Section 1.3: Graphing Equations

- Graphs
- Intercepts of Graphs
- Symmetry
- Circles

Graphs

Graphs

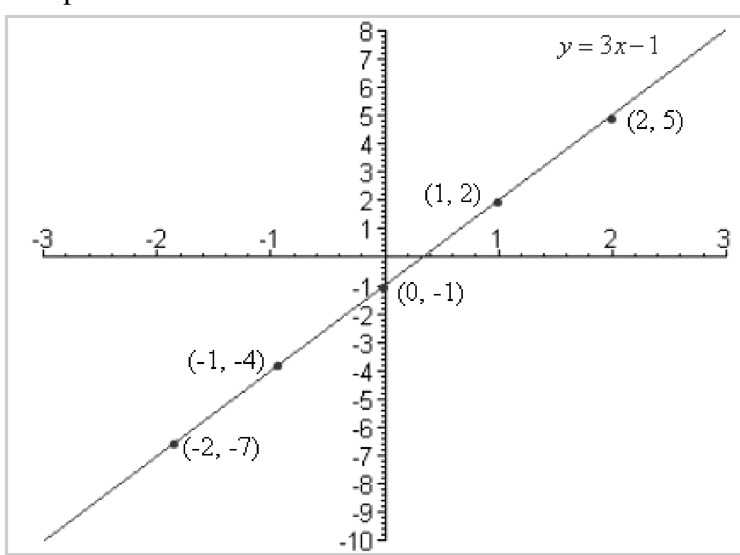
The **graph** of an equation in the variables x and y is the set of all points (x, y) in the plane that satisfy the equation. A point (x, y) will satisfy an equation in x and y if the equation is true when the coordinates of the point are substituted into the equation.

Example Problem 1:

Sketch the graph of the equation $y = 3x - 1$.

Solution:
Make a table of values and then graph the points from the table.

x	y	(x, y)
-2	$3(-2) - 1 = -7$	$(-2, -7)$
-1	$3(-1) - 1 = -4$	$(-1, -4)$
0	$3(0) - 1 = -1$	$(0, -1)$
1	$3(1) - 1 = 2$	$(1, 2)$
2	$3(2) - 1 = 5$	$(2, 5)$

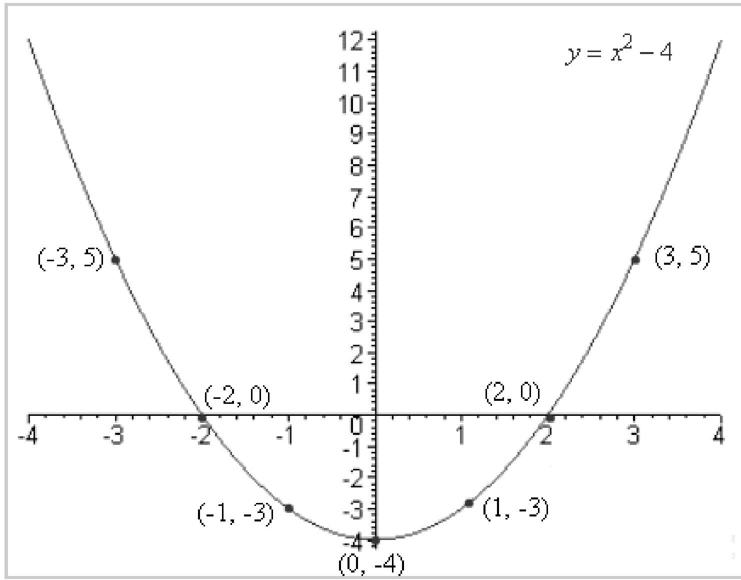


Example Problem 2:

Sketch the graph of the equation $y = x^2 - 4$.

Solution:
Make a table of values and then graph the points from the table.

x	y	(x, y)
-3	$(-3)^2 - 4 = 5$	$(-3, 5)$
-2	$(-2)^2 - 4 = 0$	$(-2, 0)$
-1	$(-1)^2 - 4 = -3$	$(-1, -3)$
0	$0^2 - 4 = -4$	$(0, -4)$
1	$1^2 - 4 = -3$	$(1, -3)$
2	$2^2 - 4 = 0$	$(2, 0)$
3	$3^2 - 4 = 5$	$(3, 5)$

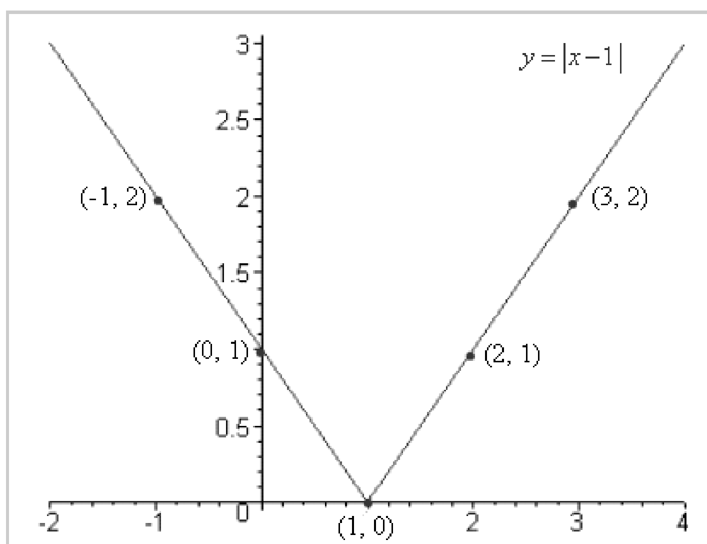


Example Problem 3:

Sketch the graph of the equation $y = |x - 1|$.

Solution:
Make a table of values and then graph the points from the table.

x	y	(x, y)
-1	$\|-1-1\| = 2$	$(-1, 2)$
0	$\|0-1\| = 1$	$(0, 1)$
1	$\|1-1\| = 0$	$(1, 0)$
2	$\|2-1\| = 1$	$(2, 1)$
3	$\|3-1\| = 2$	$(3, 2)$



Additional Example 1:

Determine which of the points $(1,-2)$, $(0,-1)$, $(3,3)$ are on the graph of the equation $1+xy=2y+3$.

Solution:

To determine if the point $(1,-2)$ is on the graph of the equation $1+xy=2y+3$, substitute $x=1$ and $y=-2$ into the equation.

$$\begin{aligned} 1+xy &= 2y+3 \\ 1+(1)(-2) &\overset{?}{=} 2(-2)+3 \\ 1+(-2) &\overset{?}{=} -4+3 \\ -1 &= -1 \end{aligned}$$

The point $(1,-2)$ lies on the graph of the equation $1+xy=2y+3$ since for $x=1$ and $y=-2$, $\text{LHS}=-1=\text{RHS}$.

To determine if the point $(0,-1)$ is on the graph of the equation $1+xy=2y+3$, substitute $x=0$ and $y=-1$ into the equation.

$$\begin{aligned} 1+xy &= 2y+3 \\ 1+(0)(-1) &\overset{?}{=} 2(-1)+3 \\ 1+0 &\overset{?}{=} -2+3 \\ 1 &= 1 \end{aligned}$$

The point $(0,-1)$ lies on the graph of the equation $1+xy=2y+3$ since for $x=0$ and $y=-1$, $\text{LHS}=1=\text{RHS}$.

To determine if the point $(3,3)$ is on the graph of the equation $1+xy=2y+3$, substitute $x=3$ and $y=3$ into the equation.

$$\begin{aligned} 1+xy &= 2y+3 \\ 1+(3)(3) &\overset{?}{=} 2(3)+3 \\ 1+9 &\overset{?}{=} 6+3 \\ 10 &\neq 9 \end{aligned}$$

The point $(3,3)$ does not lie on the graph of the equation $1+xy=2y+3$ since for $x=3$ and $y=3$, $\text{LHS}=10\neq 9=\text{RHS}$.

Additional Example 2:

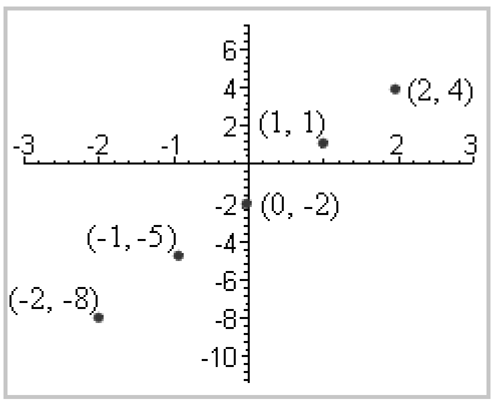
Sketch the graph of the equation $y = 3x - 2$.

Solution:

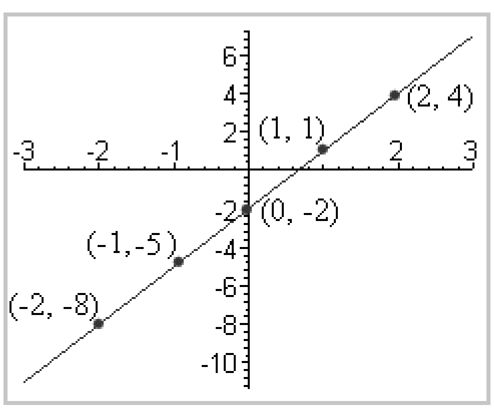
Make a table of values.

x	$y = 3x - 2$	(x, y)
-2	$3(-2) - 2 = -8$	$(-2, -8)$
-1	$3(-1) - 2 = -5$	$(-1, -5)$
0	$3(0) - 2 = -2$	$(0, -2)$
1	$3(1) - 2 = 1$	$(1, 1)$
2	$3(2) - 2 = 4$	$(2, 4)$

Plot the points shown in the table of values.



The points lie on a line. Sketch the graph by drawing a line through the plotted points.



Additional Example 3:

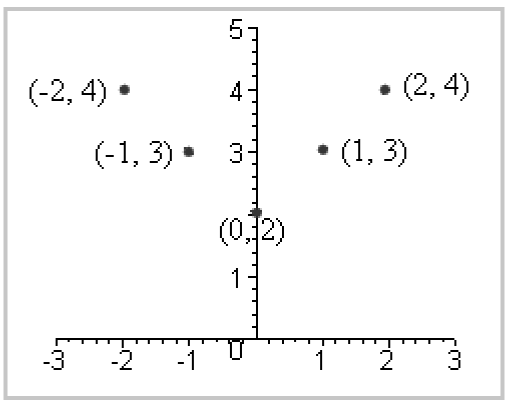
Sketch the graph of the equation $y = |x| + 2$.

Solution:

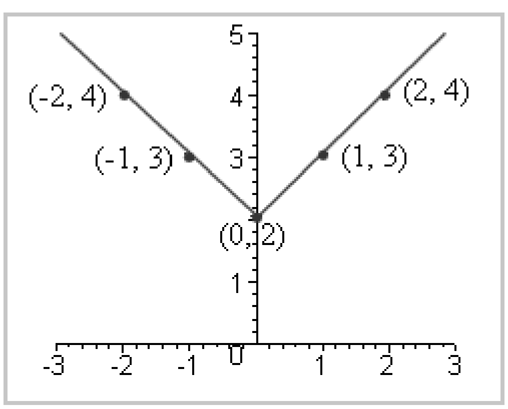
Make a table of values.

x	y	(x, y)
-2	$|-2| + 2 = 4$	$(-2, 4)$
-1	$|-1| + 2 = 3$	$(-1, 3)$
0	$|0| + 2 = 2$	$(0, 2)$
1	$|1| + 2 = 3$	$(1, 3)$
2	$|2| + 2 = 4$	$(2, 4)$

Plot the points shown in the table of values.



Plot additional points if necessary to sketch the graph.



Additional Example 4:

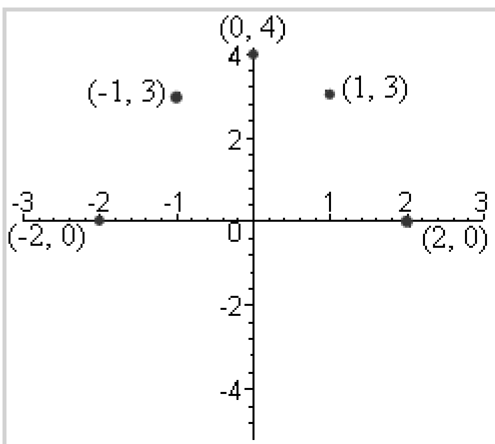
Sketch the graph of the equation $y = 4 - x^2$.

Solution:

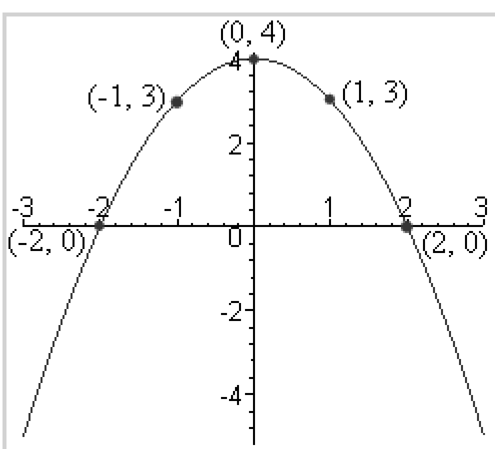
Make a table of values.

x	y	(x, y)
-2	$4-(-2)^2=0$	$(-2, 0)$
-1	$4-(-1)^2=3$	$(-1, 3)$
0	$4-0^2=4$	$(0, 4)$
1	$4-1^2=3$	$(1, 3)$
2	$4-2^2=0$	$(2, 0)$

Plot the points shown in the table of values.



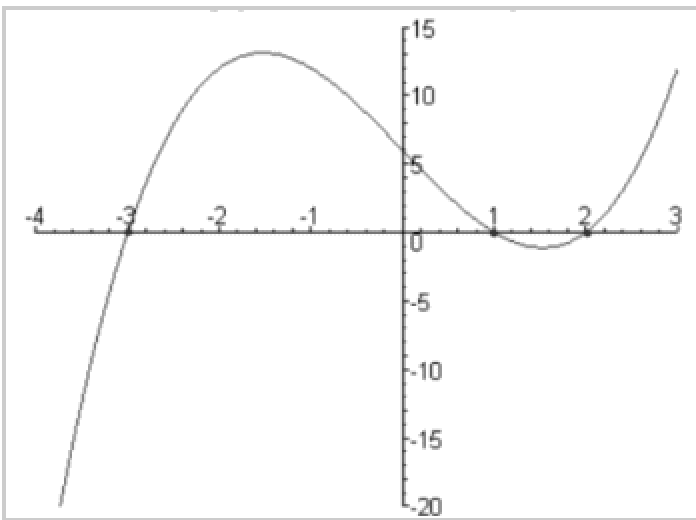
Plot additional points if necessary to sketch the graph.



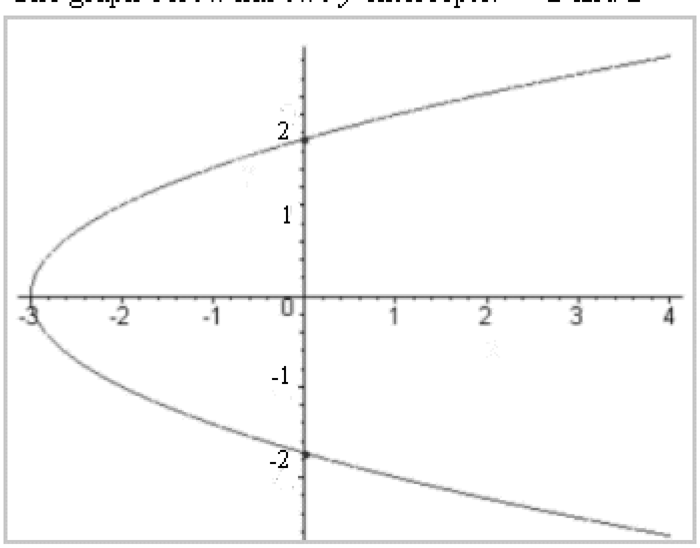
Intercepts of Graphs

Intercepts of Graphs
An **x-intercept** of a graph is the x-coordinate of a point where the graph intersects the x-axis. To find the **x-intercepts** of a graph set $y = 0$ into the equation of the graph and solve for x. A **y-intercept** of a graph is the y-coordinate of a point where the graph intersect the y-axis. To find the **y-intercepts** of a graph set $x = 0$ into the equation of the graph and solve for y.

The graph below has three x-intercepts: $-3, 1,$ and 2



The graph below has two y-intercepts: -2 and 2



Example Problem: Find the x- and y-intercepts of the graph of the equation $y = 4 - x^2$.

Solution:
Find the x-intercepts by substituting $y = 0$ into the equation $y = 4 - x^2$ and solving for x.

$y = 4 - x^2$

$0 = 4 - x^2$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

The x-intercepts are ± 2.

Find the y-intercepts by substituting $x = 0$ into the equation $y = 4 - x^2$ and solving for y.

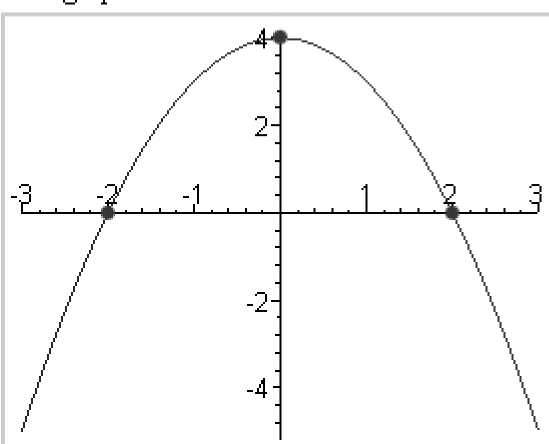
$y = 4 - x^2$

$y = 4 - 0^2$

$y = 4$

The y-intercept is 4.

The graph is shown below.



Additional Example 1:
Find the x- and y-intercepts of the graph of the equation $2x - 3y = 6$.

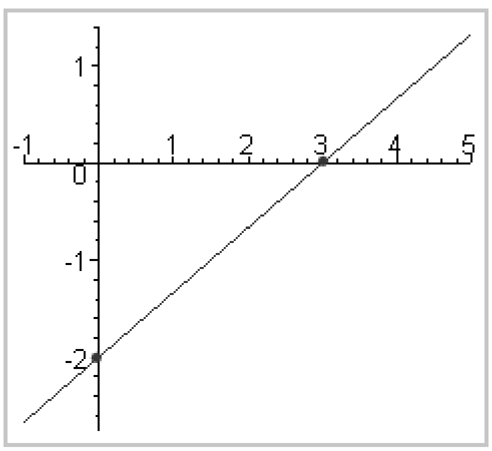
Solution:
To find the x-intercepts of the graph of the equation, substitute $y = 0$ into the equation and solve for x.

$$2x-3y=6$$
$$2x-3(0)=6$$
$$2x=6$$
$$x=3$$

To find the y-intercepts of the graph of the equation, substitute $x=0$ into the equation and solve for y.

$$2x-3y=6$$
$$2(0)-3y=6$$
$$-3y=6$$
$$y=-2$$

The x-intercepts is 3 and the y-intercept is -2. The graph of the equation $2x-3y=6$ is shown below.



Additional Example 2:
Find the x- and y-intercepts of the graph of the equation $x=1-y^2$.

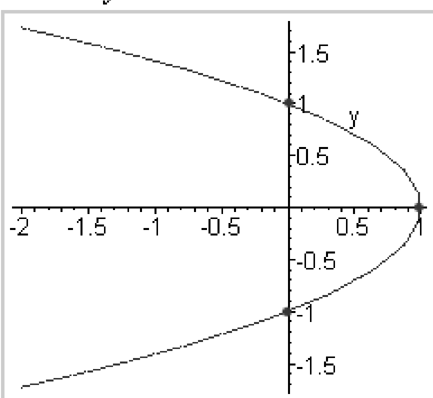
Solution:
To find the x-intercepts of the graph of the equation, substitute $y=0$ into the equation and solve for x.

$$x=1-y^2$$
$$x=1-0^2$$
$$x=1$$

To find the y-intercepts of the graph of the equation, substitute $x = 0$ into the equation and solve for y.

$$x = 1 - y^2$$
$$0 = 1 - y^2$$
$$y^2 = 1$$
$$y = \pm\sqrt{1}$$
$$y = \pm 1$$

The x-intercept is 1 and the y-intercepts are -1 and 1. The graph of the equation $x = 1 - y^2$ is shown below.



Additional Example 3:

Find the x- and y-intercepts of the graph of the equation $y = \frac{1}{2}x^2 - 2$.

Solution:

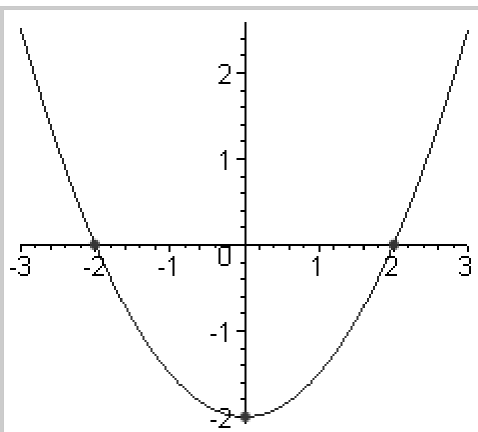
To find the x-intercepts of the graph of the equation, substitute $y = 0$ into the equation and solve for x.

$$y = \frac{1}{2}x^2 - 2$$
$$0 = \frac{1}{2}x^2 - 2$$
$$2 = \frac{1}{2}x^2$$
$$4 = x^2$$
$$\pm\sqrt{4} = x$$
$$\pm 2 = x$$

To find the y-intercepts of the graph of the equation, substitute $x = 0$ into the equation and solve for y.

$$y = \frac{1}{2}x^2 - 2$$
$$y = \frac{1}{2} \cdot 0^2 - 2$$
$$y = 0 - 2$$
$$y = -2$$

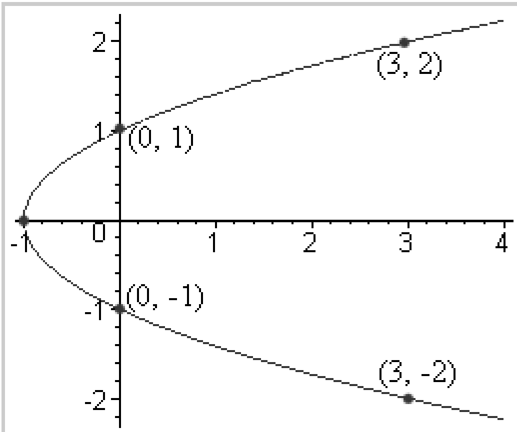
The x-intercepts are 2 and -2 and the y-intercept is -2. The graph of the equation $y = \frac{1}{2}x^2 - 2$ is shown below.



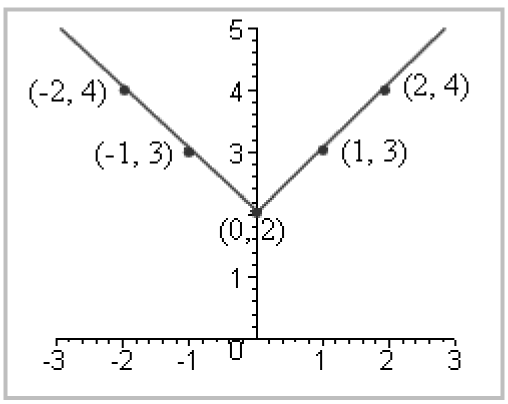
Symmetry

Symmetry
If the point $(x, -y)$ is on the graph of an equation whenever the point (x, y) is on the graph, then the graph is **symmetric with respect to the *x*-axis**. If the point $(-x, y)$ is on the graph of an equation whenever the point (x, y) is on the graph, then the graph is **symmetric with respect to the *y*-axis**. If the point $(-x, -y)$ is on the graph of an equation whenever the point (x, y) is on the graph, then the graph is **symmetric with respect to the *origin***.

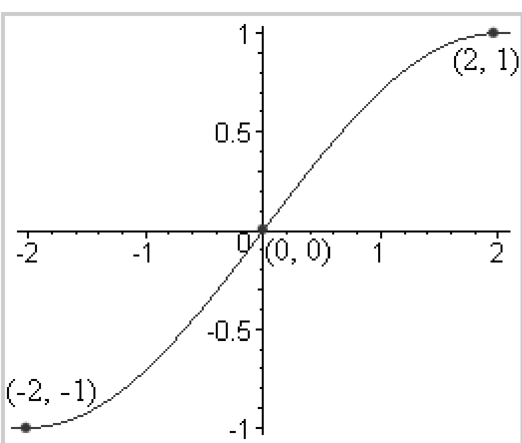
The graph below is symmetric with respect to the x-axis. The part of the graph above the x-axis is the mirror image of the part below the x-axis.



The graph below is symmetric with respect to the y-axis. The part of the graph to the left of the y-axis is the mirror image of the part to the right of the y-axis.



The graph below is symmetric with respect to the origin.



Example Problem:

Test the equation $y = x^4 + 1$ for symmetry with respect to the x-axis, the y-axis and the origin.

Solution:

To check for symmetry with respect to the x-axis, replace y by $-y$ in the equation.

$$y = x^4 + 1$$
$$-y = x^4 + 1$$

The graph is not symmetric with respect to the x-axis since the equation $-y = x^4 + 1$ is not the same as the original equation $y = x^4 + 1$.

To check for symmetry with respect to the y-axis, replace x by $-x$ in the equation.

$$y = x^4 + 1$$
$$y = (-x)^4 + 1$$
$$y = x^4 + 1$$

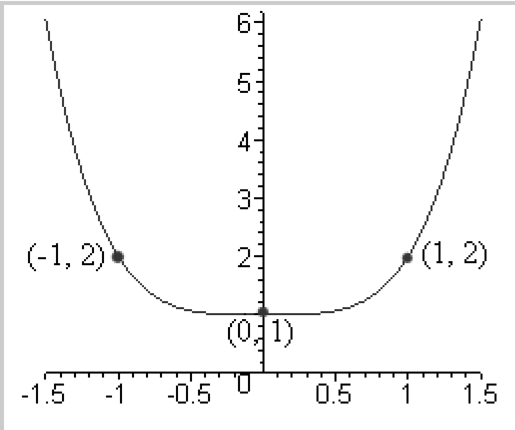
The graph is symmetric with respect to the y-axis since the original equation is unchanged.

To check for symmetry with respect to the origin, replace x by $-x$ and y by $-y$ in the equation.

$$y = x^4 + 1$$
$$-y = (-x)^4 + 1$$
$$-y = x^4 + 1$$

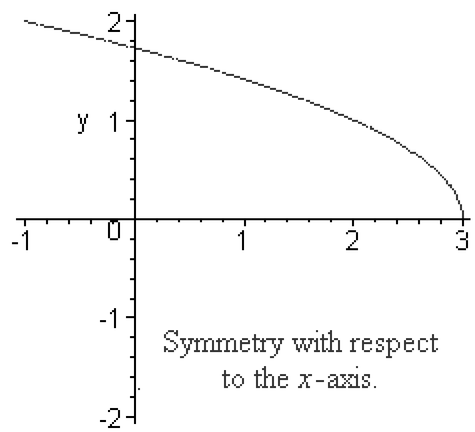
The graph is not symmetric with respect to the origin since the equation $-y = x^4 + 1$ is not the same as the original equation $y = x^4 + 1$.

The graph of the equation $y = x^4 + 1$ is shown below.



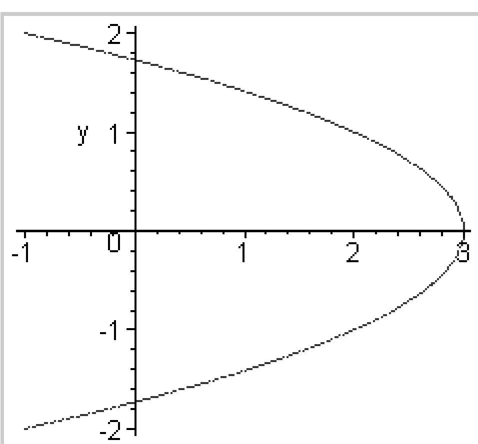
Additional Example 1:

In the following problem, only part of the graph is given. Complete the graph using the given symmetry.



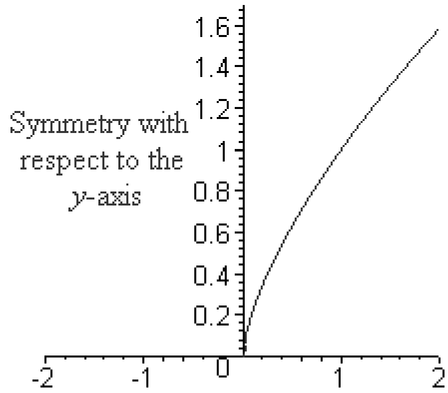
Solution:

Points on the graph are reflections of each other about the x-axis. The part of the graph below the x-axis is the mirror image of the part above the x-axis.



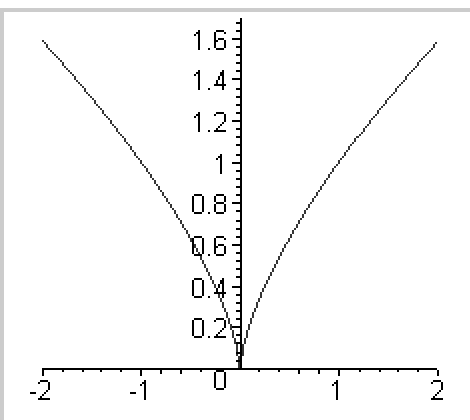
Additional Example 2:

In the following problem, only part of the graph is given. Complete the graph using the given symmetry.



Solution:

Points on the graph are reflections of each other about the y-axis. The part of the graph to the left of the y-axis is the mirror image of the part to the right of the y-axis.



Additional Example 3:

Test the equation $x^2 + y = 5$ for symmetry with respect to the x-axis, the y-axis, and the origin.

Solution:

To check for symmetry with respect to the x-axis, replace y by $-y$ in the equation.

$$x^2 + y = 5$$
$$x^2 + (-y) = 5$$
$$x^2 - y = 5$$

The graph is not symmetric with respect to the x-axis since the equation $x^2 - y = 5$ is not the same as the original equation $x^2 + y = 5$.

To check for symmetry with respect to the y-axis, replace x by $-x$ in the equation.

$$x^2 + y = 5$$
$$(-x)^2 + y = 5$$
$$x^2 + y = 5$$

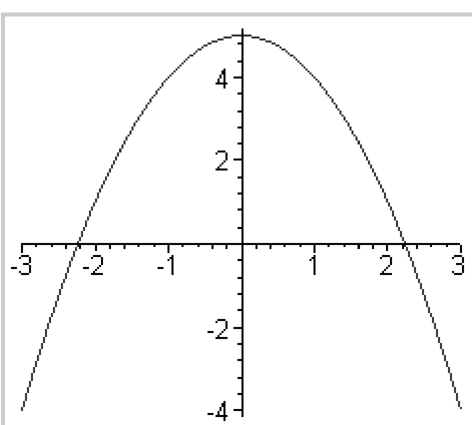
The graph is symmetric with respect to the y-axis since the original equation is unchanged.

To check for symmetry with respect to the origin, replace x by $-x$ and y by $-y$ in the equation.

$$x^2 + y = 5$$
$$(-x)^2 + (-y) = 5$$
$$x^2 - y = 5$$

The graph is not symmetric with respect to the origin since the equation $x^2 - y = 5$ is not the same as the original equation $x^2 + y = 5$.

The graph of the equation $x^2 + y = 5$ is shown below.



Additional Example 4:

Test the equation $y = x^3 + x$ for symmetry with respect to the x-axis, the y-axis, and the origin.

Solution:

To check for symmetry with respect to the x-axis, replace y by $-y$ in the equation.

$$\begin{aligned} y &= x^3 + x \\ -y &= x^3 + x \end{aligned}$$

The graph is not symmetric with respect to the x-axis since the equation $-y = x^3 + x$ is not the same as the original equation $y = x^3 + x$.

To check for symmetry with respect to the y-axis, replace x by $-x$ in the equation.

$$\begin{aligned} y &= x^3 + x \\ y &= (-x)^3 + (-x) \\ y &= -x^3 - x \end{aligned}$$

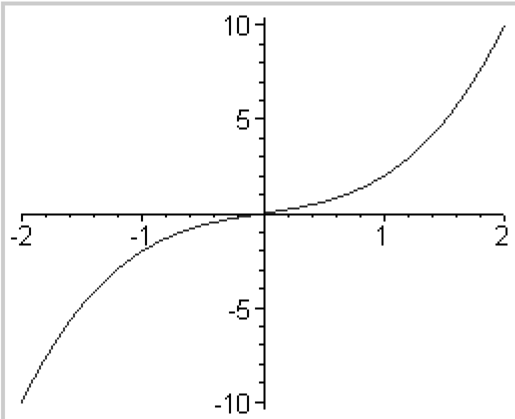
The graph is not symmetric with respect to the y-axis since the equation $y = -x^3 - x$ is not the same as the original equation $y = x^3 + x$.

To check for symmetry with respect to the origin, replace x by $-x$ and y by $-y$ in the equation.

$$\begin{aligned} y &= x^3 + x \\ -y &= (-x)^3 + (-x) \\ -y &= -x^3 - x \\ y &= x^3 + x \end{aligned}$$

The graph is symmetric with respect to the origin since the original equation is unchanged.

The graph of the equation $y = x^3 + x$ is shown below.



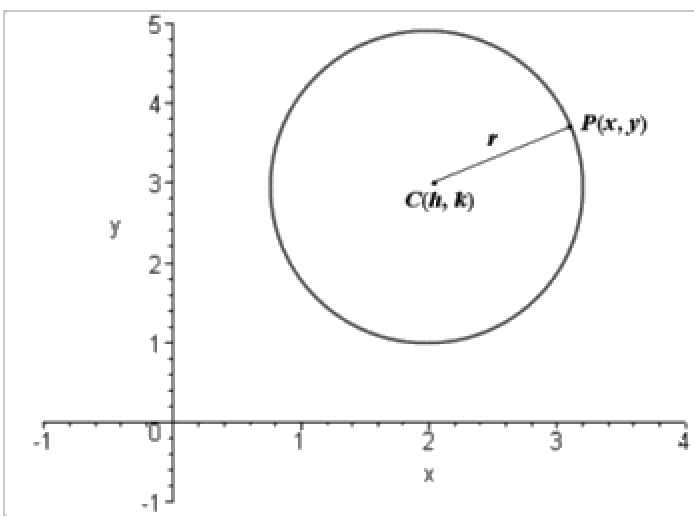
Circles

Circles

A **circle** is the set of all points $P(x, y)$ in the planes that are at a fixed distance r from a fixed point $C(h, k)$. The number r is called the **radius** of the circle. The point $C(h, k)$ is called the **center** of the circle.

The equation of a circle in **standard form** is given by

$$(x-h)^2+(y-k)^2=r^2.$$

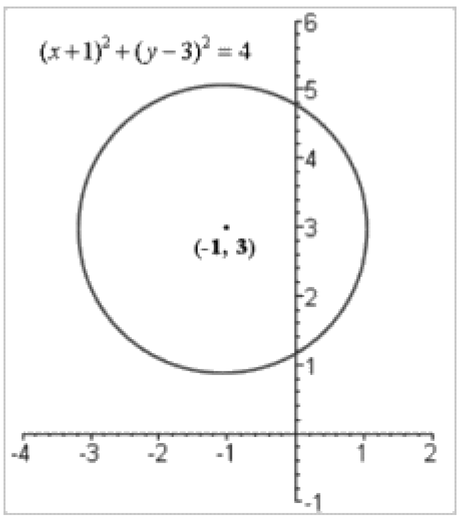


Example Problem 1: Find an equation of the circle with radius 2 and center $(-1, 3)$. Sketch the graph.

Solution:

Substitute $r = 2$, $h = -1$, and $k = 3$ into the standard form of the equation of a circle.

$$(x-(-1))^2 + (y-3)^2 = 2^2$$
$$(x+1)^2 + (y-3)^2 = 4$$



Example Problem 2: The equation $x^2 + y^2 - 6x + 10y - 2 = 0$ represents a circle. Find its center and radius.

Solution:

Group the x terms and y terms.

$(x^2 - 6x \quad) + (y^2 + 10y \quad) = 2$

Complete the square for $x^2 - 6x$ by adding $\left(\frac{1}{2}(-6)\right)^2 = (-3)^2 = 9$.

Complete the square for $y^2 + 10y$ by adding $\left(\frac{1}{2} \cdot 10\right)^2 = 5^2 = 25$.

$(x^2 - 6x + 9) + (y^2 + 10y + 25) = 2 + 9 + 25$

$(x-3)^2 + (y+5)^2 = 36$

The center is $(3, -5)$ and the radius is 6.

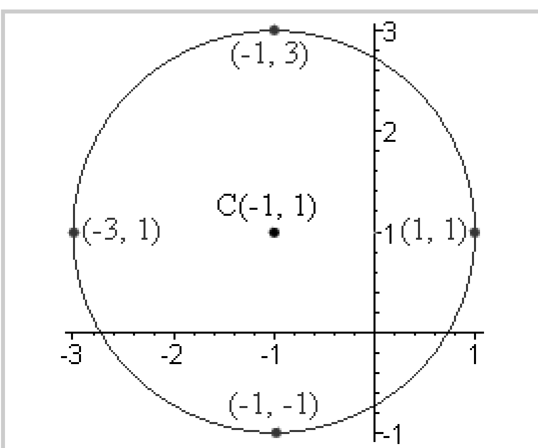
Additional Example 1:
The graph of the equation $(x+1)^2+(y-1)^2=4$ is a circle. Identify the center and radius and sketch the graph.

Solution:
The equation of a circle in standard form is given by $(x-h)^2+(y-k)^2=r^2$, where the center is the point (h,k) and the radius is r.
Rewrite the given equation in standard form.

$$(x+1)^2+(y-1)^2=4$$
$$(x-(-1))^2+(y-1)^2=2^2$$

We see that $h=-1$, $k=1$, and $r=2$. Thus, the center is the point $(-1,1)$ and the radius is 2.
To sketch the graph, begin by plotting the points $(-3,1)$, $(-1,3)$, $(1,1)$, and $(-1,-1)$. These points lie on the circle since the distance between these points and the point $(-1,1)$, the center of the circle, is 2.

The graph is shown below.



Additional Example 2:
Write an equation of the circle with center $(-3,-2)$ and radius 4.

Solution:
The equation of a circle in standard form is given by $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius.

To find an equation of the circle with center $(-3,-2)$ and radius 4, substitute $h=-3$, $k=-2$, and $r=4$ into the standard form.

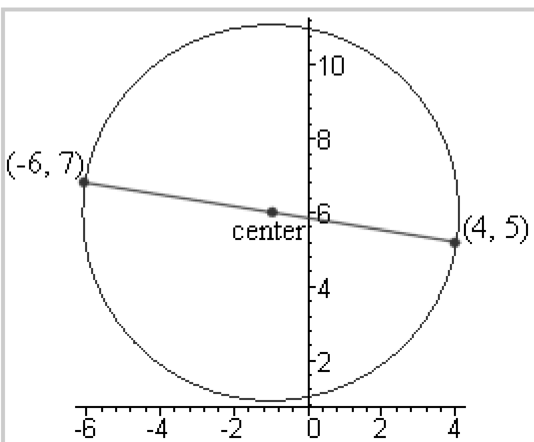
$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-3))^2+(y-(-2))^2=4^2$$
$$(x+3)^2+(y+2)^2=16$$

Additional Example 3:

Find an equation of the circle that satisfies the conditions that the endpoints of a diameter are $(-6,7)$ and $(4,5)$.

Solution:

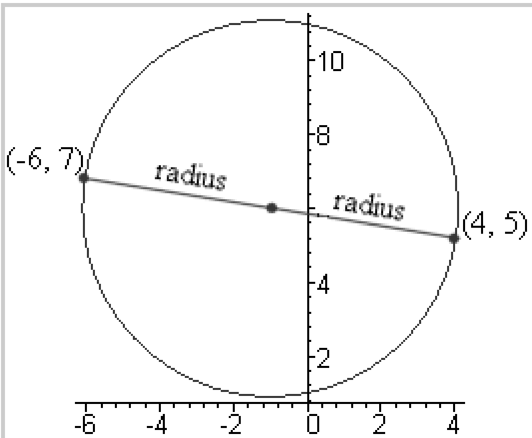
The center of the circle is the midpoint of the line segment connecting the points $(-6,7)$ and $(4,5)$.



To find the center of the circle, substitute $x_1=-6$, $y_1=7$, $x_2=4$, and $y_2=5$ into the midpoint formula.

$$\begin{aligned}\text{Center} &= \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\ &= \left(\frac{-6+4}{2},\frac{7+5}{2}\right)\\ &= \left(\frac{-2}{2},\frac{12}{2}\right)\\ &= (-1,6)\end{aligned}$$

The radius of the circle is one-half the distance between the points $(-6,7)$ and $(4,5)$.



To find the radius substitute $x_1 = -6$, $y_1 = 7$, $x_2 = 4$, and $y_2 = 5$ into the distance formula and multiply the result by $\frac{1}{2}$.

$$
\begin{aligned}
r &= \frac{1}{2} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \frac{1}{2} \cdot \sqrt{(4 - (-6))^2 + (5 - 7)^2} \\
&= \frac{1}{2} \cdot \sqrt{(4 + 6)^2 + (-2)^2} \\
&= \frac{1}{2} \cdot \sqrt{(10)^2 + 4} \\
&= \frac{1}{2} \cdot \sqrt{100 + 4} \\
&= \frac{1}{2} \cdot \sqrt{104} \\
&= \frac{1}{2} \cdot 2\sqrt{26} \\
&= \sqrt{26}
\end{aligned}
$$

The center of the circle is the point $(-1,6)$ and the radius is $\sqrt{26}$. Substitute $h = -1, k = 6,$ and $r = \sqrt{26}$ into the standard form for the equation of a circle.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-1))^2+(y-6)^2=\left(\sqrt{26}\right)^2$$
$$(x+1)^2+(y-6)^2=26$$

Additional Example 4:

The equation $x^2+y^2+6x-2y-1=0$ represents a circle. Put the equation in standard form and identify the center and the radius.

Solution:

Group the x-terms and the y-terms.

$$\left(x^2+6x\quad\right)+\left(y^2-2y\quad\right)=1$$

Complete the square for x^2+6x by adding $\left[\frac{1}{2}\cdot 6\right]^2=3^2=9.$

Complete the square for y^2-2y by adding $\left[\frac{1}{2}(-2)\right]^2=(-1)^2=1.$

$$\left(x^2+6x+9\right)+\left(y^2-2y+1\right)=1+9+1$$
$$(x+3)^2+(y-1)^2=11$$

Put the equation in the form $(x-h)^2+(y-k)^2=r^2$ to find the center (h,k) and the radius r.

$$(x-(-3))^2+(y-1)^2=\left(\sqrt{11}\right)^2$$

The center of the circle is $(-3,1)$ and the radius is $\sqrt{11}$.

Exercise Set 1.3: Graphing Equations

Determine which of the following points are on the graph of the given equation.

1. $x + y = 5$; $(1, 4)$, $(3, -7)$, $\left(\frac{3}{2}, \frac{7}{2}\right)$
2. $2x - y = 7$; $(0, -5)$, $(-3, -13)$, $\left(\frac{5}{2}, -2\right)$
3. $xy - x + 3y = 7$; $(2, -5)$, $(-1, 3)$, $\left(6, \frac{2}{3}\right)$
4. $x^2 - y^2 = x + y$; $(3, 2)$, $(5, 4)$, $(-7, -6)$
5. $x^2 y(3 + 2xy) = x + 5$; $(1,1)$, $(-2, 3)$, $(-5,0)$
6. $-2y(x^2 - xy) = 40$; $(4, -1)$, $(-3, -2)$, $(3, -5)$

Sketch the graphs of the following equations by plotting points.

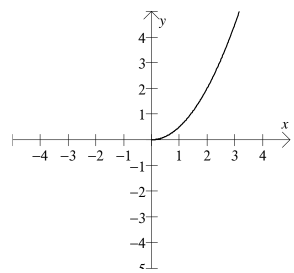
7. $y = 2x - 5$
8. $y = -3x + 4$
9. $y = |\, x + 3 \,|$
10. $y = |\, x \,| - 5$
11. $y = \sqrt{x}$
12. $y = x^2 + 1$
13. $y = 3 - x^2$
14. $y = x^3$
15. $y = \dfrac{12}{x}$
16. $y = -\dfrac{8}{x}$

Find the *x*- and *y*-intercepts of the graph of each of the following equations.

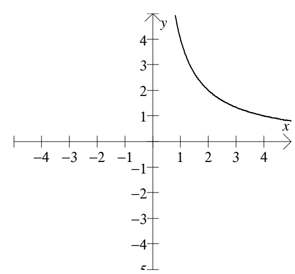
17. $y = 4x - 5$
18. $y = -3x - 7$
19. $5x + 2y = 20$
20. $3x - 4y = -24$
21. $y = x^2 - 16$
22. $x = 25 - y^2$
23. $y = \dfrac{7}{x}$
24. $xy = 6$
25. $y = x^2 + 9$
26. $y = \sqrt{x + 3}$
27. $x^2 + y^2 = 25$
28. $4x^2 - y^2 = 9$
29. $x^2 + 2xy + 3y = 12$
30. $4x^2 - 5xy + 3y = 36$

In the following questions, only part of the graph is given. Complete each graph using the given symmetry.

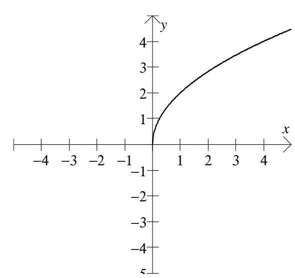
31. *x*-axis



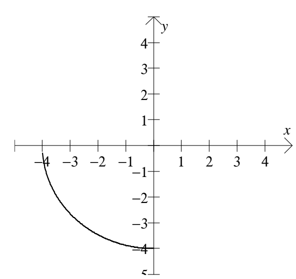
32. *y*-axis



33. origin



34. *x*-axis



Exercise Set 1.3: Graphing Equations

Test the following equations for symmetry with respect to the *x*-axis, the *y*-axis, and the origin.

35. $y = x^2 + x^4$

36. $x = y^2 + y^4$

37. $y = 5x^3$

38. $y = \frac{10}{x}$

39. $5x + x^2y^4 = y^2$

40. $x^2 + y^2 = xy$

41. $y = 3x^2 + 5x$

42. $x^2 + y^2 = 25y$

43. $|x| + 3y = 5$

44. $3x - 7|y| = 4$

For each of the following equations:

(a) Test the equation for symmetry.
(b) Find and plot the intercepts.
(c) Plot a few intermediate points in order to complete the graph. (Keep symmetry in mind to minimize the length of this step.)

45. $y = x^2 - 4$

46. $y = 5 - x^2$

47. $y = \sqrt{9 - x^2}$

48. $y = -\sqrt{25 - x^2}$

49. $y = x^3 - 4x$

50. $y = |x| + 3$

The following equations represent circles. Identify the center and radius of each circle.

51. $(x-3)^2 + (y+5)^2 = 49$

52. $(x+7)^2 + (y-1)^2 = 81$

53. $x^2 + (y-6)^2 = 28$

54. $(x+8)^2 + y^2 = 23$

Identify the center and radius, and then sketch the graph of the equation.

55. $x^2 + y^2 = 36$

56. $x^2 + y^2 = 17$

57. $(x-2)^2 + (y+3)^2 = 10$

58. $(x+2)^2 + (y-4)^2 = 4$

Write an equation of the circle that satisfies the given conditions.

59. Center (-4, -3); radius 5

60. Center (7, 0); radius 3

61. Center (-2, 5); radius $\sqrt{7}$

62. Center at the origin; radius $2\sqrt{3}$

63. Endpoints of a diameter are (-3, 6) and (5, 2)

64. Endpoints of a diameter are (5, -3) and (-9, 11)

65. Center (2, 5); passes through (-6, 4)

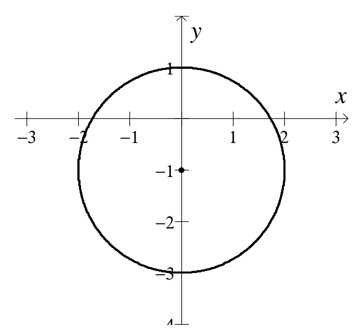
66. Center (-3, 1); passes through (5, -2)

67. Center (-2,4); tangent to the *x*-axis.

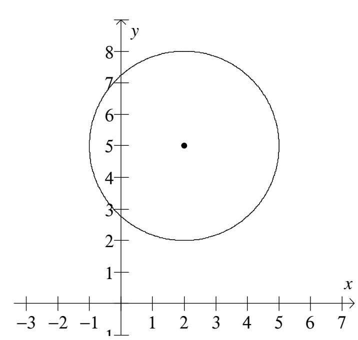
68. Center (3, -1); tangent to the *y*-axis.

Write an equation for each of the following circles.

69.



70.



Exercise Set 1.3: Graphing Equations

Show that the following equations represent circles by writing them in standard form:

$$(x-h)^2+(y-k)^2=r^2$$

Then identify the center and radius of the circle.

71. $x^2+y^2+4x-12y-9=0$

72. $x^2+y^2-2x+8y-20=0$

73. $x^2+y^2-10y+1=0$

74. $x^2+y^2-14x+39=0$

75. $x^2+y^2+x-6y+7=0$

76. $x^2+y^2-\frac{1}{3}x+\frac{1}{3}y-\frac{7}{36}=0$