Chapter 2
Solving Equations and Inequalities

Section 2.1: Linear Equations

- ➢ Solving an Equation

Solving an Equation

Solving an Equation
To solve an equation in the variable x using the **algebraic method** is to use the rules of algebra to isolate the unknown x on one side of the equation.
To solve an equation in the variable x using the **graphical method** is to move all terms to one side of the equation and set those terms equal to y. Sketch the graph to find the values of x where $y = 0$.

Algebraic Method: Solve the equation $-x - 1 = 2x + 2$ by the algebraic method.

Isolate x on one side of the equation.

$$-x - 1 = 2x + 2$$
$$-3x - 1 = 2$$
$$-3x = 3$$
$$x = -1$$

The solution is $x = -1$.

Graphical Method: Solve the equation $-x-1= 2x+2$ by the graphical method.

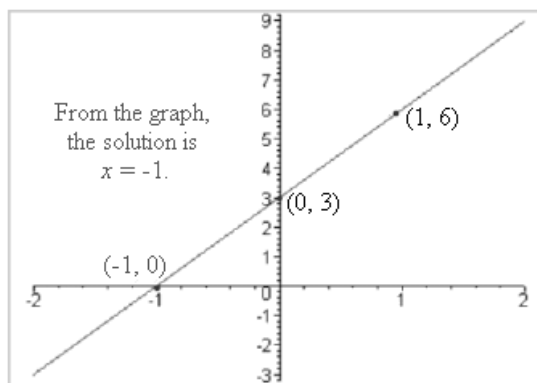Move all terms to one side of the equation.

$$-x-1= 2x+2$$
$$-1= 3x+2$$
$$0 = 3x+3$$

Set $y= 3x+3$ and graph. The graph is a line with slope 3 and y-intercept 3. Use the y-intercept 3 to plot the point (0, 3). Use the slope $3 = \dfrac{3}{1}$ to move 3 units up and 1 unit to the right to locate another point on the graph. Draw a line through the two points and extend the line until it crosses the x-axis.
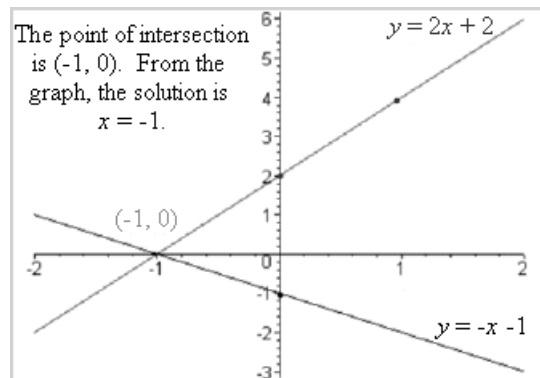
From the graph, the solution is $x = -1$.

(1, 6)

(0, 3)

(-1, 0)

Graphical Method (Another Approach): Solve the equation $-x-1= 2x+2$ by the graphical method.

Instead of moving all terms to one side of the equation, graph two equations:

$$y =-x-1 \quad \text{and} \quad y= 2x+2$$

The solution is the x-coordinate of the point of intersection of the two lines.

The point of intersection is (-1, 0). From the graph, the solution is $x = -1$.

$y= 2x +2$

(-1, 0)

$y= -x -1$

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Example Problem: Solve the equation $x^2 - 4 = 0$ both algebraically and graphically.
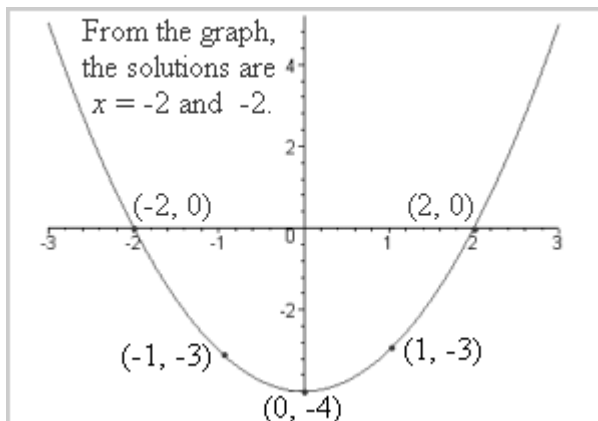
Solution:

Algebraic Method: Isolate x on one side of the equation.

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

The solutions are $x = 2$ and $x = -2$.

Graphical Method: Set $y = x^2 - 4$. The graph is a parabola. Make a table of values and sketch the graph.

Additional Example 1:

Solve the equation $6x + 3 = 4x + 33$.

Solution:

$$6x + 3 = 4x + 33$$
$$6x + 3 - 4x = 4x + 33 - 4x$$
$$2x + 3 = 33$$
$$2x + 3 - 3 = 33 - 3$$
$$2x = 30$$
$$\frac{2x}{2} = \frac{30}{2}$$
$$x = 15$$

Additional Example 2:

Solve the equation $2(x-3)+7=-4(x+1)+3$.

Solution:

$$2(x-3)+7 = -4(x+1)+3$$
$$2x-6+7 = -4x-4+3$$
$$2x+1 = -4x-1$$
$$2x+1+4x = -4x-1+4x$$
$$6x+1 = -1$$
$$6x+1-1 = -1-1$$
$$6x = -2$$
$$\frac{\cancel{6}x}{\cancel{6}} = \frac{-2}{6}$$
$$x = -\frac{1}{3}$$

Additional Example 3:

Solve the equation $2x+\dfrac{x}{12}+\dfrac{x-3}{6}=x$.

Solution:

$$2x+\frac{x}{12}+\frac{x-3}{6} = x$$

We first multiply both sides of the equation by 12 to clear the equation of fractions. Then solve as usual.

$$12(2x)+\frac{12x}{12}+\frac{12(x-3)}{6} = 12x$$
$$24x+x+2(x-3) = 12x$$
$$24x+x+2x-6 = 12x$$
$$27x-6 = 12x$$
$$27x-6-12x = 12x-12x$$
$$15x-6 = 0$$
$$15x-6+6 = 0+6$$
$$15x = 6$$
$$\frac{\cancel{15}x}{\cancel{15}} = \frac{6}{15}$$
$$x = \frac{6}{15}$$
$$x = \frac{2}{5}$$

Additional Example 4:

Solve the equation $\frac{1}{2}x + 1 = 3$ graphically.

Solution:

Move all terms to one side of the equation. To do this, subtract 3 from both sides.
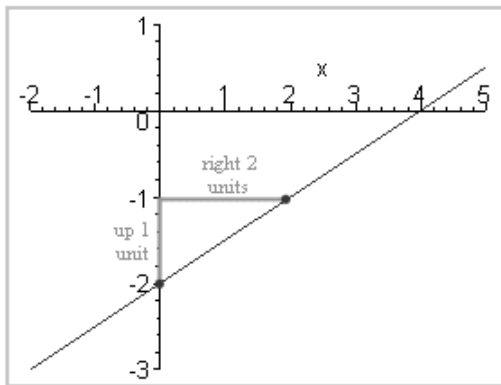
$\frac{1}{2}x + 1 = 3$

$\frac{1}{2}x - 2 = 0$
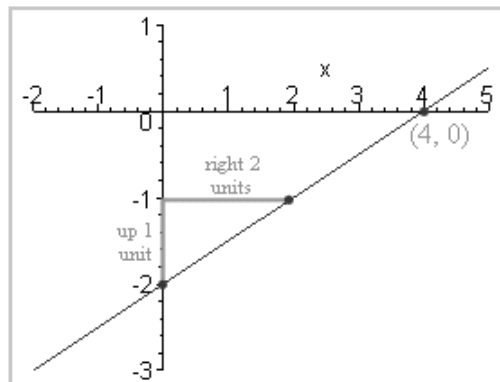
Set the LHS equal to y.

$y = \frac{1}{2}x - 2$

Sketch the graph of the equation $y = \frac{1}{2}x - 2$. The graph is a line with slope $\frac{1}{2}$ and y-intercept -2. Extend the line until it crosses the x-axis.

Look on the graph to find the value of x where $y = 0$.

The solution is $x = 4$.

Exercise Set 2.1: Linear Equations

Solve the following linear equations algebraically.

1. $-3x+7=13$

2. $5x-11=6$

3. $2x+3=4x-7$

4. $5x+2=-4x-6$

5. $3(x+2)+9=-5(x-8)-3$

6. $-4(x+3)-5=2(x-4)+3$

7. $3(2-5x)=-4(7x-3)$

8. $7+2(3-8x)=4-6(1+5x)$

9. $\dfrac{x}{5}=-7$

10. $\dfrac{x}{3}=10$

11. $\frac{2}{5}x-1=7$

12. $-\frac{3}{4}x-7=2$

13. $\frac{5}{3}(x-7)=\frac{2}{5}x+1$

14. $\frac{4}{9}x-12=-\frac{1}{6}(x-12)-3$

15. $2+\dfrac{2x}{3}-\dfrac{x+5}{7}=3x$

16. $x+\dfrac{x+7}{8}+\dfrac{5x}{6}=\dfrac{-1}{12}$

Solve the following nonlinear equations algebraically. (Note: Even though these equations are not linear, some steps in the solution will contain a linear equation.)

17. $\dfrac{2}{5x}+\dfrac{7}{4x}=-3$

18. $-\dfrac{7}{6x}+\dfrac{5}{4x}=2$

19. $\dfrac{4}{x-5}-\dfrac{1}{3}=\dfrac{-8}{3x-15}$

20. $\dfrac{7}{x+2}+\dfrac{4}{3x+6}=-\dfrac{5}{3}$

Solve each of the following linear equations graphically, and then check your answer algebraically.

21. $2x-5=3$

22. $-3x-1=5$

23. $\frac{2}{3}x-3=1$

24. $-\frac{3}{5}x-1=-4$

25. $2x+3=\frac{3}{4}x-2$

26. $\frac{2}{3}x-5=-\frac{1}{2}x+2$