Chapter 1

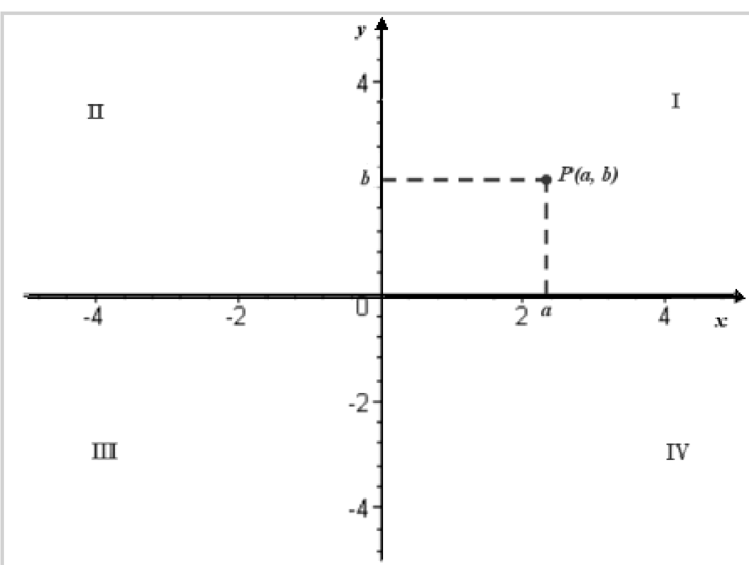
An Introduction to Graphs and Lines

Section 1.1: Points, Regions, Distance and Midpoints

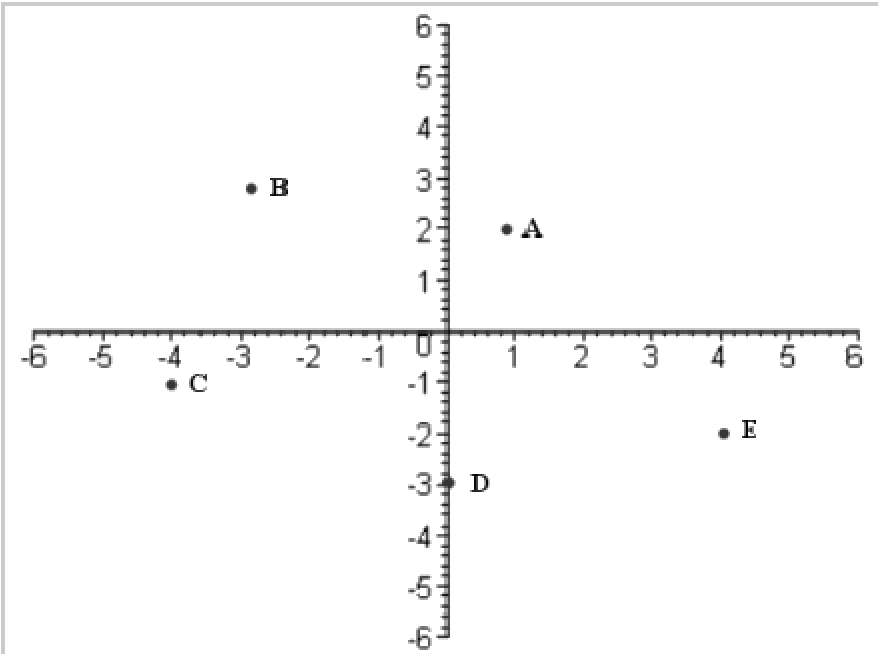
- Points in the Coordinate Plane
- Regions in the Coordinate Plane
- The Distance Formula
- The Midpoint Formula

Points in the Coordinate Plane

Points in the Coordinate Plane
A point P in the coordinate plane is located by a unique **ordered pair** of numbers (a, b). The number a is called the **x-coordinate** and the number b is called the **y-coordinate.**



Example Problem: Find the coordinates of the points shown in the figure below.



Solution:

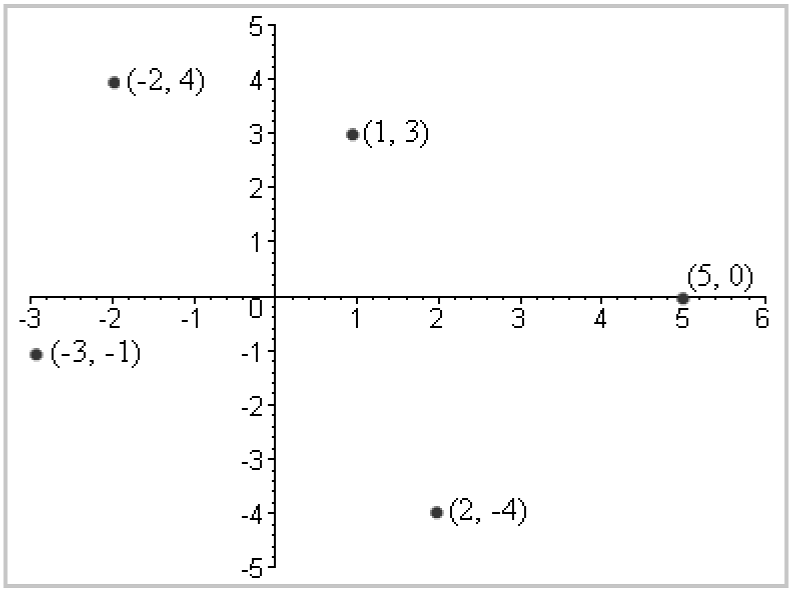
		Answer
A lies in Quadrant I.	Both coordinates are positive.	$(1,2)$
B lies in Quadrant II.	The first coordinate is negative. The second coordinate is positive.	$(-3,3)$
C lies in Quadrant III.	Both coordinates are negative.	$(-4,-1)$
D lies on the y-axis.	The first coordinate is 0.	$(0,-3)$
E lies in Quadrant IV.	The first coordinate is positive. The second coordinate is negative.	$(4,-2)$

Additional Example 1:

Plot each of the following points in a coordinate plane and identify the quadrant (or axis) in which each point is located.

$$(-2,4),\ (1,3),\ (5,0),\ (-3,-1),\text{ and }(2,-4)$$

Solution:



The point $(-2,4)$ is in Quadrant II since the first coordinate is negative and the second coordinate is positive. To plot the point, begin at the origin and move two units left and then 4 units up.

The point $(1,3)$ is in Quadrant I since both coordinates are positive. To plot the point, begin at the origin and move 1 unit right and then 3 units up.

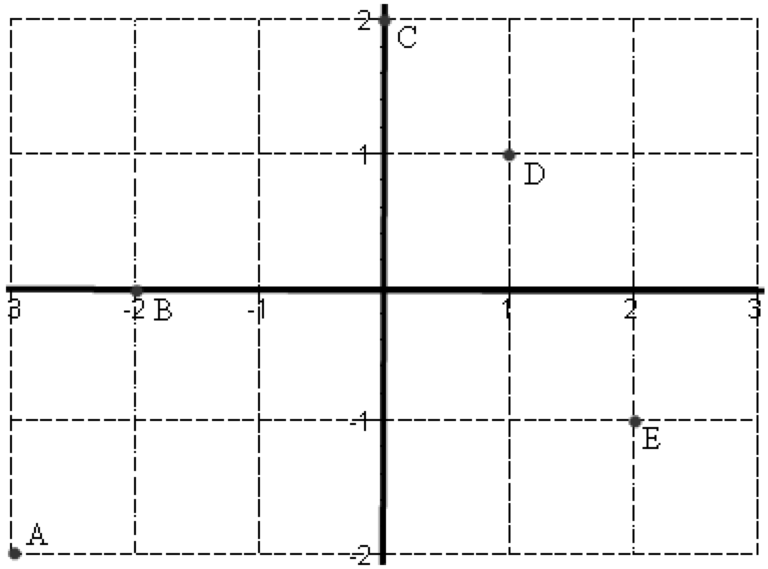
The point $(5,0)$ lies on the x-axis since the second coordinate is 0. To plot the point, begin at the origin and move 5 units right.

The point $(-3,-1)$ is in Quadrant III since both coordinates are negative. To plot the point, begin at the origin and move three units left and then 1 unit down.

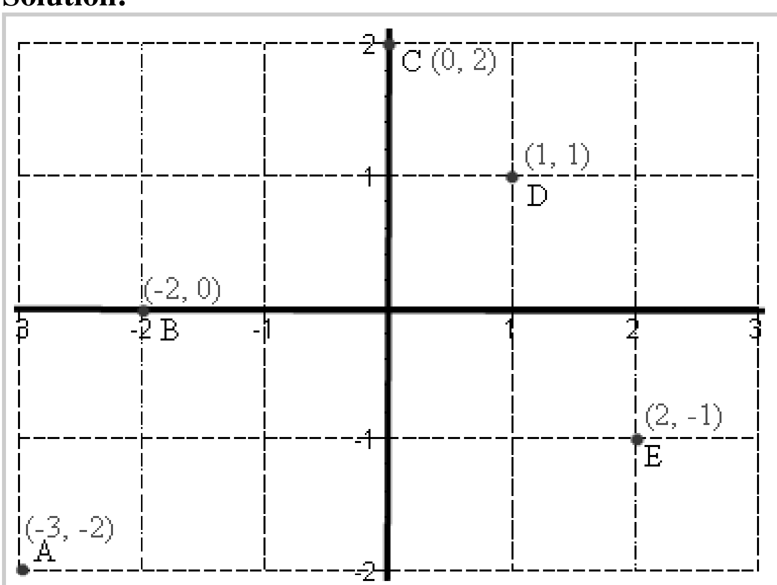
The point $(2,-4)$ is in Quadrant IV since the first coordinate is positive and the second coordinate is negative. To plot the point, begin at the origin and move two units right and then 4 units down.

Additional Example 2:

Find the coordinates of each of the points shown in the figure below and then identify the quadrant (or axis) in which each point is located.



Solution:



Point A is in Quadrant III. To get to the point, begin at the origin and move 3 units left for a first coordinate of -3 and then 2 units down for a second coordinate of -2.

Point B is on the x-axis. To get to the point, begin at the origin and move 2 units left for a first coordinate of -2. The second coordinate is 0.

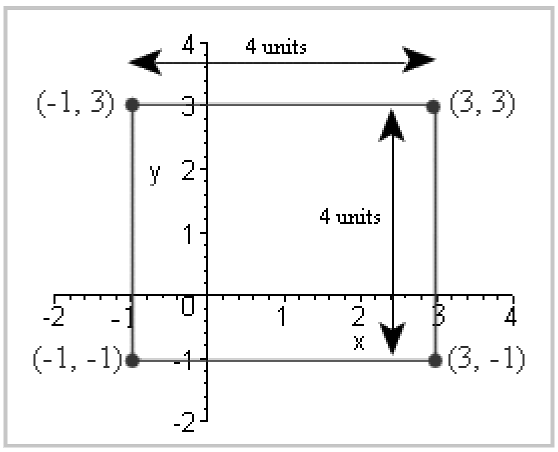
Point C is on the y-axis. To get to the point, begin at the origin and move 2 units up for a second coordinate of 2. The first coordinate is 0.

Point D is in Quadrant I. To get to the point, begin at the origin and move 1 unit right for a first coordinate of 1 and then 1 unit up for a second coordinate of 1.

Point E is in Quadrant IV. To get to the point, begin at the origin and move 2 units right for a first coordinate of 2 and then 1 unit down for a second coordinate of -1.

Additional Example 3:
Draw the square with vertices $(-1,3)$, $(-1,-1)$, $(3,3)$, and $(3,-1)$. What is the area of the square?

Solution:



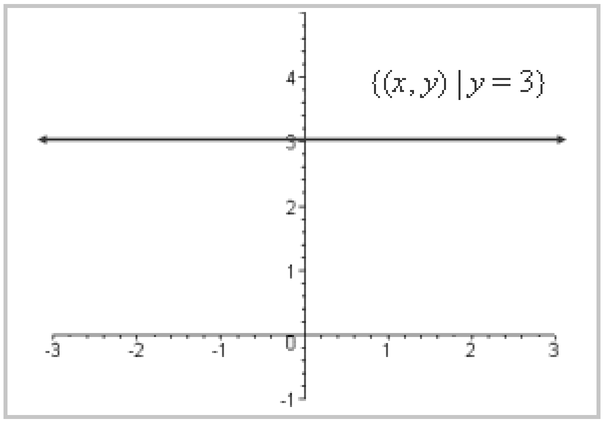
The length of each side of the square is 4. The area of the square is $4^2 = 16$.

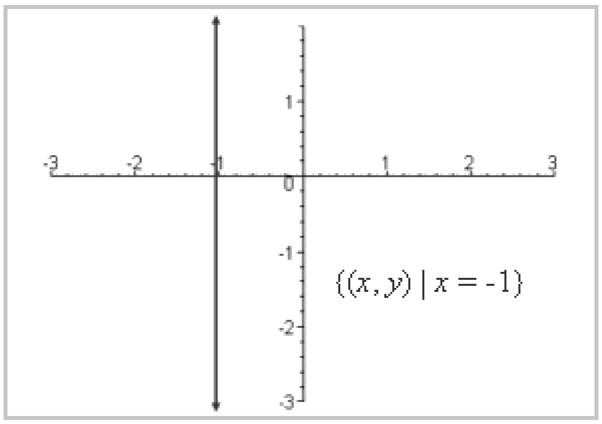
Regions in the Coordinate Plane

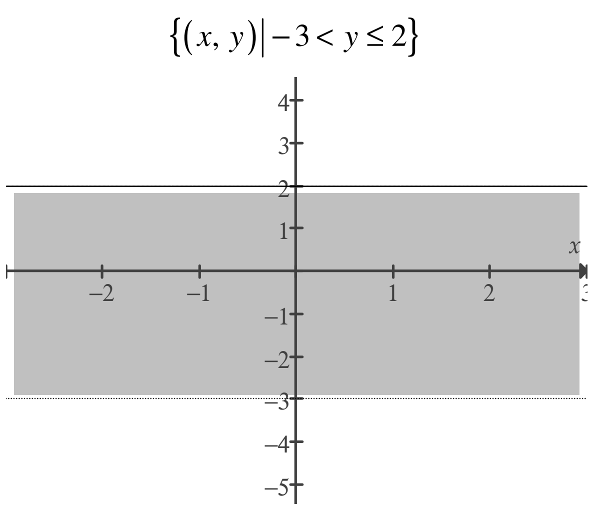
Regions in the Coordinate Plane

The set of all points in the plane with y**-coordinate** equal to the number k can be written as $\{(x, y) \mid y = k\}$. The graph is a **horizontal line**. Each point on the line has a second coordinate of k.

The set of all points in the plane with x**-coordinate** equal to the number k can be written as $\{(x, y) \mid x = k\}$. The graph is a **vertical line**. Each point on the line has a first coordinate of k.







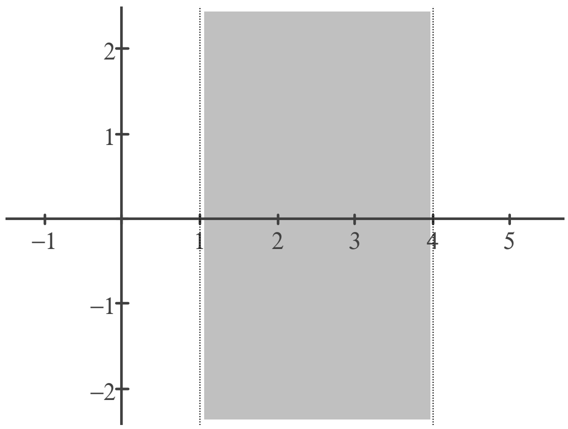
The region consists of all points that:

(a) lie on the line $\boldsymbol{y = 2}$;

(b) lie between the lines $\boldsymbol{y = 2}$ and $\boldsymbol{y = -3}$.

The broken line $\boldsymbol{y = -3}$ indicates that points on this line do not lie in the region.

$\{(x, y) | 1 < x < 4\}$



The region consists of all points that lie between the lines $\boldsymbol{x} = \mathbf{1}$ and $\boldsymbol{x} = \mathbf{4}$**.** The broken lines at $\boldsymbol{x} = \mathbf{1}$ and $\boldsymbol{x} = \mathbf{4}$ indicate that points on this line do not lie in the region.

Example Problem: Describe the region given by each set.

(1) $\{(x,y) | 0 < x < 3\}$

(2) $\{(x,y) | x = -5\}$

(3) $\{(x,y) | -1 \le y < 5\}$

Solution:

(1) The region consists of all points that lie between the vertical lines $\boldsymbol{x} = \mathbf{0}$ and $\boldsymbol{x} = \mathbf{3}$.

(2) The region consists of all points that lie on the vertical line $\boldsymbol{x} = -\mathbf{5}$.

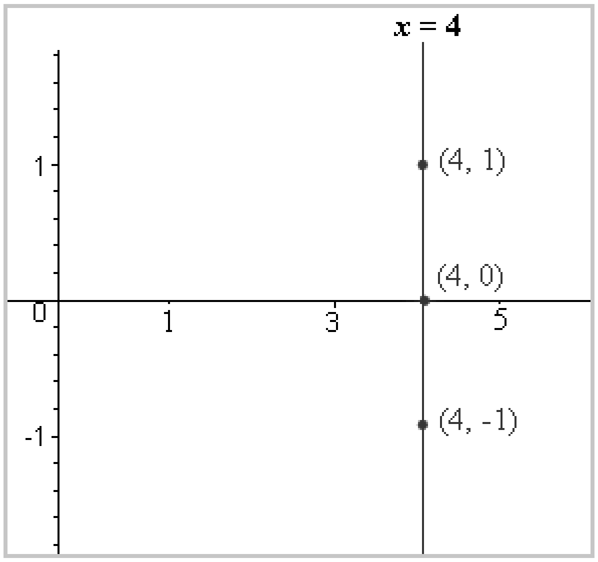
(3) The region consists of all points that lie on the horizontal line $\boldsymbol{y} = -\mathbf{1}$ together with those points that lie between the horizontal lines $\boldsymbol{y} = -\mathbf{1}$ and $\boldsymbol{y} = \mathbf{5}$.

Additional Example 1:

Sketch the region: $\{(x, y) : x = 4\}$

Solution:

Each point in the given region has a first coordinate of four. All points with a first coordinate of 4 lie on the vertical line $x = 4$.

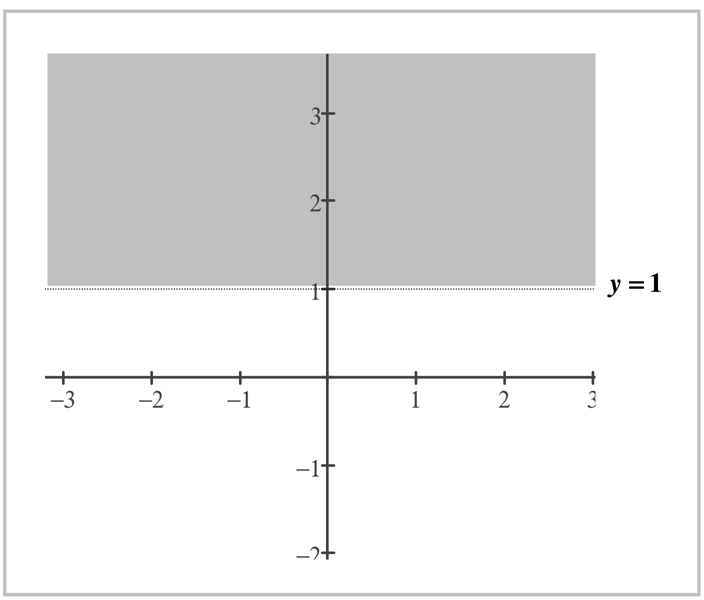


Additional Example 2:

Sketch the region: $\{(x,y): y > 1\}$

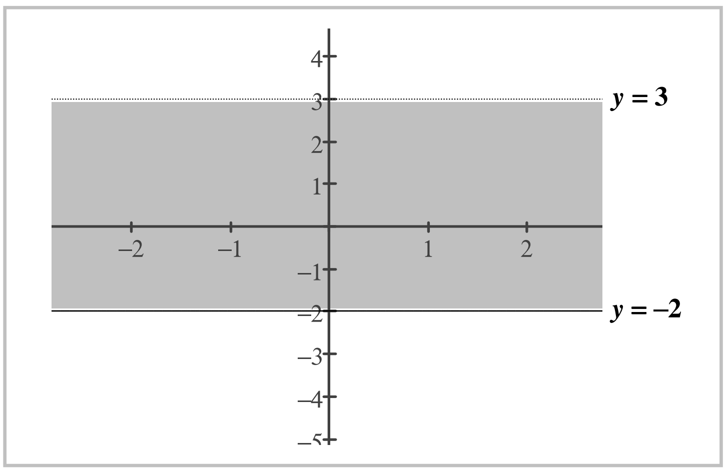
Solution:

Each point in the given region has a second coordinate that is greater than 1. All points with second coordinates greater than 1 lie above the horizontal line $y = 1$. The broken line $y = 1$ indicates that points on this line do not lie in the region.



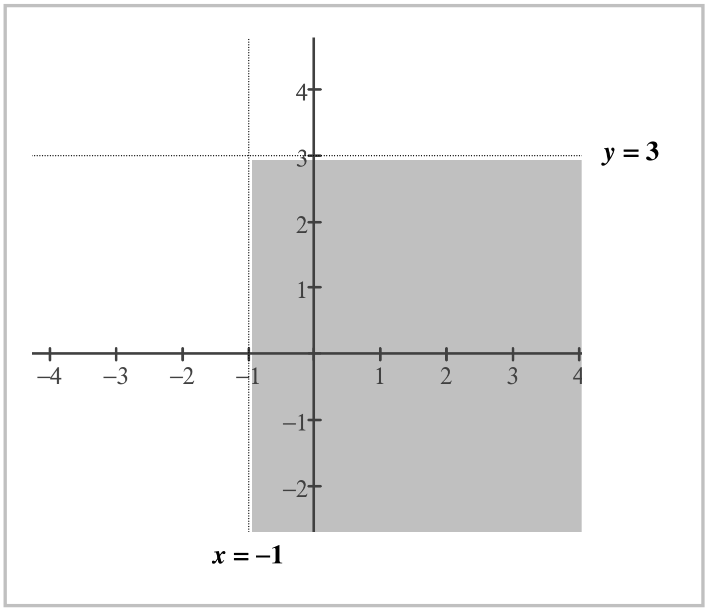
Additional Example 3:
Sketch the region: $\{(x, y): -2 \le y < 3\}$

Solution:
The given region is the intersection of two regions: $\{(x, y): y \ge -2\}$ and $\{(x, y): y < 3\}$. The first region $\{(x, y): y \ge -2\}$ is the set of all points in the plane that lie on or above the horizontal line $y = -2$. The second region $\{(x, y): y < 3\}$ is the set of all points in the plane that lie below the horizontal line $y = 3$. The intersection of the two regions is the set of all points in the plane that lie on the horizontal line $y = -2$ together with those points that lie between the two horizontal lines $y = -2$ and $y = 3$. The broken line $y = 3$ indicates that points on that line do not lie in the given region.



Additional Example 4:
Sketch the region: $\{(x, y): x > 1 \text{ and } y < 3\}$

Solution:
The given region is the intersection of two regions: $\{(x, y): x > 1\}$ and $\{(x, y): y < 3\}$. The first region $\{(x, y): x > 1\}$ is the set of all points in the plane that are to the right of the vertical line $x = 1$. The second region $\{(x, y): y < 3\}$ is the set of all points in the plane that lie below the horizontal line $y = 3$. The intersection of the two regions is the set of all points in the plane that lie to the right of the vertical line $x = 1$ and below the horizontal line $y = 3$. The broken lines indicate that no points on these lines lie in the given region.

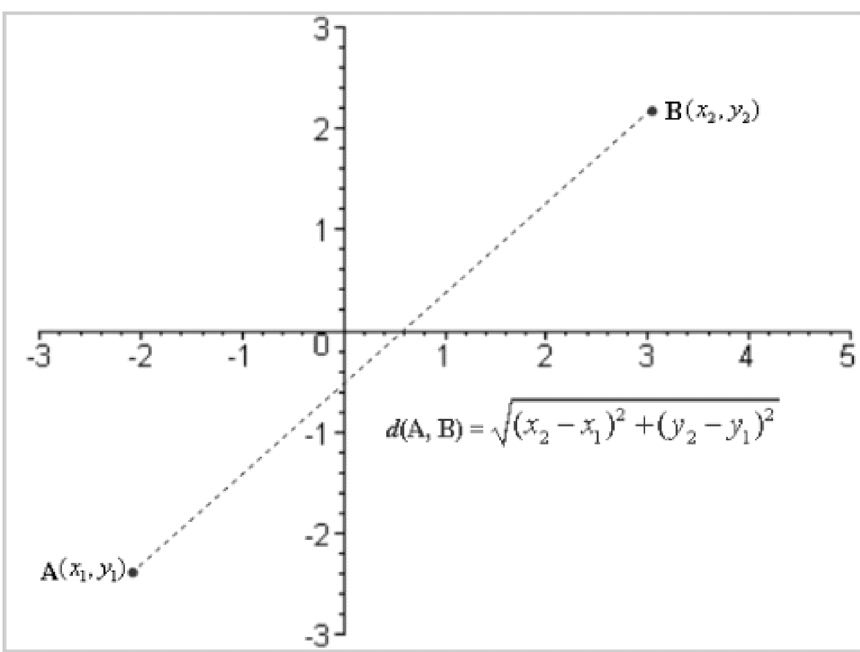


The Distance Formula

The Distance Formula

The distance between two points $\mathbf{A}(x_1, y_1)$ and $\mathbf{B}(x_2, y_2)$ in the plane is given by

$$d(\mathrm{A,B}) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example Problem: Find the distance between the points A $(3, -2)$ and B $(-1, 3)$.

Solution:
Substitute $x_1 = 3, y_1 = -2, x_2 = -1,$ and $y_2 = 3$ into the distance formula.

$$\begin{aligned} d(\mathrm{A}, \mathrm{B}) &= \sqrt{(-1-3)^2 + (3-(-2))^2} \\ &= \sqrt{(-4)^2 + (3+2)^2} \\ &= \sqrt{16+5^2} \\ &= \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

Additional Example 1:
Find the distance between the points $(1,3)$ and $(2,5)$.

Solution:
To find the distance between the points $(1,3)$ and $(2,5)$, substitute $x_1 = 1, y_1 = 3,$ $x_2 = 2,$ and $y_2 = 5$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-1)^2 + (5-3)^2} \\ &= \sqrt{1^2 + 2^2} \\ &= \sqrt{1+4} \\ &= \sqrt{5} \end{aligned}$$

Additional Example 2:
Find the distance between the points $(0,-3)$ and $(-4,5)$.

Solution:
To find the distance between the points $(0,-3)$ and $(-4,5)$, substitute $x_1 = 0,$ $y_1 = -3, x_2 = -4,$ and $y_2 = 5$ into the distance formula.

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-4-0)^2 + (5-(-3))^2} \\
&= \sqrt{(-4)^2 + (5+3)^2} \\
&= \sqrt{16+8^2} \\
&= \sqrt{16+64} \\
&= \sqrt{80} \\
&= 4\sqrt{5}
\end{aligned}
$$

Additional Example 3:

Find the distance between the points $(-2,2)$ and $(-5,1)$.

Solution:

To find the distance between the points $(-2,2)$ and $(-5,1)$, substitute $x_1 = -2$, $y_1 = 2$, $x_2 = -5$, and $y_2 = 1$ into the distance formula.

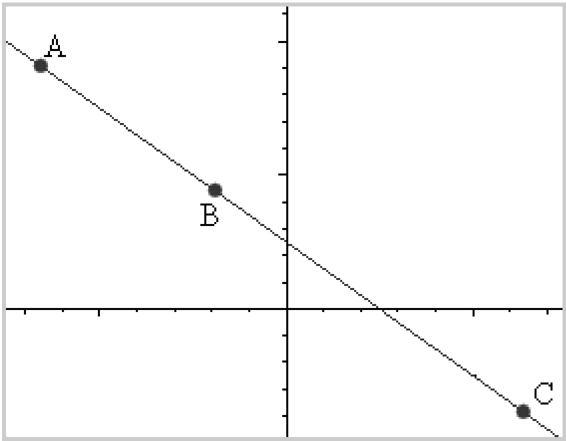
$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-5-(-2))^2 + (1-2)^2} \\
&= \sqrt{(-5+2)^2 + (-1)^2} \\
&= \sqrt{(-3)^2 + 1} \\
&= \sqrt{9+1} \\
&= \sqrt{10}
\end{aligned}
$$

Additional Example 4:

Are the points $(-1,8)$, $(1,12)$, and $(3,16)$ collinear?

Solution:

The distance formula can be used to determine if three points A, B, and C are collinear. If $d(\text{A, B}) + d(\text{B,C}) = d(\text{A,C})$, then A, B, and C are collinear. This is illustrated in the graph shown below.



To find the distance between the points $(-1,8)$ and $(1,12)$ substitute $x_1 = -1$, $y_1 = 8$, $x_2 = 1$, and $y_2 = 12$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-(-1))^2 + (12-8)^2} \\ &= \sqrt{(1+1)^2 + 4^2} \\ &= \sqrt{2^2 + 16} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

To find the distance between the points $(1,12)$ and $(3,16)$ substitute $x_1 = 1$, $y_1 = 12$, $x_2 = 3$, and $y_2 = 16$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-1)^2 + (16-12)^2} \\ &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

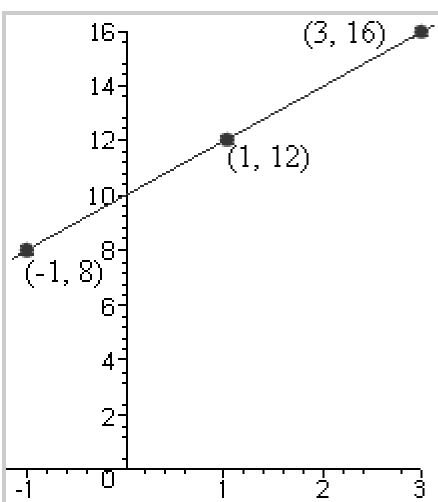
To find the distance between the points $(-1,8)$ and $(3,16)$ substitute $x_1 = -1$, $y_1 = 8$, $x_2 = 3$, and $y_2 = 16$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-1))^2 + (16-8)^2} \\ &= \sqrt{(3+1)^2 + 8^2} \\ &= \sqrt{4^2 + 64} \\ &= \sqrt{16+64} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

The distance between the points $(-1,8)$ and $(1,12)$ is $2\sqrt{5}$. The distance between the points $(1,12)$ and $(3,16)$ is $2\sqrt{5}$. The distance between the points $(-1,8)$ and $(3,16)$ is $4\sqrt{5}$.

Since $2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$, the points are collinear.

The three points are shown in the graph below.

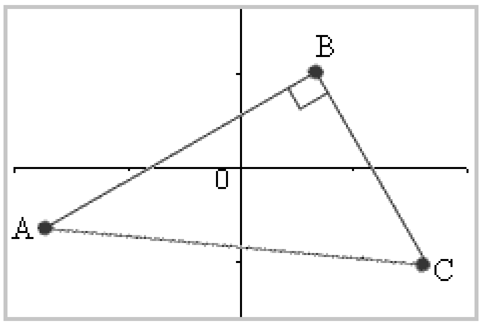


Additional Example 5:

Determine if the points $(-1,2)$, $(1,0)$, and $(3,2)$ are the vertices of a right triangle.

Solution:

If $[d(\text{A},\text{B})]^2 + [d(\text{B},\text{C})]^2 = [d(\text{A},\text{C})]^2$, then A, B, and C are the vertices of a right triangle as is illustrated in the graph shown below.



To find the distance between the points $(-1,2)$ and $(1,0)$ substitute $x_1 = -1$, $y_1 = 2$, $x_2 = 1$, and $y_2 = 0$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-(-1))^2 + (0-2)^2} \\ &= \sqrt{(1+1)^2 + (-2)^2} \\ &= \sqrt{2^2 + 4} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

To find the distance between the points $(1,0)$ and $(3,2)$ substitute $x_1 = 1$, $y_1 = 0$, $x_2 = 3$, and $y_2 = 2$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(3-1)^2+(2-0)^2} \\ &= \sqrt{2^2+2^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

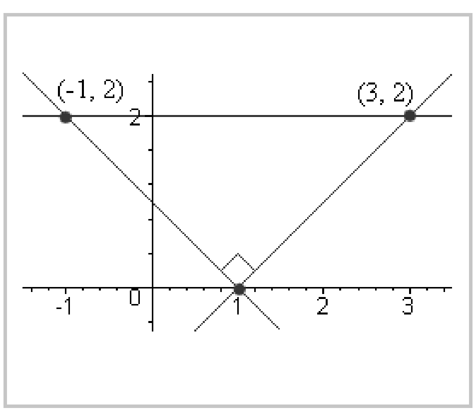
To find the distance between the points $(-1,2)$ and $(3,2)$ substitute $x_1=-1$, $y_1=2$, $x_2=3$, and $y_2=2$ into the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(3-(-1))^2+(2-2)^2} \\ &= \sqrt{(3+1)^2+0^2} \\ &= \sqrt{4^2+0} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

The distance between the points $(-1,2)$ and $(1,0)$ is $2\sqrt{2}$. The distance between the points $(1,0)$ and $(3,2)$ is $2\sqrt{2}$. The distance between the points $(-1,2)$ and $(3,2)$ is 4.

Since $\left(2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2=8+8=16=4^2$, the points are the vertices of a right triangle.

The three points are shown in the graph below.



Additional Example 6:

Determine which of the points $(-3,-1)$ or $(6,1)$ is closer to the point $(1,2)$.

Solution:

We need to find the distance between $(-3,-1)$ and $(1,2)$ and the distance between $(6,1)$ and $(1,2)$ by using the distance formula.

To find the distance between the points $(-3,-1)$ and $(1,2)$, substitute $x_1=-3$, $y_1=-1$, $x_2=1$, and $y_2=2$ into the distance formula.

$$\begin{aligned}
d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(1-(-3))^2+(2-(-1))^2} \\
&= \sqrt{(1+3)^2+(2+1)^2} \\
&= \sqrt{4^2+3^2} \\
&= \sqrt{16+9} \\
&= \sqrt{25} \\
&= 5
\end{aligned}$$

To find the distance between the points $(6,1)$ and $(1,2)$, substitute $x_1=6$, $y_1=1$, $x_2=1$, and $y_2=2$ into the distance formula.

$$\begin{aligned}
d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(1-6)^2+(2-1)^2} \\
&= \sqrt{(-5)^2+1^2} \\
&= \sqrt{25+1} \\
&= \sqrt{26}
\end{aligned}$$

The distance between $(-3,-1)$ and $(1,2)$ is 5 and the distance between $(6,1)$ and $(1,2)$ is $\sqrt{26}$.

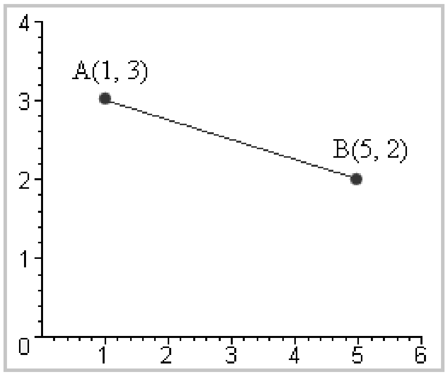
Since $5=\sqrt{25}<\sqrt{26}$, the point $(-3,-1)$ is closer to the point $(1,2)$.

Additional Example 7:

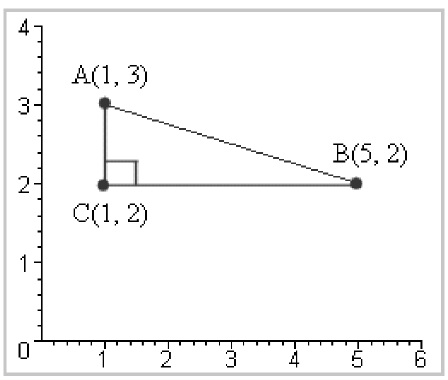
Use the Pythagorean Theorem to find the distance between the points $A(1,3)$ and $B(5,2)$.

Solution:

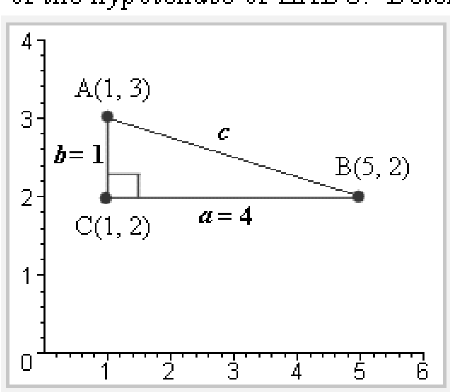
Plot the points $A(1,3)$ and $B(5,2)$ and draw line segment AB.



Find a point C such that triangle ABC is a right triangle. Draw triangle ABC.



Let a and b denote the lengths of the legs of ΔABC. Let c denote the length of the hypotenuse of ΔABC. Determine a and b from the graph.



Use the Pythagorean Theorem to determine c.

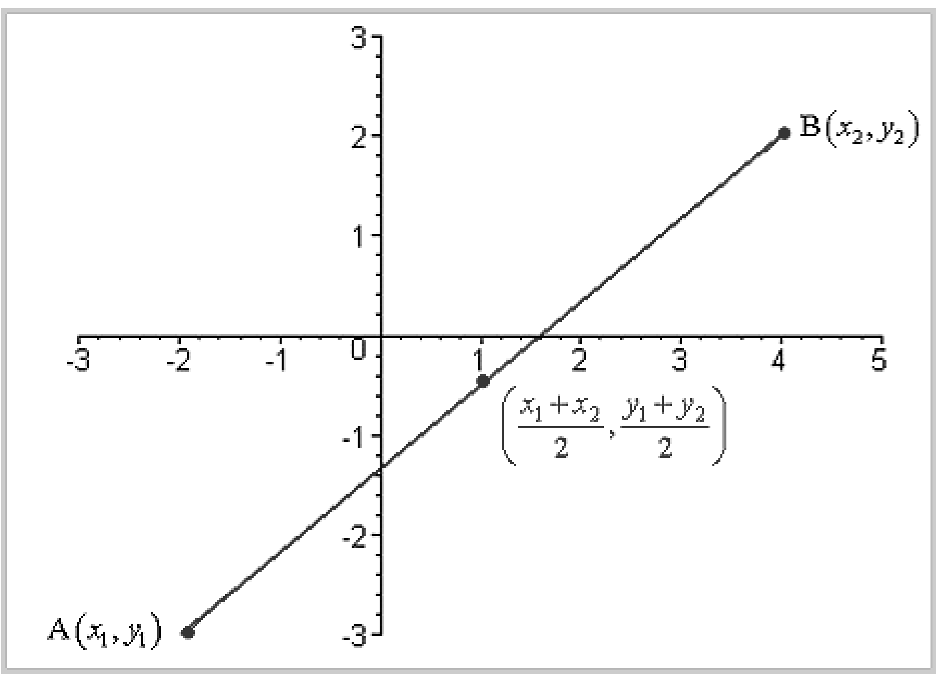
$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{4^2 + 1^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \end{aligned}$$

The Midpoint Formula

The Midpoint Formula

The midpoint of the line segment that connects two points $\mathbf{A}(x_1, y_1)$ and $\mathbf{B}(x_2, y_2)$ in the plane is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example Problem: Find the midpoint of the line segment joining the points $A(-2,-3)$ and $B(4,2)$.

Solution:

Substitute $x_1=-2, y_1=-3, x_2=4,$ and $y_2=2$ into the midpoint formula.

$$\text{Midpoint} = \left(\frac{-2+4}{2}, \frac{-3+2}{2}\right) = \left(\frac{2}{2}, \frac{-1}{2}\right) = \left(1, -\frac{1}{2}\right)$$

Additional Example 1:

Find the midpoint of the line segment connecting the points $(-2,5)$ and $(1,-3)$.

Solution:

To find the midpoint of the line segment connecting the the points $(-2,5)$ and $(1,-3)$, substitute $x_1=-2, y_1=5, x_2=1,$ and $y_2=-3$ into the midpoint formula.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\\ &= \left(\frac{-2+1}{2}, \frac{5+(-3)}{2}\right)\\ &= \left(\frac{-1}{2}, \frac{2}{2}\right)\\ &= \left(-\frac{1}{2}, 1\right)\end{aligned}$$

Additional Example 2:

Find the midpoint of the line segment connecting the points $(0,-3)$ and $(4,5)$.

Solution:

To find the midpoint of the line segment connecting the points $(0,-3)$ and $(4,5)$, substitute $x_1=0, y_1=-3, x_2=4,$ and $y_2=5$ into the midpoint formula.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\ &= \left(\frac{0+4}{2},\frac{-3+5}{2}\right)\\ &= \left(\frac{4}{2},\frac{2}{2}\right)\\ &= (2,1)\end{aligned}$$

Additional Example 3:

Find the midpoint of the line segment connecting the points $(-2,2)$ and $(-5,1)$.

Solution:

To find the midpoint of the line segment connecting the points $(-2,2)$ and $(-5,1)$, substitute $x_1=-2, y_1=2, x_2=-5,$ and $y_2=1$ into the midpoint formula.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\ &= \left(\frac{-2+(-5)}{2},\frac{2+1}{2}\right)\\ &= \left(-\frac{7}{2},\frac{3}{2}\right)\end{aligned}$$

Additional Example 4:

The point $(3,1)$ is the midpoint of the line segment AB. If A is the point $(-1,5)$, find the point B.

Solution:

Let x be the first coordinate of B. Let y be the second coordinate of B.

Find the midpoint of the line segment AB by substituting $x_1=-1, y_1=5, x_2=x,$ and $y_2=y$ into the midpoint formula.

$$\text{Midpoint of AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+x}{2}, \frac{5+y}{2}\right)$$

We are given that the midpoint of the line segment AB is the point $(3,1)$.

$$\text{Midpoint of AB} = \left(\frac{-1+x}{2}, \frac{5+y}{2}\right) = (3,1)$$

Two points in the plane are equal if their corresponding coordinates are equal. Setting first coordinates equal to each other and setting second coordinates equal to each other, we obtain two equations.

First equation: $\frac{-1+x}{2} = 3$ Second equation: $\frac{5+y}{2} = 1$

Solve the first equation for x to find the first coordinate of B.

$$\frac{-1+x}{2} = 3$$
$$-1+x = 6$$
$$x = 7$$

Solve the second equation for y to find the second coordinate of B.

$$\frac{5+y}{2} = 1$$
$$5+y = 2$$
$$y = -3$$

The point B is $(7,-3)$.

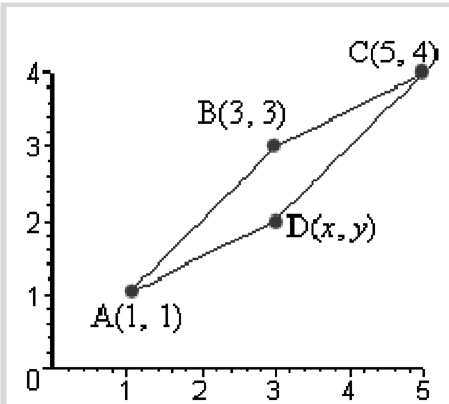
Additional Example 5:

Given the points $A(1,1)$, $B(3,3)$, and $C(5,4)$, determine the point D so that ABCD is a parallelogram.

Solution:

Let x be the first coordinate of D. Let y be the second coordinate of D.

Plot the points A, B, and C. Draw a parallelogram. The two diagonals, line segment AC and line segment BD, must have the same midpoint.



Find the midpoint of the line segment AC by substituting $x_1 = 1, y_1 = 1, x_2 = 5,$ and $y_2 = 4$ into the midpoint formula.

$$\text{Midpoint of AC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+5}{2}, \frac{1+4}{2}\right) = \left(\frac{6}{2}, \frac{5}{2}\right) = \left(3, \frac{5}{2}\right)$$

Find the midpoint of the line segment BD by substituting $x_1 = 3, y_1 = 3, x_2 = x,$ and $y_2 = y$ into the midpoint formula.

$$\text{Midpoint of BD} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3+x}{2}, \frac{3+y}{2}\right)$$

The midpoint of line segment AC is equal to the midpoint of line segment BD.

$$\left(3, \frac{5}{2}\right) = \left(\frac{3+x}{2}, \frac{3+y}{2}\right)$$

Two points in the plane are equal if their corresponding coordinates are equal. Setting first coordinates equal to each other and setting second coordinates equal to each other, we obtain two equations.

First equation: $\frac{3+x}{2}=3$ Second equation: $\frac{3+y}{2}=\frac{5}{2}$

Solve the first equation for x to find the first coordinate of D.

$$\begin{aligned}\frac{3+x}{2}&=3\\3+x&=6\\x&=3\end{aligned}$$

Solve the second equation for y to find the second coordinate of D.

$$\begin{aligned}\frac{3+y}{2}&=\frac{5}{2}\\3+y&=5\\y&=2\end{aligned}$$

The point D is $(3,2)$.

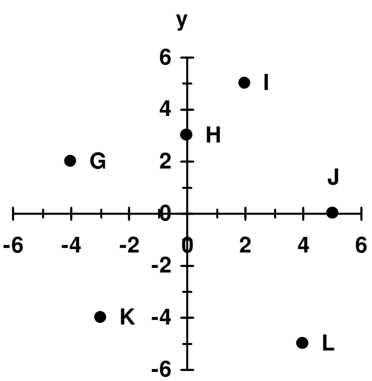
Exercise Set 1.1: Points, Regions, Distance and Midpoints

Plot the following points on a coordinate plane.

1. A(3, 4)
2. B(-2, 5)
3. C(-3, -1)
4. D(-5, 0)
5. E(-4, -6)
6. F(0, -2)

Write the coordinates of each of the points shown in the figure below. Then identify the quadrant (or axis) in which the point is located.

7. G
8. H
9. I
10. J
11. K
12. L



Answer the following.

Area of a rectangle = base x height
Area of a parallelogram = base x height
Area of a triangle = ½ (base x height)

13. Draw the rectangle with vertices *A*(3, 5), *B*(-2,5), *C*(-2, -4), and *D*(3, -4). Then find the area of rectangle *ABCD*.

14. Draw the parallelogram with vertices *A*(-2, -3), *B*(4, -3), *C*(6, 1), and *D*(0, 1). Then find the area of parallelogram *ABCD*.

15. Draw the triangle with vertices *E*(-3, 2), *F*(4, 2), and *G*(1, 5). Then find the area of triangle *EFG*.

Answer the following.

16. Given the following points:
(3, 5), (3, 1), (3, 0), (3, -2)

   (a) Plot the above points on a coordinate plane.
   (b) What do the above points have in common?
   (c) Draw a line through the above points.
   (d) What is the equation of the line drawn in part (c)?

17. Given the following points:
(-3, 4), (0, 4), (1, 4), (3, 4)

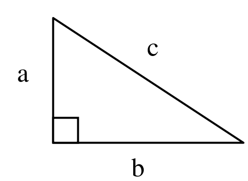
   (a) Plot the above points on a coordinate plane.
   (b) What do the above points have in common?
   (c) Draw a line through the above points.
   (d) What is the equation of the line drawn in part (c)?

Graph the following regions in a coordinate plane.

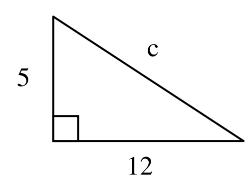
18. $\{(x, y) \mid x = 2\}$
19. $\{(x, y) \mid y = -5\}$
20. $\{(x, y) \mid y > 2\}$
21. $\{(x, y) \mid x \le 3\}$
22. $\{(x, y) \mid -1 < x \le 4\}$
23. $\{(x, y) \mid -3 \le y < 5\}$
24. $\{(x, y) \mid x \le 1 \text{ and } y > -3\}$
25. $\{(x, y) \mid x > 4 \text{ and } y < -2\}$
26. $\{(x, y) \mid x \ge 2 \text{ and } y \ge 1\}$

Use the Pythagorean Theorem to find the missing side of each of the following triangles.

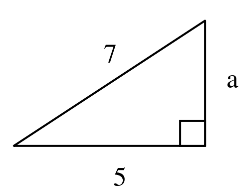
Pythagorean Theorem: In a right triangle, if a and b are the measures of the legs, and c is the measure of the hypotenuse, then $a^2 + b^2 = c^2$.



27.



28.



Exercise Set 1.1: Points, Regions, Distance and Midpoints

Answer the following.

29. Given the following points:
$A(1, 2)$ and $B(4, 7)$

(a) Plot the above points on a coordinate plane.
(b) Draw segment AB. This will be the hypotenuse of triangle ABC.
(c) Find a point C such that triangle ABC is a right triangle. Draw triangle ABC.
(d) Use the Pythagorean theorem to find the distance between A and B (the length of the hypotenuse of the triangle).

30. Given the following points:
$A(-3, 1)$ and $B(1, -5)$

(a) Plot the above points on a coordinate plane.
(b) Draw segment AB. This will be the hypotenuse of triangle ABC.
(c) Find a point C such that triangle ABC is a right triangle. Draw triangle ABC.
(d) Use the Pythagorean theorem to find the distance between A and B (the length of the hypotenuse of the triangle).

Use the distance formula to find the distance between the two given points. *(You may also use the method from the previous two problems to verify your answer.)*

31. (3, 6) and (5, 9)

32. (4, 7) and (2, 3)

33. (-5, 0) and (-2, 6)

34. (9, -4) and (2, -3)

35. (4, 0) and (0, -7)

36. (-4, -7) and (-10, -2)

Find the midpoint of the line segment joining points *A* and *B*.

37. $A(7, 6)$ and $B(3, 8)$

38. $A(5, 9)$ and $B(1, 3)$

39. $A(-7, 0)$ and $B(-4, 8)$

40. $A(7, -5)$ and $B(4, -3)$

41. $A(3, 0)$ and $B(0, -9)$

42. $A(-6, -7)$ and $B(-10, -6)$

Answer the following.

43. If $M(3, -5)$ is the midpoint of the line segment joining points A and B, and A has coordinates (-1, -2), find the coordinates of B.

44. If $M(-2, 4)$ is the midpoint of the line segment joining points A and B, and A has coordinates (-1, -2), find the coordinates of B.

45. Determine which of the following points is closer to the origin: $A(5, -6)$ or $B(-3, 7)$?

46. Determine which of the following points is closer to the point (4, -1): $A(-2, 3)$ or $B(6, 6)$?

47. Determine whether or not the triangle formed by the following vertices is a right triangle:
$A(3, 2)$, $B(8, 5)$, and $C(6, 10)$

48. Determine whether or not the triangle formed by the following vertices is a right triangle:
$A(4, 3)$, $B(5,0)$, and $C(-1, -2)$

49. Determine if the points $A(-5,2)$, $B(-1, 8)$, and $C(1, 11)$ are collinear.

50. Determine if the points $A(-1,3)$, $B(2, 4)$, and $C(6, 5)$ are collinear.

51. Given the points $A(-3,2)$, $B(2, 5)$, and $C(9, 4)$, determine the point D so that $ABCD$ is a parallelogram.

52. A circle has a diameter with endpoints $A(-5, -9)$ and $B(3, 5)$.

(a) Find the coordinates of the center of the circle.
(b) Find the length of the radius of the circle.