{17}{17}$$

$$x = 1$$

Substitute $x = 1$ into the first equation of the original system and solve for y.

$$\begin{aligned} 3x-y&=-1\\ 3(1)-y&=-1\\ 3-y&=-1\\ 3-y-3&=-1-3\\ -y&=-4\\ (-1)(-y)&=(-1)(-4)\\ y&=4 \end{aligned}$$

The system has one solution. The lines intersect at the point $(1,4)$. The solution is the ordered pair $(1,4)$; that is, $x=1$ and $y=4$.

Example Problem 3: Solving by Substitution

Solve the following system by the substitution method.

$$\begin{aligned} x+\ y&=4\\ 3x+2y&=7 \end{aligned}$$

Solution:

Solve the first equation for y.

$$\begin{aligned} x+y&=4\\ x+y-x&=4-x\\ y&=4-x \end{aligned}$$

Now substitute $4-x$ for y in the second equation.

$$\begin{aligned} 3x+2y&=7\\ 3x+2(4-x)&=7\\ 3x+8-2x&=7\\ x+8&=7\\ x+8-8&=7-8\\ x&=-1 \end{aligned}$$

Substitute $x=-1$ for x in the equation $y=4-x$.

$$\begin{aligned} y&=4-x\\ y&=4-(-1)=4+1=5 \end{aligned}$$

The system has one solution. The lines intersect at the point $(-1,5)$. The solution is the ordered pair $(-1,5)$; that is, $x=-1$ and $y=5$.

Additional Example 1:

Solve the following system of linear equations by graphing.

$$4x + y = 8$$
$$8x + 2y = -4$$

Solution:

To graph the first line, find the x-intercept and the y-intercept.

Find the x-intercept by substituting $y = 0$ into the equation and solving for x.

$$4x + y = 8$$
$$4x + 0 = 8$$
$$4x = 8$$
$$\frac{4x}{4} = \frac{8}{4}$$
$$x = 2$$

Find the y-intercept by substituting $x = 0$ into the equation and solving for y.

$$4x + y = 8$$
$$4(0) + y = 8$$
$$0 + y = 8$$
$$y = 8$$

To graph the second line, find the x-intercept and the y-intercept.

Find the x-intercept by substituting $y = 0$ into the equation and solving for x.

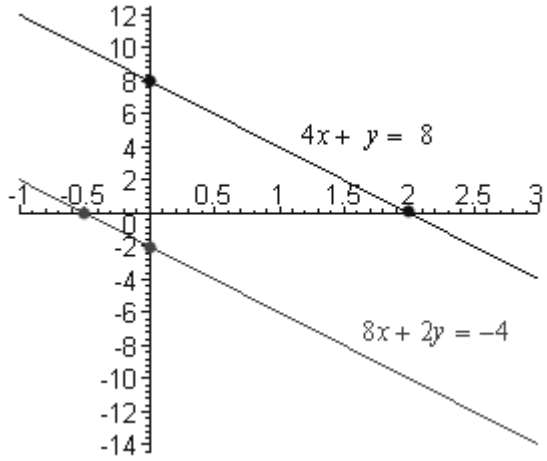
$$8x + 2y = -4$$
$$8x + 2(0) = -4$$
$$8x + 0 = -4$$
$$8x = -4$$
$$\frac{8x}{8} = \frac{-4}{8}$$
$$x = -\frac{1}{2}$$

Find the y-intercept by substituting $x = 0$ into the equation and solving for y.

$$8x + 2y = -4$$
$$8(0) + 2y = -4$$

$$\begin{aligned} 0+2y &= -4 \\ 2y &= -4 \\ \frac{2y}{2} &= \frac{-4}{2} \\ y &= -2 \end{aligned}$$

For the first line, the x-intercept is 2 and the y-intercept is 8. For the second line, the x-intercept is $-\frac{1}{2}$ and the y-intercept is -2. Graph the lines.



The lines are parallel. The system has no solution.

Additional Example 2:

Solve the following system of linear equations by using the elimination method.

$$\begin{aligned} 3x+\ y &= 5 \\ x-2y &= 4 \end{aligned}$$

Solution:

We will eliminate the variable y. Multiply the first equation by 2.

$$6x+2y=10$$

Add the equations together.

$$\begin{array}{r} 6x+2y=10 \\ x-2y=\ 4 \\ \hline 7x\quad\ =14 \end{array}$$

Solve the equation $7x = 14$ by dividing both sides by 7.

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

Substitute 2 for x in the first equation to find y.

$$\begin{aligned} 3x + y &= 5 \\ 3(2) + y &= 5 \\ 6 + y &= 5 \\ 6 + y - 6 &= 5 - 6 \\ y &= -1 \end{aligned}$$

The system has one solution. The lines intersect at the point $(2, -1)$.
The solution is the ordered pair $(2, -1)$; that is, $x = 2$ and $y = -1$.

Additional Example 3:
Solve the following system of linear equations by using the substitution method.

$$\begin{aligned} 4x + y &= 9 \\ 3x - 2y &= -7 \end{aligned}$$

Solution:
Solve the first equation for y.

$$\begin{aligned} 4x + y &= 9 \\ 4x + y - 4x &= 9 - 4x \\ y &= 9 - 4x \end{aligned}$$

Substitute $9 - 4x$ for y in the second equation.

$$\begin{aligned} 3x - 2y &= -7 \\ 3x - 2(9 - 4x) &= -7 \\ 3x - 18 + 8x &= -7 \\ 11x - 18 &= -7 \\ 11x - 18 + 18 &= -7 + 18 \\ 11x &= 11 \\ \frac{11x}{11} &= \frac{11}{11} \\ x &= 1 \end{aligned}$$

Substitute $x = 1$ in the equation $y = 9 - 4x$ to find y.

$$y = 9 - 4(1) = 9 - 4 = 5$$

The system has one solution. The lines intersect at the point $(1, 5)$.

The solution is the ordered pair $(1, 5)$; that is, $x = 1$ and $y = 5$.

Additional Example 4:

Solve the following system of linear equations by using the elimination method.

$$3x + y = -1$$
$$15x + 5y = 2$$

Solution:

We will eliminate the variable x. Multiply the first equation by -5.

$$-15x - 5y = 5$$

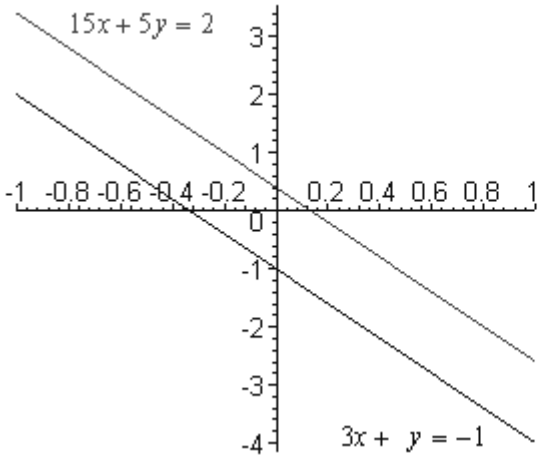
Add the equations together.

$$\begin{array}{r} -15x - 5y = 5 \\ 15x + 5y = 2 \\ \hline 0 = 7 \end{array}$$

$0 = 7$ false statement

There are no values of x and y for which $0 = 7$. The system has no solution.

The lines are parallel as is illustrated in the graph shown below.



Additional Example 5:

Solve the following system of linear equations by using the substitution method.

$$\begin{aligned} x - 2y &= 4 \\ -3x + 6y &= -12 \end{aligned}$$

Solution:

Solve the first equation for x.

$$\begin{aligned} x - 2y &= 4 \\ x - 2y + 2y &= 4 + 2y \\ x &= 4 + 2y \end{aligned}$$

Substitute $4 + 2y$ for x in the second equation.

$$\begin{aligned} -3x + 6y &= -12 \\ -3(4 + 2y) + 6y &= -12 \\ -12 - 6y + 6y &= -12 \\ \underbrace{-12 = -12}_{\text{true statement}} & \end{aligned}$$

The equations given in the system are just different ways of representing the equation of a single line; that is, the lines coincide. There are infinitely many solutions.

Write $x - 2y = 4$ in slope-intercept form by solving for y.

$$\begin{aligned} x - 2y &= 4 \\ x - 2y - x &= 4 - x \\ -2y &= 4 - x \\ \frac{-2y}{-2} &= \frac{4 - x}{-2} \\ y &= \frac{1}{2}x - 2 \end{aligned}$$

The coordinates of any point on the line $y = \frac{1}{2}x - 2$ represent a solution to the system.

The solution is $\left(x, \frac{1}{2}x - 2\right)$, where x can be any real number.

Exercise Set 6.1: 2x2 Linear Systems

Graph each of the following pairs of lines on a single set of axes. Determine whether or not the lines are parallel, intersect in one point, or represent the same line. If the lines intersect in one point, then state their point of intersection.

1. $x - y = -3$
$-3x + y = -7$

2. $2x + y = 7$
$4x + 2y - 14 = 0$

3. $-15x + 3y = 12$
$5x - y = 1$

4. $2x + 3y = -12$
$-x + y = 6$

5. $6x - 2y = 8$
$9x - 3y = 12$

6. $3x + y = 2$
$6x + 2y = -8$

Solve the following systems of linear equations by graphing. If there are infinitely many solutions, give your answer in the form $(x, f(x))$, where $f(x)$ represents the equation of the line in the form $f(x) = mx + b$.

7. $2x + y = 3$
$x - 2y = 14$

8. $5x - y = 0$
$6x - 3y = -9$

9. $2x + y = 1$
$4x + 2y = 10$

10. $-2x + 4y = -10$
$3x - 6y = 15$

11. $2x - y = 8$
$3x + y = 7$

12. $2x + y = 11$
$x + 3y = 3$

13. $3x - 2y = 6$
$6x - 4y = 12$

14. $8x - 2y = 10$
$-12x + 3y = 6$

Solve the following systems of linear equations by using the substitution method. If there are infinitely many solutions, give your answer in the form $(x, f(x))$, where $f(x)$ represents the equation of the line in the form $f(x) = mx + b$.

15. $2x - 7y = -6$
$x = 5y$

16. $2x + 3y = -12$
$y = -2x$

17. $3x - 2y = -5$
$2x + y = 13$

18. $5x - 7y = -4$
$x - 4y = 7$

19. $2x + y = 10$
$6x + 3y = 30$

20. $15x + 3y = -1$
$5x + y = 4$

21. $x - 3y = 3$
$2x + 9y = 11$

22. $y - 2 = 3x$
$9x - 3y = -6$

23. $5x - 4y = -5$
$3x + y = 14$

24. $x + 4y + 2 = 0$
$-2x + 12y = 9$

25. $-2x + y - 5 = 0$
$-6x + 3y = 21$

26. $y - 7 = 2x$
$4x + 3y = 31$

Exercise Set 6.1: 2x2 Linear Systems

Solve the following systems of linear equations by using the elimination method. If there are infinitely many solutions, give your answer in the form $(x, f(x))$, where $f(x)$ represents the equation of the line in the form $f(x) = mx + b$.

27. $4x - 5y = 24$
$3x + 4y = -13$

28. $2x - 5y = 10$
$6x - 15y = 13$

29. $3x + 5y = 11$
$2x + 3y = 7$

30. $3x + 4y = -25$
$2x - 3y = 6$

31. $4x + 8y = -14$
$-2x - 4y = 7$

32. $2x - 3y = 8$
$-5x + 2y = 13$

33. $4x + 3y - 8 = 0$
$3x - 5y = -23$

34. $-6x - 3y = -33$
$4x + 3y = 19$

35. $-8x + 6y = 11$
$12x - 9y = 15$

36. $5x - 2y = 21$
$3x + 8y = 8$

37. $7x + 4y = 11$
$5x + 8y = 4$

38. $8x - 8y - 16 = 0$
$3x - 3y = 6$

Solve the following systems of equations by using the method of your choice.

39. $-4x + 3y = 10$
$7x + y = 20$

40. $-6x + 7y = 20$
$18x - 21y = -10$

41. $4x + 5y - 7 = 0$
$6x - 2y = -18$

42. $x + 4y - 8 = 0$
$2x - 5y = 29$

43. $4x + 10y = 6$
$6x + 15y = 9$

44. $5x + 6y = 14$
$3x + 5y = 7$