Section 2.3: Quadratic Equations

- Solving by Factoring
- Solving by Completing the Square
- Solving by the Quadratic Formula
- The Discriminant

Solving by Factoring

Solving by Factoring

A **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$.

To solve a quadratic equation by factoring, rewrite the equation, if necessary, so that one side is equal to 0 and use the **Zero-Product Property:**

$$ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0$$

Example Problem 1: Solve the equation $2x^2 + 7x + 3 = 0$ by factoring.

Solution:

The equation is in the correct form since the right-hand side is 0. Factor the left-hand side and use the zero-product property.

$2x^2 + 7x + 3 = 0$

$(2x+1)(x+3) = 0$ Factor

$2x+1 = 0$ or $x+3 = 0$ Zero-Product Property

$2x = -1$ or $x = -3$

$x = -\frac{1}{2}$ or $x = -3$

The solutions are $x = -\frac{1}{2}$ and $x = -3$.

Example Problem 2: Solve the equation $x^2 = 5(x+100)$ by factoring.

Solution:
Rewrite the equation so that one side is equal to 0. Next factor and use the zero-product property.

$$x^2 = 5(x+100)$$
$$x^2 = 5x+500$$
$$x^2 - 5x - 500 = 0$$
$$(x-25)(x+20) = 0 \qquad \text{Factor}$$
$$x-25=0 \quad \text{or} \quad x+20=0 \qquad \text{Zero-Product Property}$$
$$x = 25 \quad \text{or} \quad x = -20$$

The solutions are $x = 25$ and $x = -20$.

Additional Example 1:
Solve the equation $x^2 + 8x = -12$ by factoring.

Solution:

$$x^2 + 8x = -12$$
$$x^2 + 8x + 12 = -12 + 12$$
$$x^2 + 8x + 12 = 0$$
$$(x+6)(x+2) = 0$$

Apply the Zero-Product Property

$$x+6=0 \quad \text{or} \quad x+2=0$$

Solve both equations.

First equation:	Second equation:
$x+6=0$	$x+2=0$
$x+6-6=0-6$	$x+2-2=0-2$
$x=-6$	$x=-2$

The solutions are $x = -6$ and $x = -2$.

Additional Example 2:

Solve the equation $5x^2 = -14x + 3$ by factoring.

Solution:

$$5x^2 = -14x+3$$
$$5x^2+14x = -14x+3+14x$$
$$5x^2+14x = 3$$
$$5x^2+14x-3 = 3-3$$
$$5x^2+14x-3 = 0$$
$$(5x-1)(x+3) = 0$$

Apply the Zero-Product Property

$5x-1=0$ or $x+3=0$

Solve both equations.

First equation:

$$5x-1=0$$
$$5x-1+1=0+1$$
$$5x=1$$
$$\frac{\cancel{5}x}{\cancel{5}}=\frac{1}{5}$$
$$x=\frac{1}{5}$$

Second equation:

$$x+3=0$$
$$x+3-3=0-3$$
$$x=-3$$

The solutions are $x=\frac{1}{5}$ and $x=-3$.

Additional Example 3:

Solve the equation $4x^2 = 81$ by factoring.

Solution:

$$4x^2 = 81$$
$$4x^2-81 = 81-81$$
$$4x^2-81 = 0$$
$$(2x+9)(2x-9) = 0$$

Apply the Zero-Product Property

$2x+9=0$ or $2x-9=0$

Solve both equations.

First equation:

$$2x+9=0$$
$$2x+9-9=0-9$$
$$2x=-9$$
$$\frac{\not{2}x}{\not{2}}=\frac{-9}{2}$$
$$x=-\frac{9}{2}$$

Second equation:

$$2x-9=0$$
$$2x-9+9=0+9$$
$$2x=9$$
$$\frac{\not{2}x}{\not{2}}=\frac{9}{2}$$
$$x=\frac{9}{2}$$

The solutions are $x=-\frac{9}{2}$ and $x=\frac{9}{2}$.

Additional Example 4:

Solve the equation $6x^2-x=4x$ by factoring.

Solution:

$$6x^2-x=4x$$
$$6x^2-x-4x=4x-4x$$
$$6x^2-5x=0$$
$$x(6x-5)=0$$

Apply the Zero-Product Property

$x=0$ or $6x-5=0$

From the first equation, we see that one solution is $x=0$. Solve the second equation.

Second equation:

$$6x-5=0$$
$$6x-5+5=0+5$$
$$6x=5$$
$$\frac{\not{6}x}{\not{6}}=\frac{5}{6}$$
$$x=\frac{5}{6}$$

The solutions are $x=0$ and $x=\frac{5}{6}$.

Additional Example 5:

The length of a rectangle is 3 feet longer than its width. If the area of the rectangle is 40 square feet, what is its width?

Solution:

Let $x =$ width of the rectangle (in feet).

Then $x+3 =$ the length of the rectangle.

Write an equation. (width)(length) = area

$$x(x+3)=40$$
$$x^2+3x=40$$
$$x^2+3x-40=40-40$$
$$x^2+3x-40=0$$
$$(x+8)(x-5)=0$$

Apply the Zero-Product Property.

$x+8=0$ or $x-5=0$

Solve both equations.

First equation:

$$x+8=0$$
$$x+8-8=0-8$$
$$x=-8$$

Second equation:

$$x-5=0$$
$$x-5+5=0+5$$
$$x=5$$

We must rule out -8 for the width since the width cannot be negative.

Check the results.

$x=5$

$x+3=5+3=8$

(Width) 5 ft

(Length) 8 ft

Area = (5 ft)(8 ft) = 40 sq ft ✓

Answer the problem in a complete sentence.

The width of the rectangle is 5 feet.

Solving by Completing the Square

Solving by Completing the Square
To solve a quadratic equation $x^2+bx+c=0$ by the method of **completing the square**, rewrite the equation as $x^2+bx=-c$ and make the left-hand side a perfect square. To make x^2+bx a perfect square, add $\left(\frac{b}{2}\right)^2$, the square of half the coefficient of x, to get $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$.
To solve $ax^2+bx+c=0$ (where $a \neq 0$ and $a \neq 1$) by the method of **completing the square**, rewrite the equation as $ax^2+bx=-c$ and then divide both sides by a to get $x^2+\frac{b}{a}x=-\frac{c}{a}$. Next make $x^2+\frac{b}{a}x$ a perfect square by adding $\left(\frac{b}{2a}\right)^2$, the square of half the coefficient of x.

Example Problem 1: Solve the equation $x^2-10x+3=0$ by completing the square.

Solution:

$x^2-10x+3=0$

$x^2-10x=-3$ Subtract 3 from both sides.

$x^2-10x+25=-3+25$ Complete the square by adding $\left(\frac{-10}{2}\right)^2=25$.

$(x-5)^2=22$ Factor the perfect square on the LHS.

$x-5=\pm\sqrt{22}$

$x=5\pm\sqrt{22}$

The solutions are $x=5+\sqrt{22}$ and $x=5-\sqrt{22}$.

Example Problem 2: Solve the equation $2x^2+8x+1=0$ by completing the square.

Solution:

$2x^2+8x+1=0$

$2x^2+8x=-1$ Subtract 1 from both sides.

$x^2+4x=-\frac{1}{2}$ Divide both sides by 2.

$x^2+4x+4=-\frac{1}{2}+4$ Complete the square by adding $\left(\frac{4}{2}\right)^2=4$.

$x^2+4x+4=\frac{7}{2}$ Simplify on the RHS.

$(x+2)^2=\frac{7}{2}$ Factor the perfect square on the LHS.

$x+2=\pm\sqrt{\frac{7}{2}}$

$x+2=\pm\frac{\sqrt{14}}{2}$ $\sqrt{\frac{7}{2}}=\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{7}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{14}}{2}$

$x=-2\pm\frac{\sqrt{14}}{2}$

The solutions are $x=-2-\frac{\sqrt{14}}{2}$ and $x=-2+\frac{\sqrt{14}}{2}$.

Additional Example 1:

Solve the equation $x^2-4x-5=0$ by completing the square.

Solution:

$x^2-4x-5=0$

$x^2-4x-5+5=0+5$

$x^2-4x=5$

$x^2-4x+4=5+4$ Complete the square for x^2-4x by adding $\left(\frac{1}{2}\cdot(-4)\right)^2=4$.

$x^2-4x+4=9$

$(x-2)^2=9$

$x-2=\pm\sqrt{9}$

$x-2=\pm3$

$x-2=3$ or $x-2=-3$

Solve each equation by adding 2 to both sides.

First equation:

$$x-2=3$$
$$x-2+2=3+2$$
$$x=5$$

Second equation:

$$x-2=-3$$
$$x-2+2=-3+2$$
$$x=-1$$

The solutions are $x=5$ and $x=-1$.

Additional Example 2:

Solve the equation $x^2+10x+13=0$ by completing the square.

Solution:

$$x^2+10x+13=0$$
$$x^2+10x+13-13=0-13$$
$$x^2+10x=-13$$
$$x^2+10x+25=-13+25$$

Complete the square for x^2+10x by adding $\left(\frac{1}{2}\cdot 10\right)^2=25$.

$$x^2+10x+25=12$$
$$(x+5)^2=12$$
$$x+5=\pm\sqrt{12}$$
$$x+5=\pm 2\sqrt{3}$$
$$x+5-5=\pm 2\sqrt{3}-5$$
$$x=-5\pm 2\sqrt{3}$$

The solutions are $x=-5-2\sqrt{3}$ and $x=-5+2\sqrt{3}$.

Additional Example 3:

Solve the equation $x^2=1-\frac{2}{3}x$ by completing the square.

Solution:

$$x^2=1-\frac{2}{3}x$$
$$x^2+\frac{2}{3}x=1-\frac{2}{3}x+\frac{2}{3}x$$
$$x^2+\frac{2}{3}x=1$$

$x^2+\frac{2}{3}x+\frac{1}{9}=1+\frac{1}{9}$ Complete the square for $x^2+\frac{2}{3}x$ by adding $\left(\frac{1}{2}\cdot\frac{2}{3}\right)^2=\frac{1}{9}$

$$x^2+\frac{2}{3}x+\frac{1}{9}=\frac{10}{9}$$

$$\left(x+\frac{1}{3}\right)^2=\frac{10}{9}$$

$$x+\frac{1}{3}=\pm\sqrt{\frac{10}{9}}$$

$x+\frac{1}{3}=\pm\frac{\sqrt{10}}{3}$ Simplify the radical. $\sqrt{\frac{10}{9}}=\frac{\sqrt{10}}{\sqrt{9}}=\frac{\sqrt{10}}{3}$

$$x+\frac{1}{3}-\frac{1}{3}=\pm\frac{\sqrt{10}}{3}-\frac{1}{3}$$

$$x=-\frac{1}{3}\pm\frac{\sqrt{10}}{3}$$

The solutions are $x=-\frac{1}{3}-\frac{\sqrt{10}}{3}$ and $x=-\frac{1}{3}+\frac{\sqrt{10}}{3}$.

Additional Example 4:

Solve the equation $2x^2+8x+3=0$ by completing the square.

Solution:

$$2x^2+8x+3=0$$

$$2x^2+8x+3-3=0-3$$

$$2x^2+8x=-3$$

$$\frac{2x^2+8x}{2}=\frac{-3}{2}$$

$$x^2+4x=-\frac{3}{2}$$

$x^2+4x+4=-\frac{3}{2}+4$ Complete the square for x^2+4x by adding $\left(\frac{1}{2}\cdot 4\right)^2=4$.

$$x^2+4x+4=\frac{5}{2}$$

$$(x+2)^2=\frac{5}{2}$$

$$x+2=\pm\sqrt{\frac{5}{2}}$$

$$x+2=\pm\frac{\sqrt{10}}{2}$$ Simplify the radical. $\sqrt{\frac{5}{2}}=\frac{\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{5}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{10}}{2}$

$$x+2-2=\pm\frac{\sqrt{10}}{2}-2$$

$$x=-2\pm\frac{\sqrt{10}}{2}$$

The solutions are $x=-2-\frac{\sqrt{10}}{2}$ and $x=-2+\frac{\sqrt{10}}{2}$.

Solving by the Quadratic Formula

Solving by the Quadratic Formula

The solutions of the equation $ax^2+bx+c=0$, where $a\neq 0$, can be found by using the **quadratic formula:**

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example Problem 1: Solve the equation $5x^2-2x-2=0$ by the quadratic formula.

Solution:

Substitute $a=5$, $b=-2$, and $c=-2$ into the quadratic formula.

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(5)(-2)}}{2(5)}=\frac{2\pm\sqrt{4+40}}{10}=\frac{2\pm\sqrt{44}}{10}=\frac{2\pm2\sqrt{11}}{10}=\frac{1\pm\sqrt{11}}{5}$$

The solutions are $x=\frac{1+\sqrt{11}}{5}$ and $x=\frac{1-\sqrt{11}}{5}$.

Example Problem 2: Solve the equation $4x^2 - x = 3$ by the quadratic formula.

Solution:
Rewrite the equation so that the right-hand side is 0.

$$4x^2 - x = 3$$
$$4x^2 - x - 3 = 0$$

Substitute $a = 4$, $b = -1$, and $c = -3$ into the quadratic formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-3)}}{2(4)} = \frac{1 \pm \sqrt{1+48}}{8} = \frac{1 \pm \sqrt{49}}{8} = \frac{1 \pm 7}{8}$$

$$x = \frac{1-7}{8} = \frac{-6}{8} = -\frac{3}{4}$$
or
$$x = \frac{1+7}{8} = \frac{8}{8} = 1$$

The solutions are $x = -\frac{3}{4}$ and $x = 1$.

Additional Example 1:

Solve the equation $x^2 - 4x = 5$ by the quadratic formula.

Solution:

To solve the equation using the quadratic formula, rewrite the equation so that the RHS is equal to 0.

$$x^2 - 4x = 5$$
$$x^2 - 4x - 5 = 0$$

Substitute $a = 1$, $b = -4$, and $c = -5$ into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+20}}{2}$$
$$= \frac{4 \pm \sqrt{36}}{2}$$
$$= \frac{4 \pm 6}{2}$$

$$x = \frac{4+6}{2} = 5 \quad \text{or} \quad x = \frac{4-6}{2} = -1$$

The solutions are $x = 5$ and $x = -1$.

Additional Example 2:

Solve the equation $x^2 + 10x + 13 = 0$ by the quadratic formula.

Solution:

Substitute $a = 1$, $b = 10$, and $c = 13$ into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{(10)^2 - 4(1)(13)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{100-52}}{2}$$
$$= \frac{-10 \pm \sqrt{48}}{2}$$
$$= \frac{-10 \pm 4\sqrt{3}}{2}$$
$$= -5 \pm 2\sqrt{3}$$

The solutions are $x = -5 - 2\sqrt{3}$ and $x = -5 + 2\sqrt{3}$.

Additional Example 3:

Solve the equation $2x^2+8x+3=0$ by the quadratic formula.

Solution:

Substitute $a=2$, $b=8$, and $c=3$ into the quadratic formula.

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-8\pm\sqrt{8^2-4(2)(3)}}{2(2)} \\ &= \frac{-8\pm\sqrt{64-24}}{4} \\ &= \frac{-8\pm\sqrt{40}}{4} \\ &= \frac{-8\pm2\sqrt{10}}{4} \\ &= -2\pm\frac{1}{2}\sqrt{10} \end{aligned}$$

The solutions are $x=-2-\frac{1}{2}\sqrt{10}$ and $x=-2+\frac{1}{2}\sqrt{10}$.

Additional Example 4:

The sum of the squares of two positive consecutive even integers is 1460. What is the smaller integer?

Solution:

Let x = the smaller integer
Then $x+2$ = the larger integer

Write an equation.
(square of the smaller integer) + (square of the larger integer) = 1460

$$\begin{aligned} x^2+(x+2)^2 &= 1460 \\ x^2+x^2+4x+4 &= 1460 \\ 2x^2+4x+4 &= 1460 \\ 2x^2+4x+4-1460 &= 1460-1460 \\ 2x^2+4x-1456 &= 0 \end{aligned}$$

Substitute $a = 2,\ b = 4,$ and $c = -1456$ into the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4(2)(-1456)}}{2(2)} \\ &= \frac{-4 \pm \sqrt{16 + 11648}}{4} \\ &= \frac{-4 \pm \sqrt{11664}}{4} \\ &= \frac{-4 \pm 108}{4} \end{aligned}$$

Determine the solutions to the quadratic equation $2x^2 + 4x - 1456 = 0$.

$$x = \frac{-4 - 108}{4} = \frac{-112}{4} = -28 \quad \text{or} \quad x = \frac{-4 + 108}{4} = \frac{104}{4} = 26$$

We must rule out the solution $x = -28$ since the problem calls for positive integers.

Check the results.

$x = 26$
$x + 2 = 28$
$(26)^2 = 676$
$(28)^2 = 784$
$676 + 784 = 1460$ ✓

Answer the problem in a complete sentence.

The smaller integer is 26.

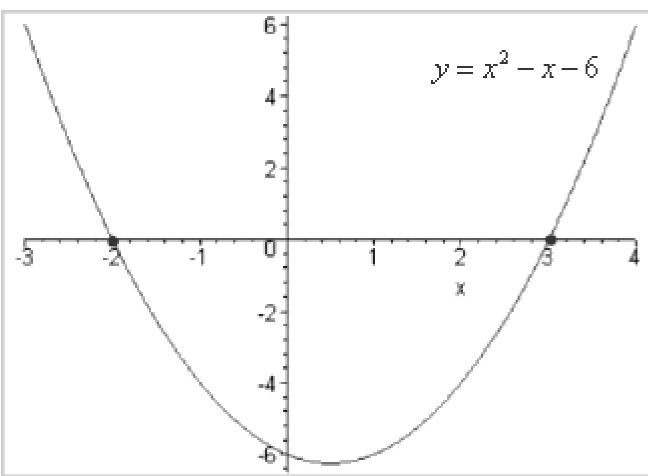
The Discriminant

The Discriminant
The **discriminant** of the equation $ax^2+bx+c=0$ $(a \neq 0)$ is given by $$D=b^2-4ac.$$ If $D>0$, then the equation $ax^2+bx+c=0$ has two distinct real solutions. If $D=0$, then the equation $ax^2+bx+c=0$ has exactly one real solution. If $D<0$, then the equation $ax^2+bx+c=0$ has no real solution.

Example Problem 1: Without solving the equation $x^2-x-6=0$, determine how many real solutions the equation has.

Solution:
Find the discriminant.
$D=(-1)^2-4(1)(-6)=1+24=25>0$. The equation has two distinct real solutions.

The figure below shows why the equation $x^2-x-6=0$ has two real solutions.

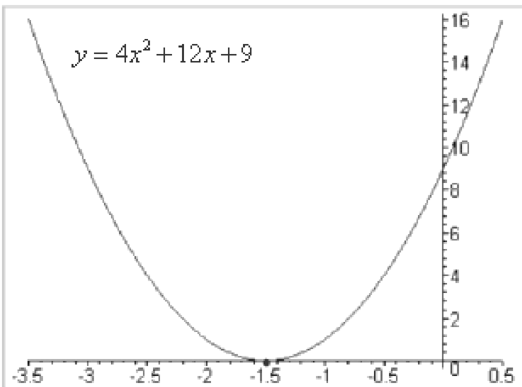


The two solutions to the equation $x^2-x-6=0$ are $x=-2$ and $x=3$, which are the x-intercepts of the graph of the equation $y=x^2-x-6$.

Example Problem 2: Without solving the equation $4x^2 + 12x + 9 = 0$, determine how many real solutions the equation has.

Solution:
Find the discriminant.

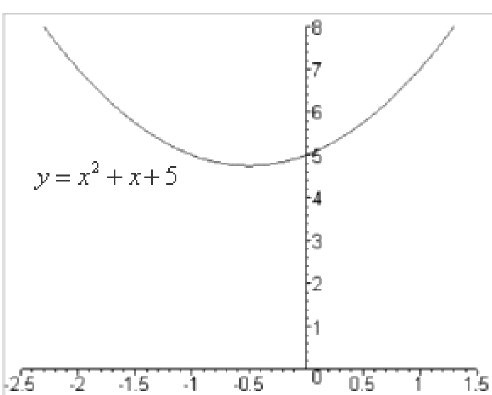
$D = (12)^2 - 4(4)(9) = 144 - 144 = 0$. The equation has exactly one real solution.
The figure below shows why the equation $4x^2 + 12x + 9 = 0$ has one real solution.



The solution to the equation $4x^2 + 12x + 9 = 0$ is $x = -1.5$, which is the x-intercept of the graph of the equation $y = 4x^2 + 12x + 9$.

Example Problem 3: Without solving the equation $x^2 + x + 5 = 0$, determine how many real solutions the equation has.

Solution:
Find the discriminant.
$D = 1^2 - 4(1)(5) = 1 - 20 = -19 < 0$. The equation has no real solution.
The figure below shows why the equation $x^2 + x + 5 = 0$ has no real solution.



The equation $x^2 + x + 5 = 0$ has no real solution. The graph of the equation $y = x^2 + x + 5$ does not cross the x-axis. The graph has no x-intercepts.

Additional Example 1:

Use the discriminant to determine the number of real solutions to the equation $4x^2 = 5 - 3x$. (Do not solve the equation.)

Solution:

Rewrite the equation so that the RHS is equal to 0.

$$\begin{aligned} 4x^2 &= 5 - 3x \\ 4x^2 + 3x &= 5 - 3x + 3x \\ 4x^2 + 3x &= 5 \\ 4x^2 + 3x - 5 &= 5 - 5 \\ 4x^2 + 3x - 5 &= 0 \end{aligned}$$

Substitute $a = 4,\ b = 3,$ and $c = -5$ into the discriminant.

$$\begin{aligned} D &= b^2 - 4ac \\ &= 3^2 - 4(4)(-5) \\ &= 9 + 80 \\ &= 89 \end{aligned}$$

The equation has two distinct real solutions since $D = 89 > 0$.

Additional Example 2:

Use the discriminant to determine the number of real solutions to the equation $x^2 + 6x = -13$. (Do not solve the equation.)

Solution:

Rewrite the equation so that the RHS is equal to 0.

$$\begin{aligned} x^2 + 6x &= -13 \\ x^2 + 6x + 13 &= -13 + 13 \\ x^2 + 6x + 13 &= 0 \end{aligned}$$

Substitute $a = 1,\ b = 6,$ and $c = 13$ into the discriminant.

$$\begin{aligned} D &= b^2 - 4ac \\ &= 6^2 - 4(1)(13) \\ &= 36 - 52 \\ &= -16 \end{aligned}$$

The equation has no real solutions since $D = -16 < 0$.

Additional Example 3:

Use the discriminant to determine the number of real solutions to the equation $25x^2 + 60x = -36$. (Do not solve the equation.)

Solution:

Rewrite the equation so that the RHS is equal to 0.

$$25x^2 + 60x = -36$$
$$25x^2 + 60x + 36 = -36 + 36$$
$$25x^2 + 60x + 36 = 0$$

Substitute $a = 25$, $b = 60$, and $c = 36$ into the discriminant.

$$\begin{aligned} D &= b^2 - 4ac \\ &= (60)^2 - 4(25)(36) \\ &= 3600 - 3600 \\ &= 0 \end{aligned}$$

The equation has exactly one real solution since $D = 0$.

Additional Example 4:

Use the discriminant to find all values of k so that the equation $2kx^2 + 8x + k = 0$ has exactly one real solution.

Solution:

Substitute $a = 2k$, $b = 8$, and $c = k$ into the discriminant.

$$\begin{aligned} D &= b^2 - 4ac \\ &= 8^2 - 4(2k)(k) \\ &= 64 - 8k^2 \end{aligned}$$

The equation has exactly one solution when $D = 0$. Set $D = 0$ and solve the resulting equation for k.

$$0 = 64 - 8k^2$$
$$0 + 8k^2 = 64 - 8k^2 + 8k^2$$
$$8k^2 = 64$$
$$\frac{\cancel{8}k^2}{\cancel{8}} = \frac{64}{8}$$
$$k^2 = 8$$
$$k = \pm\sqrt{8}$$
$$k = \pm 2\sqrt{2}$$

Exercise Set 2.3: Quadratic Equations

Solve the following equations by factoring.

1. $x^2 - 10x + 21 = 0$
2. $x^2 + 13x + 40 = 0$
3. $x(x-2) = 35$
4. $x(x+8) = 20$
5. $x^2 + 14x = 72$
6. $x^2 - 60 = 11x$
7. $2x^2 - 7x - 15 = 0$
8. $3x^2 - 7x + 4 = 0$
9. $6x^2 + 17x = -12$
10. $10x^2 - 7x = 6$
11. $x^2 - 25 = 0$
12. $x^2 - 49 = 0$
13. $4x^2 - 9 = 0$
14. $36x^2 - 25 = 0$

Solve the following equations by factoring. To simplify the process, remember to first factor out the Greatest Common Factor (GCF).

15. $x^2 - 8x = 0$
16. $x^2 + 10x = 0$
17. $-3x^2 + 21x = 0$
18. $5x^2 - 30x = 0$
19. $3x^2 - 12 = 0$
20. $-7x^2 + 7 = 0$
21. $-5x^2 - 15x + 90 = 0$
22. $-4x^2 + 20x + 24 = 0$
23. $80x^2 + 230x - 30 = 0$
24. $12x^2 - 75x + 18 = 0$

Find all real solutions of the following equations by completing the square.

25. $x^2 + 8x + 12 = 0$
26. $x^2 - 6x - 40 = 0$
27. $x(x-10) = -18$
28. $x^2 + 4x = -8$
29. $x^2 + 14x + 60 = 0$
30. $x(x+12) = -28$
31. $x^2 + 5x - 5 = 0$
32. $x^2 - 7x + 2 = 0$
33. $3x^2 - 12x - 5 = 0$
34. $2x^2 + 12x = -3$
35. $-4x^2 + 8x = -6$
36. $-5x^2 - 40x - 78 = 0$
37. $3x^2 - 5x = -2$
38. $8x^2 - 6x - 5 = 0$

Find all real solutions of the following equations by using the quadratic formula.

39. $x^2 + 5x + 2 = 0$
40. $x^2 - 7x + 3 = 0$
41. $x^2 - 6x + 8 = 0$
42. $x^2 - 2x = 15$
43. $x^2 + 5x + 7 = 0$
44. $x^2 - 8x = -16$
45. $x^2 + 10x + 25 = 0$
46. $4x^2 - 6x + 5 = 0$
47. $3x^2 - 5x = -1$
48. $2x^2 + 3x - 6 = 0$
49. $-2x^2 - 6x - 3 = 0$
50. $-5x^2 + 8x - 1 = 0$

Exercise Set 2.3: Quadratic Equations

Find all real solutions of the following equations by using a method of your choice.

51. $x^2 - 10x + 16 = 0$

52. $x^2 - 6x - 8 = 0$

53. $2x^2 - 4x = 5$

54. $3x^2 + 12x = 36$

Use the discriminant to determine the number of real solutions of each equation. (Do not solve the equation.)

55. $x^2 - 5x + 2 = 0$

56. $2x^2 + 3x + 7 = 0$

57. $x^2 + 4x = -4$

58. $3x^2 - 6x = -3$

59. $x^2 + 4x = -5$

60. $3x^2 + 7x - 2 = 0$

61. $x^2 - cx - d = 0$, where $d > 0$

62. $3x^2 - rx + t = 0$, where $t < 0$

Use the discriminant to find all values of *k* so that each of the following equations has exactly one solution.

63. $kx^2 + 6x + 3k = 0$

64. $4x^2 + kx + 49 = 0$

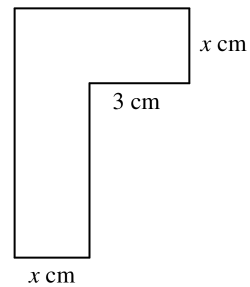
For each of the following problems:
- **(a) Model the situation by writing appropriate equation(s).**
- **(b) Solve the equation(s) and then answer the question posed in the problem.**

65. The length of a rectangular frame is 5 cm longer than its width. If the area of the frame is 36 cm^2, find the length and width of the frame.

66. The height of a right triangle is 4 inches longer than its base. If its diagonal measures 20 inches, find the base and the height of the triangle.

67. The height of a triangle is 3 cm shorter than its base. If the area of the triangle is 90 cm^2, find the base and height of the triangle.

68. Find x if the area of the figure below is 26cm^2. *(Note that the figure may not be drawn to scale.)*



69. Find two consecutive odd integers that have a product of 255.

70. Find two numbers that have a sum of 39 and a product of 350.

71. John can paint the fence in 3 hours less time than Chris. If it takes them $\frac{4}{5}$ of an hour when working together, how long does it take each of them to paint the fence individually?

72. Marie takes 1 hour more time to clean the house than Tina. If it takes them $1\frac{1}{5}$ hours when working together, how long would it take each of them to clean the house individually?