Section 1.4: Exponents and Radicals

- Evaluating Exponential Expressions
- Square Roots

Evaluating Exponential Expressions

Let n be a natural number. Then the exponential expression x^n is defined by

$x^n = \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ factors}}$. The expression is read as "x to the n^{th} power."

The number x is called the **base** and n is called the **exponent**. The exponent n gives the number of factors that the base x is used in a product.

Two Rules for Exponential Expressions:

Let n and m be natural numbers.

(1) Product rule: $x^m x^n = x^{m+n}$

If two exponential expressions with the same base are multiplied, keep the common base and add the exponents.

(2) Power rule: $\left(x^m\right)^n = x^{mn}$

If an exponential expression is raised to a power, keep the base and multiply the exponents.

Example:

Identify the base and exponent for each of the following exponential expressions. Then evaluate each expression.

(a) 9^2

(b) -9^2

Solution:

(a) The base is 9 and the exponent is 2. This tells us that the base of 9 is used as a factor 2 times in a product.

Thus,

$$\begin{aligned} 9^2 &= 9 \cdot 9 \\ &= 81. \end{aligned}$$

(b) The base is 9 and the exponent is 2. This tells us that the base of 9 is used as a factor 2 times in a product.

Thus,

$$\begin{aligned} -9^2 &= -9 \cdot 9 \\ &= -81. \end{aligned}$$

Example:

Write each of the following as a base and exponent. Do not evaluate.

(a) $3^4 \cdot 3^6$

(b) $\left(3^5\right)^2$

Solution:

(a) Use the product rule for exponential expressions. Keep the common base and find the sum of the exponents.

$$\begin{aligned} 3^4 \cdot 3^6 &= 3^{4+6} \\ &= 3^{10} \end{aligned}$$

(b) Use the power rule for exponential expressions. Keep the base and find the product of the exponents.

$$\begin{aligned} \left(3^5\right)^2 &= 3^{5(2)} \\ &= 3^{10} \end{aligned}$$

Additional Properties for Exponential Expressions:

Two Definitions:

Let n be a natural number. We have the following definitions:

(1) If $x \neq 0$, then $x^0 = 1$.

(2) (Negative exponents) If $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $x^n = \frac{1}{x^{-n}}$.

Quotient Rule for Exponential Expressions:

Quotient Rule: If m and n are natural numbers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If two exponential expressions with the same base are divided, keep the common base and subtract the exponents.

From the definition of negative exponents, it follows that the Product Rule, the Power Rule, and the Quotient Rule hold for all exponents that are integers.

Exponential Expressions with Bases of Products:

If n is an integer, $(xy)^n = x^n y^n$.

Exponential Expressions with Bases of Fractions:

If n is an integer, $x \neq 0$, and $y \neq 0$, then $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$.

Note that $\left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$.

Example:
Evaluate each of the following:

(a) 2^{-3} (b) $\frac{5^9}{5^6}$ (c) $\left(\frac{2}{5}\right)^{-3}$

Solution:

(a) If n is an integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2\cdot 2\cdot 2} = \frac{1}{8}$$

(b) If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

Using this rule, rewrite the given expression and then evaluate.

$$\frac{5^9}{5^6} = 5^{9-6} = 5^3 = 5\cdot 5\cdot 5 = 125$$

(c) If n is an integer, $x \neq 0$, and $y \neq 0$, then $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3} = \frac{5}{2}\cdot\frac{5}{2}\cdot\frac{5}{2} = \frac{5\cdot 5\cdot 5}{2\cdot 2\cdot 2} = \frac{125}{8}$$

Additional Example 1:

Identify the base and exponent for each of the following exponential expressions. Then evaluate each expression.

(a) 6^3

(b) -6^2

(c) $\left(\frac{1}{6}\right)^3$

(d) $\left(-\frac{1}{6}\right)^2$

Solution:

(a) The base is 6 and the exponent is 3. This tells us that the base of 6 is used as a factor 3 times in a product.

Thus,

$$\begin{aligned} 6^3 &= 6 \cdot 6 \cdot 6 \\ &= 36 \cdot 6 \\ &= 216. \end{aligned}$$

(b) The base is 6 and the exponent is 2. This tells us that the base of 6 is used as a factor 2 times in a product.

Thus,

$$\begin{aligned} -6^2 &= -6 \cdot 6 \\ &= -36. \end{aligned}$$

(c) The base is $\frac{1}{6}$ and the exponent is 3. This tells us that the base of $\frac{1}{6}$ is used as a factor 3 times in a product.

Thus,

$$\begin{aligned} \left(\frac{1}{6}\right)^3 &= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \\ &= \frac{1}{36} \cdot \frac{1}{6} \\ &= \frac{1}{216}. \end{aligned}$$

(d) The base is $-\frac{1}{6}$ and the exponent is 2. This tells us that the base of $-\frac{1}{6}$ is used as a factor 2 times in a product.

Thus,

$$\left(-\frac{1}{6}\right)^2 = \left(-\frac{1}{6}\right)\left(-\frac{1}{6}\right)$$
$$= \frac{1}{36}.$$

Additional Example 2:

Write each of the following as a base and exponent. Do not evaluate.

(a) $4^3 \cdot 4^5$

(b) $4^{-2} \cdot 4^7$

(c) $\frac{4^8}{4^3}$

(d) $\left(4^2\right)^6$

Solution:

(a) Use the product rule for exponential expressions. Keep the common base and find the sum of the exponents.

$$4^3 \cdot 4^5 = 4^{3+5}$$
$$= 4^8$$

(b) Use the product rule for exponential expressions. Keep the common base and find the sum of the exponents.

$$4^{-2} \cdot 4^7 = 4^{-2+7}$$
$$= 4^5$$

(c) Use the quotient rule for exponential expressions. Keep the common base and subtract the exponents.

$$\frac{4^8}{4^3} = 4^{8-3}$$
$$= 4^5$$

(d) Use the power rule for exponential expressions. Keep the base and find the product of the exponents.

$$\begin{aligned}\left(4^2\right)^6 &= 4^{2(6)}\\ &= 4^{12}\end{aligned}$$

Additional Example 3:

Rewrite each expression so that it contains positive exponents rather than negative exponents, and then evaluate the expression.

(a) 8^{-2}

(b) $\frac{1}{7^{-3}}$

(c) $\left(\frac{4}{9}\right)^{-3}$

(d) $\left(-\frac{2}{3}\right)^{-2}$

Solution:

(a) If n is an integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$\begin{aligned}8^{-2} &= \frac{1}{8^2}\\ &= \frac{1}{8 \cdot 8}\\ &= \frac{1}{64}\end{aligned}$$

(b) If n is an integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$\frac{1}{7^{-3}} = 7^3$$
$$= 7 \cdot 7 \cdot 7$$
$$= 343$$

(c) If n is an integer, $x \neq 0$, and $y \neq 0$, then $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$\left(\frac{4}{9}\right)^{-3} = \left(\frac{9}{4}\right)^{3}$$
$$= \frac{9}{4} \cdot \frac{9}{4} \cdot \frac{9}{4}$$
$$= \frac{729}{64}$$

(d) If n is an integer, $x \neq 0$, and $y \neq 0$, then $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$.

Using this rule, rewrite the given expression so that it contains a positive exponent. Then evaluate the resulting exponential expression.

$$\left(-\frac{2}{3}\right)^{-2} = \left(-\frac{3}{2}\right)^{2}$$
$$= \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)$$
$$= \frac{9}{4}$$

Square Roots

Definitions:

A number y is called a square root of a number x provided that $y^2 = x$. We see that -9 and 9 are square roots of 81 since $(-9)^2 = 9^2 = 81$. In general, if $x > 0$, then x has two square roots, one is negative and one is positive.

The prinicipal square root of x is the positive square root of x and is denoted by $\sqrt{x}$. The expression $\sqrt{x}$ is an example of a radical expression and is read "the square root of x."

Also, the principal square root of 0 is 0: $\sqrt{0} = 0$. Moreover, the square root of a negative number is not a real number.

In the example above, we have $\sqrt{81} = 9$ and $-\sqrt{81} = -9$.

In the expression $\sqrt{x}$, the symbol $\sqrt{\ }$ is called a radical sign and x is called the radicand.

Two Rules for Square Roots:

(1) Product Rule: If $x \geq 0$ and $y \geq 0$, then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

(2) Quotient Rule: If $x \geq 0$ and $y > 0$, then $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$.

Writing Radical Expressions in Simplest Radical Form:

The Product Rule and Quotient Rule for square roots can be used to write radical expressions in simplest radical form.

A square root is not in simplest radical form if the radicand contains a perfect square factor.

Examples of perfect integer squares are $36 = 6^2$, $64 = 8^2$, and $121 = 11^2$. (Their square roots are integers.) For example, $\sqrt{24}$ is not in simplest radical form since 24 contains a perfect square factor of 4.

Example:
Write $\sqrt{24}$ in simplest radical form.

Solution:
First, factor 24 into the product of two numbers so that one of the factors is the largest perfect square factor that divides 24: $\sqrt{24} = \sqrt{4 \cdot 6}$

Next, use the product rule for square roots: $\sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6}$

Thus,

$$\begin{aligned}\sqrt{24} &= \sqrt{4 \cdot 6} \\ &= \sqrt{4} \cdot \sqrt{6} \\ &= 2\sqrt{6}.\end{aligned}$$

($\sqrt{4} = 2$ since $2^2 = 4$ and 2 is positive.)

A square root is not in simplest radical form if the radicand contains a fraction. In addition, a radical expression is not in simplest radical form if there is a radical in the denominator.

For example, $\dfrac{1}{\sqrt{3}}$ is not in simplest radical form. In this case, we can *rationalize* the denominator by multiplying both numerator and denominator by $\sqrt{3}$ and use the definition of a square root to note that $\sqrt{3} \cdot \sqrt{3} = 3$.

In general, if $x \geq 0$, then $\sqrt{x} \cdot \sqrt{x} = x$.

Example:
Write $\sqrt{\dfrac{64}{5}}$ in simplest radical form.

Solution:

First, use the quotient property of square roots: $\sqrt{\frac{64}{5}} = \frac{\sqrt{64}}{\sqrt{5}}$

Then

$$\sqrt{\frac{64}{5}} = \frac{\sqrt{64}}{\sqrt{5}}$$

$$= \frac{8}{\sqrt{5}} \qquad \text{Simplify in the numerator.}$$

$$= \frac{8 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} \qquad \text{Multiply numerator and denominator by } \sqrt{5}.$$

$$= \frac{8\sqrt{5}}{5}. \qquad \text{Simplify. (Note that if } x \geq 0, \text{ then } \sqrt{x} \cdot \sqrt{x} = x.)$$

Exponential Form:

By the following definition, a square root can be written in exponential form.

$$\text{If } x \geq 0, \text{ then } x^{\frac{1}{2}} = \sqrt{x}.$$

For example, $36^{\frac{1}{2}} = \sqrt{36} = 6$.

Additional Example 1:

Write each expression in simplest radical form.

(a) $\sqrt{49}$

(b) $(100)^{\frac{1}{2}}$

(c) $\sqrt{28}$

(d) $\left(\frac{25}{81}\right)^{\frac{1}{2}}$

Solution:

(a) $\sqrt{49}$ is the principal square root of 49. Note that $7^2 = 49$ and 7 is positive.
Thus,

$$\sqrt{49} = 7.$$

(b) First, rewrite the expression in radical form: $(100)^{\frac{1}{2}} = \sqrt{100}$

$\sqrt{100}$ is the principal square root of 100. Note that $10^2 = 100$ and 10 is positive.

Thus,

$$\begin{aligned}(100)^{\frac{1}{2}} &= \sqrt{100}\\ &= 10.\end{aligned}$$

(c) First, factor 28 into the product of two numbers so that one of the factors is the largest perfect square that divides 28: $\sqrt{28} = \sqrt{4 \cdot 7}$

Next, use the product rule for square roots: $\sqrt{4 \cdot 7} = \sqrt{4} \cdot \sqrt{7}$

Thus,

$$\begin{aligned}\sqrt{28} &= \sqrt{4 \cdot 7}\\ &= \sqrt{4} \cdot \sqrt{7}\\ &= 2\sqrt{7}.\end{aligned}$$

(d) First, rewrite the expression in radical form: $\left(\frac{25}{81}\right)^{\frac{1}{2}} = \sqrt{\frac{25}{81}}$

Next, use the quotient rule for square roots: $\sqrt{\frac{25}{81}} = \frac{\sqrt{25}}{\sqrt{81}}$

Thus,

$$\begin{aligned}\left(\frac{25}{81}\right)^{\frac{1}{2}} &= \sqrt{\frac{25}{81}}\\ &= \frac{\sqrt{25}}{\sqrt{81}}\\ &= \frac{5}{9}.\end{aligned}$$

Additional Example 2:

Write each expression in simplest radical form.

(a) $\frac{7}{\sqrt{3}}$

(b) $\sqrt{\frac{4}{5}}$

Solution:

(a) First, multiply numerator and denominator by $\sqrt{3}$. Then simplify by noting that if $x \geq 0$, $\sqrt{x} \cdot \sqrt{x} = x$.

Thus,

$$\frac{7}{\sqrt{3}} = \frac{7 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$
$$= \frac{7\sqrt{3}}{3}.$$

(b) $\sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}}$ Use the quotient rule for square roots.

$= \frac{2}{\sqrt{5}}$ Simplify in the numerator.

$= \frac{2 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$ Multiply numerator and denominator by $\sqrt{5}$.

$= \frac{2\sqrt{5}}{5}$ Simplify. Note that if $x \geq 0$, then $\sqrt{x} \cdot \sqrt{x} = x$.

Additional Example 3:

Write each expression in simplest radical form.

(a) $-\frac{\sqrt{18}}{6}$

(b) $-\frac{4}{\sqrt{12}}$

Solution:

(a) First, factor 18 into the product of two numbers so that one of the factors is the largest perfect square that divides 18: $-\frac{\sqrt{18}}{6} = -\frac{\sqrt{9 \cdot 2}}{6}$

Then

$$-\frac{\sqrt{18}}{6} = -\frac{\sqrt{9 \cdot 2}}{6}$$

$= -\frac{\sqrt{9} \cdot \sqrt{2}}{6}$ Use the product rule for square roots.

$= -\frac{3\sqrt{2}}{6}$ Simplify in the numerator.

$$= -\frac{\overset{1}{\cancel{3}}\sqrt{2}}{2\cdot\underset{1}{\cancel{3}}}$$ Divide out common factors.

$$= -\frac{\sqrt{2}}{2}.$$ Simplify.

(b) First, factor 12 into the product of two numbers so that one of the factors is the largest perfect square that divides 12: $-\frac{4}{\sqrt{12}} = -\frac{4}{\sqrt{4\cdot 3}}$

Then

$$-\frac{4}{\sqrt{12}} = -\frac{4}{\sqrt{4\cdot 3}}$$

$$= -\frac{4}{\sqrt{4}\cdot\sqrt{3}}$$ Use the product rule for square roots.

$$= -\frac{4}{2\sqrt{3}}$$ Simplify in the denominator.

$$= -\frac{4\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}}$$ Multiply numerator and denominator by $\sqrt{3}$.

$$= -\frac{4\sqrt{3}}{2\cdot 3}$$ Simplify. (Note that if $x \geq 0$, $\sqrt{x}\cdot\sqrt{x} = x$.)

$$= -\frac{\overset{1}{\cancel{2}}\cdot 2\cdot\sqrt{3}}{\underset{1}{\cancel{2}}\cdot 3}$$ Divide out common factors.

$$= -\frac{2\sqrt{3}}{3}.$$ Simplify.

Exercise Set 1.4: Exponents and Radicals

Write each of the following products instead as a base and exponent. (For example, $6 \cdot 6 = 6^2$)

1. **(a)** $7 \cdot 7 \cdot 7$ **(b)** $10 \cdot 10$
(c) $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ **(d)** $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

2. **(a)** $9 \cdot 9 \cdot 9$ **(b)** $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
(c) $5 \cdot 5 \cdot 5 \cdot 5$ **(d)** $17 \cdot 17$

Fill in the appropriate symbol from the set $\{<, >, =\}$.

3. -7^2 ______ 0

4. $(-9)^4$ ______ 0

5. $(-8)^6$ ______ 0

6. -8^6 ______ 0

7. -10^2 ______ $(-10)^2$

8. -10^3 ______ $(-10)^3$

Evaluate the following.

9. **(a)** 3^1 **(b)** 3^2 **(c)** 3^3
(d) -3^1 **(e)** -3^2 **(f)** -3^3
(g) $(-3)^1$ **(h)** $(-3)^2$ **(i)** $(-3)^3$
(j) 3^0 **(k)** -3^0 **(l)** $(-3)^0$
(m) 3^4 **(n)** -3^4 **(o)** $(-3)^4$

10. **(a)** 5^0 **(b)** $(-5)^0$ **(c)** -5^0
(d) 5^1 **(e)** $(-5)^1$ **(f)** -5^1
(g) 5^2 **(h)** $(-5)^2$ **(i)** -5^2
(j) 5^3 **(k)** $(-5)^3$ **(l)** -5^3
(m) 5^4 **(n)** $(-5)^4$ **(o)** -5^4

11. **(a)** $(0.5)^2$ **(b)** $\left(\frac{1}{5}\right)^2$ **(c)** $\left(-\frac{1}{9}\right)^2$

12. **(a)** $(0.03)^2$ **(b)** $\left(\frac{1}{3}\right)^4$ **(c)** $-\left(\frac{1}{12}\right)^2$

Write each of the following products instead as a base and exponent. (Do not evaluate; simply write the base and exponent.) No answers should contain negative exponents.

13. **(a)** $5^2 \cdot 5^6$ **(b)** $5^{-2} \cdot 5^6$

14. **(a)** $3^8 \cdot 3^5$ **(b)** $3^8 \cdot 3^{-5}$

15. **(a)** $\frac{6^9}{6^2}$ **(b)** $\frac{6^9}{6^{-2}}$

16. **(a)** $\frac{7^9}{7^5}$ **(b)** $\frac{7^9}{7^{-5}}$

17. **(a)** $\frac{4^7 \cdot 4^3}{4^8}$ **(b)** $\frac{4^{11} \cdot 4^{-3}}{4^8 \cdot 4^{-5}}$

18. **(a)** $\frac{8^{12}}{8^5 \cdot 8^4}$ **(b)** $\frac{8^{-4} \cdot 8^9}{8^4 \cdot 8^{-1}}$

19. **(a)** $\left(7^3\right)^6$ **(b)** $\left(\left(5^2\right)^4\right)^3$

20. **(a)** $\left(3^2\right)^4$ **(b)** $\left(\left(2^3\right)^5\right)^4$

Rewrite each expression so that it contains positive exponent(s) rather than negative exponent(s), and then evaluate the expression.

21. **(a)** 5^{-1} **(b)** 5^{-2} **(c)** 5^{-3}

22. **(a)** 3^{-1} **(b)** 3^{-2} **(c)** 3^{-3}

23. **(a)** 2^{-3} **(b)** 2^{-5}

24. **(a)** 7^{-2} **(b)** 10^{-4}

25. **(a)** $\left(\frac{1}{5}\right)^{-1}$ **(b)** $\left(\frac{2}{3}\right)^{-1}$

26. **(a)** $\left(\frac{1}{7}\right)^{-1}$ **(b)** $\left(\frac{6}{5}\right)^{-1}$

27. **(a)** -5^{-2} **(b)** $(-5)^{-2}$

28. **(a)** $(-8)^{-2}$ **(b)** -8^{-2}

Exercise Set 1.4: Exponents and Radicals

Evaluate the following.

29. (a) $\dfrac{-2^3}{2^8}$ **(b)** $\dfrac{2^{-2}}{2^{-6}}$

30. (a) $\dfrac{5^{-1}}{5^2}$ **(b)** $\dfrac{-5^{-1}}{5^{-3}}$

31. (a) $\left(\left(2^3\right)^0\right)^2$ **(b)** $\left(\left(2^{-3}\right)^{-1}\right)^{-2}$

32. (a) $\left(\left(3^{-1}\right)^{-2}\right)^2$ **(b)** $\left(\left(3^{-2}\right)^{-1}\right)^0$

Simplify the following. No answers should contain negative exponents.

33. (a) $\left(3x^3y^4z^{-2}\right)^3$ **(b)** $\left(3x^3y^4z^{-2}\right)^{-3}$

34. (a) $\left(6x^{-5}y^3z^4\right)^2$ **(b)** $\left(6x^{-5}y^3z^4\right)^{-2}$

35. $\dfrac{\left(x^{-3}x^{-4}x^{-6}\right)^{-1}}{x^{-7}}$

36. $\dfrac{x^2x^{-3}x^{-4}}{\left(x^4x^{-1}\right)^{-1}}$

37. $\dfrac{k^3m^2}{k^{-1}\left(m^{-2}\right)^{-3}}$

38. $\dfrac{a^{-4}\left(b^{-3}\right)^4c^7}{a^3b^{-5}c^9}$

39. $\dfrac{-2a^4b^{-3}}{4^{-1}a^0b^{-9}}$

40. $\dfrac{5d^{-7}e^0}{-3^{-1}d^{-2}e^4}$

41. $\dfrac{a^0+b^0}{(a+b)^0}$

42. $\dfrac{c^0-d^0}{(c-d)^0}$

43. $\left(-\dfrac{3a^3b^6}{2a^{-3}b^2}\right)^{-2}$

44. $\left(\dfrac{5a^{-2}b^2}{-6a^2b}\right)^{-3}$

Write each of the following expressions in simplest radical form or as a rational number (if appropriate). If it is already in simplest radical form, say so.

45. (a) $(36)^{\frac{1}{2}}$ **(b)** $\sqrt{7}$ **(c)** $\sqrt{18}$

46. (a) $\sqrt{20}$ **(b)** $\sqrt{49}$ **(c)** $(32)^{\frac{1}{2}}$

47. (a) $(50)^{\frac{1}{2}}$ **(b)** $\sqrt{14}$ **(c)** $\sqrt{\dfrac{81}{16}}$

48. (a) $(19)^{\frac{1}{2}}$ **(b)** $\sqrt{\dfrac{16}{49}}$ **(c)** $\sqrt{55}$

49. (a) $\sqrt{28}$ **(b)** $\sqrt{72}$ **(c)** $(27)^{\frac{1}{2}}$

50. (a) $(45)^{\frac{1}{2}}$ **(b)** $\sqrt{48}$ **(c)** $\sqrt{500}$

51. (a) $\sqrt{54}$ **(b)** $(80)^{\frac{1}{2}}$ **(c)** $\sqrt{60}$

52. (a) $\sqrt{120}$ **(b)** $\sqrt{180}$ **(c)** $(84)^{\frac{1}{2}}$

53. (a) $\sqrt{\dfrac{1}{5}}$ **(b)** $\left(\dfrac{3}{4}\right)^{\frac{1}{2}}$ **(c)** $\sqrt{\dfrac{2}{7}}$

54. (a) $\sqrt{\dfrac{1}{3}}$ **(b)** $\sqrt{\dfrac{5}{9}}$ **(c)** $\left(\dfrac{2}{5}\right)^{\frac{1}{2}}$

55. (a) $\dfrac{7}{\sqrt{4}}$ **(b)** $\dfrac{1}{\sqrt{10}}$ **(c)** $\dfrac{3}{\sqrt{11}}$

56. (a) $\dfrac{1}{\sqrt{6}}$ **(b)** $\dfrac{11}{\sqrt{9}}$ **(c)** $\dfrac{5}{\sqrt{2}}$

57. **(a)** $\sqrt{3^5}$ **(b)** $\sqrt{x^4y^5z^7}$

58. **(a)** $\sqrt{2^7}$ **(b)** $\sqrt{a^2b^9c^5}$

Evaluate the following.

59. **(a)** $\left(\sqrt{5}\right)^2$ **(b)** $\left(\sqrt{6}\right)^4$ **(c)** $\left(\sqrt{2}\right)^6$

60. **(a)** $\left(\sqrt{7}\right)^2$ **(b)** $\left(\sqrt{3}\right)^4$ **(c)** $\left(\sqrt{10}\right)^6$

We can evaluate radicals other than square roots. With square roots, we know, for example, that $\sqrt{49}=7$, since $7^2=49$, and $\sqrt{-49}$ is not a real number. (There is no real number that when squared gives a value of -49, since 7^2 and $(-7)^2$ give a value of 49, not -49. The answer is a complex number, which will not be addressed in this course.) In a similar fashion, we can compute the following:

<u>Cube Roots</u>

$\sqrt[3]{125}=5$, **since** $5^3=125$.

$\sqrt[3]{-125}=-5$, **since** $(-5)^3=-125$.

<u>Fourth Roots</u>

$\sqrt[4]{10,000}=10$, **since** $10^4=10,000$.

$\sqrt[4]{-10,000}$ **is not a real number.**

<u>Fifth Roots</u>

$\sqrt[5]{32}=2$, **since** $2^5=32$.

$\sqrt[5]{-32}=-2$, **since** $(-2)^5=-32$.

<u>Sixth Roots</u>

$\sqrt[6]{\frac{1}{64}}=\frac{1}{2}$, **since** $\left(\frac{1}{2}\right)^6=64$.

$\sqrt[6]{-\frac{1}{64}}$ **is not a real number.**

Evaluate the following. If the answer is not a real number, state "Not a real number."

61. **(a)** $\sqrt{64}$ **(b)** $\sqrt{-64}$ **(c)** $-\sqrt{64}$

62. **(a)** $\sqrt{25}$ **(b)** $\sqrt{-25}$ **(c)** $-\sqrt{25}$

63. **(a)** $\sqrt[3]{8}$ **(b)** $\sqrt[3]{-8}$ **(c)** $-\sqrt[3]{8}$

64. **(a)** $\sqrt[4]{81}$ **(b)** $\sqrt[4]{-81}$ **(c)** $-\sqrt[4]{81}$

65. **(a)** $\sqrt[6]{1,000,000}$ **(b)** $\sqrt[6]{-1,000,000}$ **(c)** $-\sqrt[6]{1,000,000}$

66. **(a)** $\sqrt[5]{32}$ **(b)** $\sqrt[5]{-32}$ **(c)** $-\sqrt[5]{32}$

67. **(a)** $\sqrt[4]{\frac{1}{16}}$ **(b)** $\sqrt[4]{-\frac{1}{16}}$ **(c)** $-\sqrt[4]{\frac{1}{16}}$

68. **(a)** $\sqrt[3]{\frac{1}{27}}$ **(b)** $\sqrt[3]{-\frac{1}{27}}$ **(c)** $-\sqrt[3]{\frac{1}{27}}$

69. **(a)** $\sqrt[5]{\frac{1}{100,000}}$ **(b)** $\sqrt[5]{-\frac{1}{100,000}}$ **(c)** $-\sqrt[5]{\frac{1}{100,000}}$

70. **(a)** $\sqrt[6]{1}$ **(b)** $\sqrt[6]{-1}$ **(c)** $-\sqrt[6]{1}$