Chapter 1

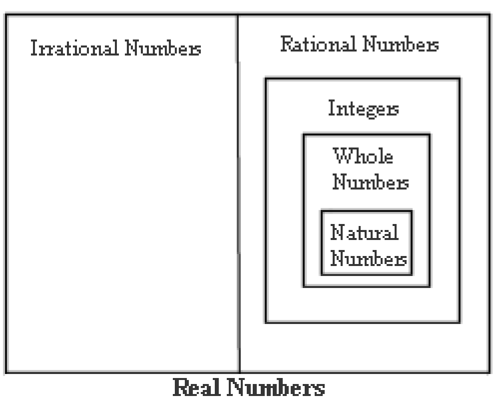
Introductory Information and Review

Section 1.1: Numbers

- Types of Numbers
- Order on a Number Line

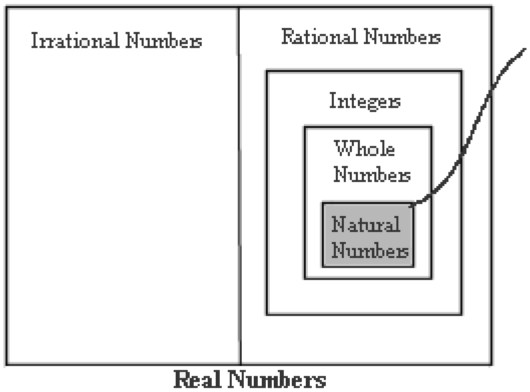
Types of Numbers

In this course, we will work with real numbers. The following diagram shows the types of numbers that form the set of real numbers.



Natural Numbers:

We begin with the right side of the diagram and describe the natural numbers.



The set of natural numbers is $\{1, 2, 3, 4, \ldots\}$.

A natural number is prime if it is greater than 1 and its only factors are itself and 1. A natural number greater than 1 that is not prime is called composite.

Example:
Classify each of the following natural numbers as either prime or composite.
(a) 37
(b) 18

Solution:
(a) Since 37 is a natural number greater than 1 and its only factors are 1 and itself, we see that 37 is a prime number.

(b) Since 18 is a natural number greater than 1 and the factors of 18 are 1, 2, 3, 6, 9, and 18, we see that 18 is a composite number.

Even/Odd Natural Numbers:

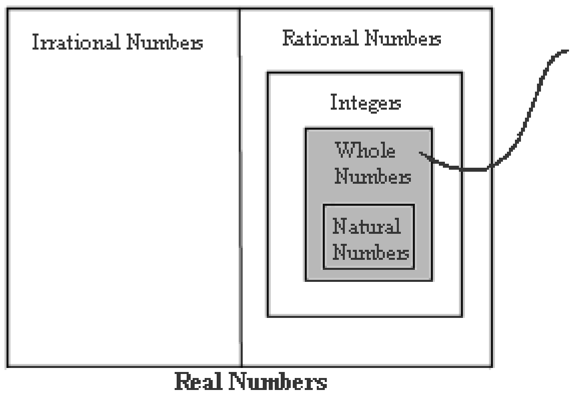
An even natural number is one that can be written in the form $2n$, where n is a natural number. For example, 38 is an even natural number: $38 = 2 \cdot 19$.

The set of even natural number is $\{2, 4, 6, 8, 10, \ldots\}$.

An odd natural number is one that can be written in the form $2n-1$, where n is a natural number. For example, 37 is an odd natural number: $37 = 2 \cdot 19 - 1$.

The set of odd natural number is $\{1, 3, 5, 7, 9, \ldots\}$.

Whole Numbers:



The set of whole numbers is $\{0, 1, 2, 3, \ldots\}$.

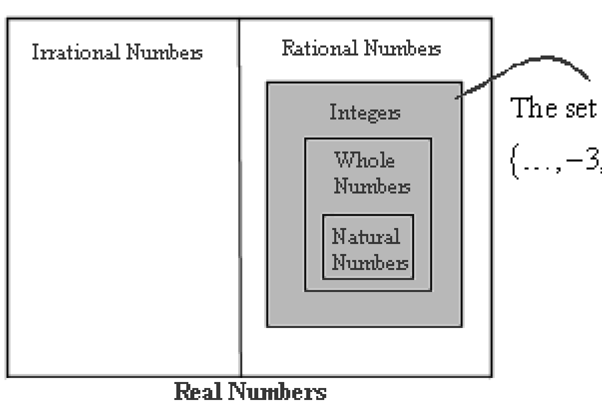
Every natural number is also a whole number.

$$\overbrace{0,\quad \underbrace{1, 2, 3, \ldots}_{\text{natural numbers}}}^{\text{whole numbers}}$$

Example:
True or False: Every whole number is also a natural number.

Solution:
False: 0 is a whole number, but 0 is not a natural number since it does not belong to the set $\{1, 2, 3, \ldots\}$.

Integers:



The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Every whole number is also an integer.

$$\overbrace{\ldots,-3,-2,-1,\underbrace{0,1,2,3,\ldots}_{\text{whole numbers}}}^{\text{integers}}$$

Example:
True or False: Every integer is also a natural number.

Solution:
False: -12 is an integer, but -12 is not a natural number since it does not belong to the set $\{1,2,3,\ \ldots\}$.

Even/Odd Integers:

The integers can be qualified as even or odd:

$\{\ldots,-6,-4,-2,0,2,4,6,\ldots\}$ is the set of even integers; even integers are divisible by 2.

$\{\ldots,-5,-3,-1,1,3,5,\ldots\}$ is the set of odd integers; odd integers are not divisible by 2.

Example:
Classify each of the following integers as either even or odd.

(a) 38

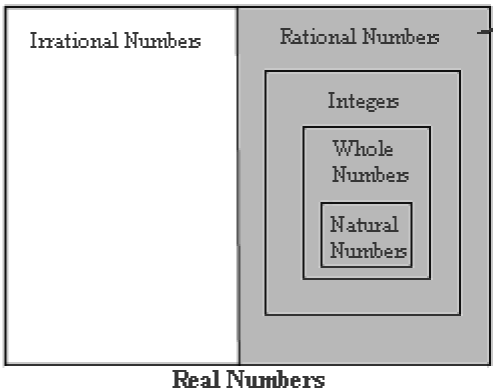
(b) -15

Solution:

(a) The integer 38 is even since it belongs in the set $\{\ldots,-6,-4,-2,0,2,4,6,\ldots\}$.

(b) The integer -15 is odd since it belongs in the set $\{\ldots,-5,-3,-1,1,3,5,\ldots\}$.

Rational Numbers:



A rational number is a number that can be put in the form p/q, where p and $q \neq 0$ are integers.

Every integer is also a rational number. Each integer m can be expressed as the ratio of integers by writing $m = \frac{m}{1}$.

Terminating decimals and repeating decimals are rational numbers since they can be written as a ratio of integers.

Example:
Express each of the following rational numbers as a ratio of integers.

(a) $4\frac{2}{3}$

(b) 1.31

(c) $2.\overline{3}$

(d) 29

Solution:

(a) $4\frac{2}{3}$ is an example of a mixed number. It is the sum of the whole number 4 and the proper fraction $\frac{2}{3}$. (A proper fraction is one in which the numerator is less than the denominator.)

We can write a mixed number as an improper fraction. (An improper fraction is one in which the numerator is greater than or equal to the denominator.)

$$4\frac{2}{3} = \frac{(3)(4)+2}{3} = \frac{12+2}{3} = \frac{14}{3}$$

(b) 1.31 is an example of a terminating decimal.

$$1.31 = 1\frac{31}{100} = \frac{131}{100}$$

(c) $2.\overline{3}$ is an example of a repeating decimal. $2.\overline{3}$ can also be written as 2.33333… . (The three dots are used to indicate that the digits repeat in the pattern indicated.)

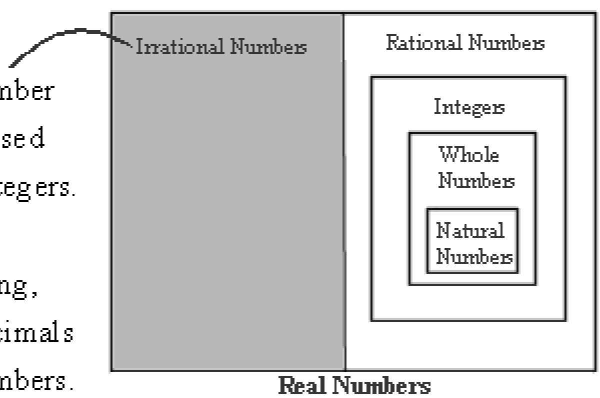
$$2.\overline{3} = 2\frac{1}{3} = \frac{7}{3}$$

(d) $29 = \frac{29}{1}$

Irrational Numbers:

An irrational number cannot be expressed as the ratio of integers.

All nonterminating, nonrepeating decimals are irrational numbers.



Examples of irrational numbers are π, e, $\sqrt{2}$, and $\sqrt{3}$. (The square root of any prime number is an irrational number.)

Approximate values for the four irrational numbers given above are shown below:

$$\pi \approx 3.142$$
$$e \approx 2.718$$
$$\sqrt{2} \approx 1.414$$
$$\sqrt{3} \approx 1.732$$

(The symbol $\approx$ is read "is approximately equal to.")

Real Numbers:

A real number is a number that is either rational or irrational (but not both). Every real number can be written as either a terminating decimal, a repeating decimal, or a nonterminating, nonrepeating decimal.

A real number x is positive if it is greater than 0. This is written as $x > 0$. A real number y is negative if it is less than 0. This is written as $y < 0$. The real number 0 is neither negative nor positive.

Example:
Circle all of the words that can be used to describe the each of the numbers below.

(a) 26 (b) $\sqrt{29}$

Even, Odd, Positive, Negative, Prime, Composite,
Natural, Whole, Integer, Rational, Irrational, Real

Solution:
(a) 26 is a natural number since it belongs to the set $\{1, 2, 3, \ldots\}$.

Thus, 26 is also a whole number, an integer, a rational number, and a real number.

Since 26 is a natural number greater than 1 and the factors of 26 are 1, 2, 13, and 26, we see that 26 is a composite number.

Since $26 > 0$, we see that 26 is positive.

The integer 26 is even since it belongs in the set $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

(b) Since 29 is a prime number, we see that $\sqrt{29}$ is an irrational number. Thus, $\sqrt{29}$ is also a real number.

Note that a real number is either rational or irrational but not both.

Since $\sqrt{29} > \sqrt{25} = 5 > 0$, we see that $\sqrt{29}$ is positive.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

Note About Division Involving Zero:

For each number $m \neq 0$, we have $\frac{0}{m} = 0$. Division by 0 is undefined.

Additional Example 1:
State whether each of the following numbers is prime, composite, or neither. If composite, then list all the factors of the number.

(a) 29

(b) 42

(c) -14

Solution:
The set of natural numbers is $\{1, 2, 3, 4, \ldots\}$. Prime numbers and composite numbers must be natural numbers. A natural number is prime if it is greater than 1 and its only factors are itself and 1. Otherwise, the natural number is composite.

(a) 29 is a natural number since it belongs to the set $\{1, 2, 3, 4, \ldots\}$. The only factors of 29 are itself and 1. Therefore, 29 is a prime number.

(b) 42 is a natural number since it belongs to the set $\{1, 2, 3, 4, \ldots\}$. Note that $42 = 1 \cdot 42 = 2 \cdot 21 = 6 \cdot 7 = 14 \cdot 3$. Thus, the factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42. Therefore, 42 is a composite number.

(c) Prime numbers and composite numbers must belong to the set of natural numbers. -14 is a not a natural number since it does not belong to the set $\{1, 2, 3, 4, \ldots\}$. Therefore, -14 is neither prime nor composite.

Additional Example 2:

Circle all of the words that can be used to describe each of the numbers below.

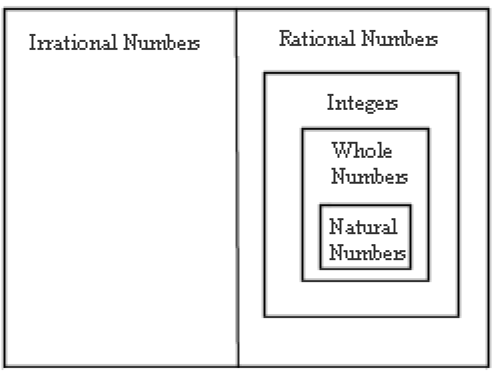
(a) 10 (b) -13

Even, Odd, Positive, Negative, Prime, Composite,
Natural, Whole, Integer, Rational, Irrational, Real

Solution:

We will use the diagram and the definitions given below:

The diagram below illustrates how the natural numbers, whole numbers, integers, rational numbers, and irrational numbers form the set of real numbers.



Real Numbers

Natural Numbers:

The set of natural numbers is $\{1, 2, 3, 4, \ldots\}$.

Whole Numbers:

The set of whole numbers is $\{0, 1, 2, 3, \ldots\}$.

Integers:

The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Prime/Composite Numbers:

A natural number is prime if it is greater than 1 and its only factors are itself and 1; otherwise, it is composite.

Positive/Negative Numbers:

A real number x is positive if it is greater than 0. This is written as $x > 0$. A real number y is negative if it is less than 0. This is written as $y < 0$. The real number 0 is neither negative nor positive.

Even/Odd Numbers:

The integers can be qualified as even or odd:

$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$ is the set of even integers; even integers are divisible by 2.

$\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$ is the set of odd integers; odd integers are not divisible by 2.

Rational Numbers:

A number that can be written in the form p/q, where p and $q \neq 0$ are integers, is a rational number.

(a) 10 is a natural number since it belongs to the set $\{1, 2, 3, \ldots\}$.

From the diagram shown above, we see that 10 is also a whole number, an integer, a rational number, and a real number.

Since 10 is a natural number greater than 1 and the factors of 10 are 1, 2, 5, and 10, we see that 10 is a composite number.

Since $10 > 0$, we see that 10 is positive.

The integer 10 is even since it belongs in the set $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

(b) -13 is an integer since it belongs to the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
From the diagram shown above, -13 is also a rational number and a real number.

Note that -13 does not belong to the following sets of numbers:

$\{0, 1, 2, 3, \ldots\}$ (whole numbers)

$\{1, 2, 3, \ldots\}$ (natural numbers)

Since $-13 < 0$, we see that -13 is negative.

The integer -13 is odd since it belongs in the set $\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

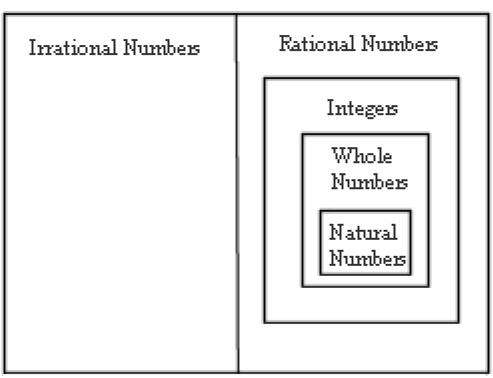
Additional Example 3:
Circle all of the words that can be used to describe each of the numbers below.

(a) $\frac{3}{8}$ (b) $\sqrt{7}$

Even, Odd, Positive, Negative, Prime, Composite,
Natural, Whole, Integer, Rational, Irrational, Real

Solution:
We will use the diagram and the definitions given below:

The diagram below illustrates how the natural numbers, whole numbers, integers, rational numbers, and irrational numbers form the set of real numbers.



Real Numbers

<u>Natural Numbers:</u>
The set of natural numbers is $\{1, 2, 3, 4, \ldots\}$.

<u>Whole Numbers:</u>
The set of whole numbers is $\{0, 1, 2, 3, \ldots\}$.

Integers:

The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Prime/Composite Numbers:

A natural number is prime if it is greater than 1 and its only factors are itself and 1; otherwise, it is composite.

Positive/Negative Numbers:

A real number x is positive if it is greater than 0. This is written as $x > 0$. A real number y is negative if it is less than 0. This is written as $y < 0$. The real number 0 is neither negative nor positive.

Even/Odd Numbers:

The integers can be qualified as even or odd:

$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$ is the set of even integers; even integers are divisible by 2.

$\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$ is the set of odd integers; odd integers are not divisible by 2.

Rational Numbers:

A number that can be written in the form p/q, where p and $q \neq 0$ are integers, is a rational number.

(a) Since 3 and 8 are integers, we see that $\frac{3}{8}$ is a rational number.

Note that $\frac{3}{8}$ does not belong to the following sets of numbers:

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ (integers)

$\{0, 1, 2, 3, \ldots\}$ (whole numbers)

$\{1, 2, 3, \ldots\}$ (natural numbers)

From the diagram shown above, we see that $\frac{3}{8}$ is also a real number.

Since $\frac{3}{8} > 0$, we see that $\frac{3}{8}$ is positive.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

(b) If m is a prime number, then $\sqrt{m}$ is an irrational number. Since 7 is a prime number, we see that $\sqrt{7}$ is an irrational number.

From the diagram shown above, we see that $\sqrt{7}$ is also a real number.

Note that a real number is either rational or irrational but not both.

Since $\sqrt{7} > \sqrt{4} = 2 > 0$, we see that $\sqrt{7}$ is positive.

The results are circled below:

Even, Odd, Positive, Negative,
Prime, Composite, Natural,
Whole, Integer, Rational,
Irrational, Real

Additional Example 4:

Which elements of $\left\{-10.2, -8, 0, \frac{7}{8}, 1.\overline{23}, \sqrt{17}, 23, 25\frac{1}{8}\right\}$ belong to the categories listed below?

(a) Natural, (b) Prime, (c) Composite, (d) Whole, (e) Integer, (f) Even, (g) Odd, (h) Rational, (i) Irrational, (j) Real, (k) Positive, (l) Negative

Solution:

(a) Identify the elements in the given set that are natural numbers.

23 is a natural number since it belongs to the set $\{1, 2, 3, \ldots\}$.

(b/c) Decide which natural numbers are prime and which are composite.

23 is a prime number since it is a natural number greater than 1 and its only factors are itself and 1. There are no composite numbers in the given set.

(d) Identify the elements in the given set that are whole numbers.

0 and 23 are whole numbers since they belong to the set $\{0, 1, 2, 3, \ldots\}$.

(e) Identify the elements in the given set that are integers.

$-8, 0,$ and 23 are integers since they belong to the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

(f/g) Decide which integers are even and which are odd.

The integers -8 and 0 are even since they belong to the set

$$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}.$$

The integer 23 is odd since it belongs to the set

$$\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}.$$

(h) Identify the elements in the given set that are rational numbers.

Recall that a number that can be written in the form p/q, where p and $q \neq 0$ are integers, is a rational number.

All terminating and all repeating decimals are rational numbers since they can be expressed as the ratio of two integers.

$-10.2, -8, 0, \frac{7}{8}, 1.\overline{23}, 23, 25\frac{1}{8}$

are rational numbers:

-10.2 (terminating decimal)

$$-10\frac{2}{10} = -\frac{102}{10} = \frac{-102}{10}$$

$$-8 = \frac{-8}{1}$$

$0 = \frac{0}{1}$

$1.\overline{23}$ (repeating decimal)

$1.\overline{23} = \frac{122}{99}$

$23 = \frac{23}{1}$

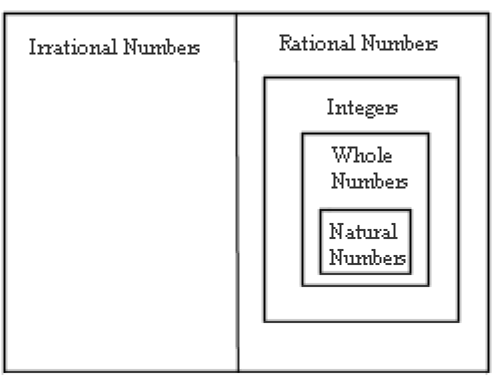
$25\frac{1}{8}$ (mixed number)

$25\frac{1}{8} = \frac{201}{8}$

(i) Identify the elements in the given set that are irrational numbers.

$\sqrt{17}$ is irrational since the square root of a prime number is irrational. Its decimal representation is nonterminating, nonrepeating.

(j) Identify the elements in the given set that are real numbers. Use the diagram below.



Real Numbers

From the diagram shown above, all of the numbers $-10.2, -8, 0, \frac{7}{8}, 1.\overline{23}$, $\sqrt{17}$, 23, and $25\frac{1}{8}$ are real numbers.

(k/l) Identify the elements in the given set that are negative (less than 0) and identify those elements that are positive (greater than 0). Recall that 0 is neither negative nor positive.

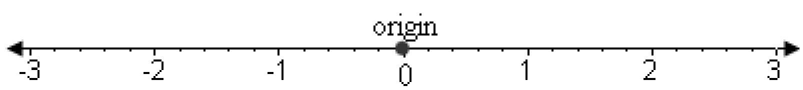
Those elements that are less than 0 and consequently negative are -10.2 and -8.

Those elements that are greater than 0 and consequently positive are $\frac{7}{8}, 1.\overline{23}, \sqrt{17}, 23,$ and $25\frac{1}{8}$.

Order on a Number Line

The Real Number Line:

We can graph real numbers on a number line. For each real number, there corresponds exacly one point on the line. Also, for each point on the line there corresponds exactly exactly one real number. This number is called the coordinate of the point. The point on the real number line whose coordinate is 0 is called the origin.



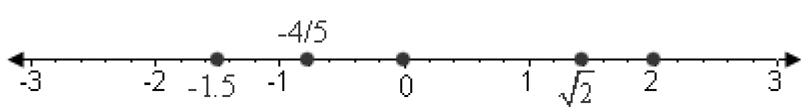
Points to the right of the origin have positive coordinates. Points to the left of the origin have negative coordinates.

Example:
Graph the following numbers on the number line:

$$-1.5, -\frac{4}{5}, 0, \sqrt{2}, 2$$

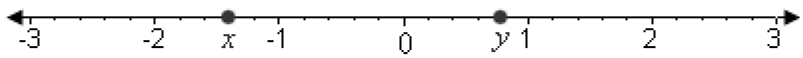
(Hint: Use the approximation: $\sqrt{2} \approx 1.41$.)

Solution:



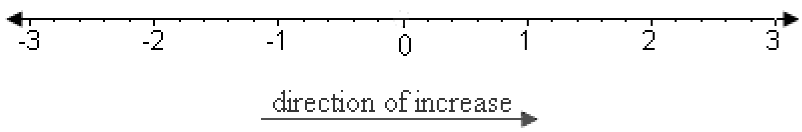
Inequality Symbols:

If a real number x is less than a real number y, then we write $x < y$. The inequality symbol $<$ is read "is less than." On a number line the graph of x is to the left of the graph of y.



$x < y$

The coordinates of points increase as we move left to right on the number line.



The following table describes additional inequality symbols.

Symbol	Meaning	Some Examples
$>$	is greater than	$0 > -4$, $\pi > 3$, $\sqrt{3} > \sqrt{2}$
$\geq$	is greater than or equal to	$2.\overline{4} \geq 2.4$, $\frac{3}{2} \geq 1.5$, $\sqrt{6} \geq \sqrt{5}$
$\leq$	is less than or equal to	$\sqrt{3} \leq 2$, $-1.2 \leq -\frac{6}{5}$, $\sqrt{5} \leq \sqrt{6}$

Example:
For each pair of real numbers, place one of the symbols $<$, $=$, or $>$ in the blank provided.

(a) $-\frac{7}{2}$ ___ $-3\frac{1}{2}$

(b) $\frac{1}{6}$ ___ $\frac{1}{5}$

(c) $\sqrt{5}$ ___ 2

(d) $\frac{5}{2}$ ___ 2.6

Solution:

(a) $-\frac{7}{2} = -3\frac{1}{2}$ Express $-\frac{7}{2}$ as a mixed number: $-\frac{7}{2} = -3\frac{1}{2}$

(b) $\frac{1}{6} < \frac{1}{5}$ Express $\frac{1}{5}$ and $\frac{1}{6}$ as decimals: $\frac{1}{5} = 0.2$ and $\frac{1}{6} = 0.1\overline{6}$

(c) $\sqrt{5} > 2$ Compare squareroots: $\sqrt{5} > \sqrt{4} = 2$

(d) $\frac{5}{2} < 2.6$ Express $\frac{5}{2}$ as a decimal: $\frac{5}{2} = 2\frac{1}{2} = 2.5.$

Example:
Find all natural numbers less than $\sqrt{35}$.

Solution:
Note that $5 = \sqrt{25} < \sqrt{35}$ and $\sqrt{35} < \sqrt{36} = 6$. Thus, $\sqrt{35}$ is between 5 and 6.

Thus, the natural numbers that are less than $\sqrt{35}$ are those natural numbers that are less than 6: 1, 2, 3, 4, 5 .

Example:
Find all even integers between -2.75 and 9.8.

Solution:
The integers between -2.75 and 9.8 must be greater than -2.75 and less than 9.8. They are $-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9.

The even integers on the list are $-2, 0, 2, 4, 6,$ and 8.

Additional Example 1:
List the prime numbers between 30 and 40.

Solution:
A prime number is a natural number greater than 1 whose only factors are itself and 1.

The set of natural numbers is $\{1, 2, 3, 4, \ldots\}$. The natural numbers between 30 and 40 are greater than 30 and less than 40:

31, 32, 33, 34, 35, 36, 37, 38, 39

Omit the even natural numbers between 30 and 40. They are not prime since they each have a factor of 2.

31, $\cancel{32}$, 33, $\cancel{34}$, 35, $\cancel{36}$, 37, $\cancel{38}$, 39

Find the factors of the odd natural numbers between 30 and 40.

Odd Natural Number	Factors
31	1, 31
33	1, 3, 11, 33
35	1, 5, 7, 35
37	1, 37
39	1, 3, 13, 39

Using the results in the table we see that the prime numbers between 30 and 40 are 31 and 37.

The numbers 32, 33, 34, 35, 36, 38, and 39 are composite numbers.

Additional Example 2:
List the composite numbers between $\sqrt{12}$ and 5π and list all the factors of these numbers. (Hint: Use the approximation: $\pi \approx 3.14$.)

Solution:
The given numbers are irrational.

Note that $3 = \sqrt{9} < \sqrt{12}$ and $\sqrt{12} < \sqrt{16} = 4$. Thus, $\sqrt{12}$ is between 3 and 4.

5π means $5 \cdot \pi$. From the given hint, $5\pi \approx 15.7$.

List the natural numbers between $\sqrt{12}$ and 5π. They must be greater than $\sqrt{12}$ and less than 5π.

The natural numbers between $\sqrt{12}$ and 5π are $4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,$ and 15.

Omit the prime numbers from the list: $4, \cancel{5}, 6, \cancel{7}, 8, 9, 10, \cancel{11}, 12, \cancel{13}, 14, 15$

The remaining numbers in the list are composite numbers. Each of these natural numbers has factors other than itself and 1.

The composite numbers between $\sqrt{12}$ and 5π are 4, 6, 8, 9, 10, 12, 14, and 15.

The factors are shown in the table below.

Composite Number	Factors
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
12	1, 2, 3, 4, 6, 12
14	1, 2, 7, 14
15	1, 3, 5, 15

Additional Example 3:

List the composite numbers between $\sqrt{12}$ and 5π and list all the factors of these numbers. (Hint: Use the approximation: $\pi \approx 3.14$.)

Solution:

The given numbers are irrational.

Note that $3 = \sqrt{9} < \sqrt{12}$ and $\sqrt{12} < \sqrt{16} = 4$. Thus, $\sqrt{12}$ is between 3 and 4.

5π means $5 \cdot \pi$. From the given hint, $5\pi \approx 15.7$.

List the natural numbers between $\sqrt{12}$ and 5π. They must be greater than $\sqrt{12}$ and less than 5π.

The natural numbers between $\sqrt{12}$ and 5π are 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15.

Omit the prime numbers from the list: $4, \cancel{5}, 6, \cancel{7}, 8, 9, 10, \cancel{11}, 12, \cancel{13}, 14, 15$

The remaining numbers in the list are composite numbers. Each of these natural numbers has factors other than itself and 1.

The composite numbers between $\sqrt{12}$ and 5π are 4, 6, 8, 9, 10, 12, 14, and 15.

The factors are shown in the table below.

Composite Number	Factors
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
12	1, 2, 3, 4, 6, 12
14	1, 2, 7, 14
15	1, 3, 5, 15

Additional Example 4:

Fill in the appropriate symbol from the set $\{<, >, =\}$.

(a) $\sqrt{11}$ ______ 11

(b) $-\sqrt{36}$ ______ -6

(c) $-\frac{1}{8}$ ______ $-\frac{1}{7}$

(d) 6.32 ______ $\frac{63}{10}$

Solution:

(a) Note that $\sqrt{11} < \sqrt{16} = 4$. Thus, $\sqrt{11} \quad < \quad 11$.

(b) Note that $\sqrt{36} = 6$. Thus, $-\sqrt{36} \quad = \quad -6$ since $-\sqrt{36} = (-1)\sqrt{36} = -6$.

(c) Represent $-\frac{1}{8}$ and $-\frac{1}{7}$ as decimals. Since both numbers are rational, the decimals will either be terminating or repeating.

$$-\frac{1}{7} = -0.\overline{142857}$$

$$-\frac{1}{8} = -0.125$$

Thus, $-\frac{1}{8} > -\frac{1}{7}$.

(d) Representing $\frac{63}{10}$ as a decimal, we see that $\frac{63}{10} = 6\frac{3}{10} = 6.3$.

Thus, $6.32 > \frac{63}{10}$.

Exercise Set 1.1: Numbers

State whether each of the following numbers is prime, composite, or neither. If composite, then list all the factors of the number.

1. **(a)** 8 **(b)** 5 **(c)** 1 **(d)** -7 **(e)** 12

2. **(a)** 11 **(b)** -6 **(c)** 15 **(d)** 0 **(e)** -2

Answer the following.

3. In (a)-(e), use long division to change the following fractions to decimals.

(a) $\frac{1}{9}$ **(b)** $\frac{2}{9}$ **(c)** $\frac{3}{9}$
(d) $\frac{4}{9}$ **(e)** $\frac{5}{9}$ *Note:* $\frac{3}{9}=\frac{1}{3}$

Notice the pattern above and use it as a shortcut in (f)-(m) to write the following fractions as decimals without performing long division.

(f) $\frac{6}{9}$ **(g)** $\frac{7}{9}$ **(h)** $\frac{8}{9}$
(i) $\frac{9}{9}$ **(j)** $\frac{10}{9}$ **(k)** $\frac{14}{9}$
(l) $\frac{25}{9}$ **(m)** $\frac{29}{9}$ *Note:* $\frac{6}{9}=\frac{2}{3}$

4. Use the patterns from the problem above to change each of the following decimals to either a proper fraction or a mixed number.

(a) $0.\overline{4}$ **(b)** $0.\overline{7}$ **(c)** $2.\overline{3}$
(d) $1.\overline{2}$ **(e)** $4.\overline{5}$ **(f)** $7.\overline{6}$

State whether each of the following numbers is rational or irrational. If rational, then write the number as a ratio of two integers. (If the number is already written as a ratio of two integers, simply rewrite the number.)

5. **(a)** 0.7 **(b)** $\sqrt{5}$ **(c)** $\frac{3}{7}$
(d) -5 **(e)** $\sqrt{16}$ **(f)** $0.\overline{3}$
(g) 12 **(h)** $\frac{2.3}{3.5}$ **(i)** e
(j) $-\sqrt{4}$ **(k)** 0.04004000400004...

6. **(a)** $\sqrt{\pi}$ **(b)** $0.\overline{6}$ **(c)** $\sqrt{8}$
(d) $\frac{1.3}{4.7}$ **(e)** $-\frac{4}{5}$ **(f)** $-\sqrt{9}$
(g) 3.1 **(h)** -10 **(i)** 0
(j) $\frac{7}{9}$ **(k)** 0.03003000300003…

Circle all of the words that can be used to describe each of the numbers below.

7. -9

Even	Odd	Positive	Negative
Prime	Composite	Natural	Whole
Integer	Rational	Irrational	Real
Undefined			

8. $0.\overline{7}$

Even	Odd	Positive	Negative
Prime	Composite	Natural	Whole
Integer	Rational	Irrational	Real
Undefined			

9. $\sqrt{2}$

Even	Odd	Positive	Negative
Prime	Composite	Natural	Whole
Integer	Rational	Irrational	Real
Undefined			

10. $-\frac{4}{7}$

Even	Odd	Positive	Negative
Prime	Composite	Natural	Whole
Integer	Rational	Irrational	Real
Undefined			

Answer the following.

11. Which elements of the set $\left\{-8,\ -2.1,\ -0.\overline{4},\ 0,\ \sqrt{7},\ \pi,\ \frac{15}{4},\ 5,\ 12\right\}$ belong to each category listed below?

(a) Even **(b)** Odd
(c) Positive **(d)** Negative
(e) Prime **(f)** Composite
(g) Natural **(h)** Whole
(i) Integer **(j)** Real
(k) Rational **(l)** Irrational
(m) Undefined

Exercise Set 1.1: Numbers

12. Which elements of the set $\left\{-6.25,\ -4\tfrac{3}{4},\ -3,\ -\sqrt{5},\ -1,\ \tfrac{2}{5},\ 1,\ 2,\ 10\right\}$ belong to each category listed below?

(a) Even	**(b)** Odd
(c) Positive	**(d)** Negative
(e) Prime	**(f)** Composite
(g) Natural	**(h)** Whole
(i) Integer	**(j)** Real
(k) Rational	**(l)** Irrational
(m) Undefined	

Fill in each of the following tables. Use "Y" for yes if the row name applies to the number or "N" for no if it does not.

13.

	$\frac{\sqrt{25}}{0}$	1	$5\frac{3}{10}$	-55	$13.\overline{3}$
Undefined					
Natural					
Whole					
Integer					
Rational					
Irrational					
Prime					
Composite					
Real					

14.

	2.36	$\frac{0}{5}=0$	$\frac{\sqrt{2}}{2}$	$\frac{2}{7}$	$\sqrt{9}=3$
Undefined					
Natural					
Whole					
Integer					
Rational					
Irrational					
Prime					
Composite					
Real					

Answer the following. If no such number exists, state "Does not exist."

15. Find a number that is both prime and even.
16. Find a rational number that is a composite number.
17. Find a rational number that is not a whole number.
18. Find a prime number that is negative.
19. Find a real number that is not a rational number.
20. Find a whole number that is not a natural number.
21. Find a negative integer that is not a rational number.
22. Find an integer that is not a whole number.
23. Find a prime number that is an irrational number.
24. Find a number that is both irrational and odd.

Answer True or False. If False, justify your answer.j

25. All natural numbers are integers.
26. No negative numbers are odd.
27. No irrational numbers are even.
28. Every even number is a composite number.
29. All whole numbers are natural numbers.
30. Zero is neither even nor odd.
31. All whole numbers are integers.
32. All integers are rational numbers.
33. All nonterminating decimals are irrational numbers.
34. Every terminating decimal is a rational number.

Answer the following.

35. List the prime numbers less than 10.
36. List the prime numbers between 20 and 30.
37. List the composite numbers between 7 and 19.
38. List the composite numbers between 31 and 41.
39. List the even numbers between $\sqrt{13}$ and $\sqrt{97}$.
40. List the odd numbers between $\sqrt{29}$ and $\sqrt{123}$.

Exercise Set 1.1: Numbers

Fill in the appropriate symbol from the set $\{<, >, =\}$.

41. $\sqrt{7}$ ______ 7

42. 3 ______ $\sqrt{3}$

43. $-\sqrt{7}$ ______ -7

44. -3 ______ $-\sqrt{3}$

45. $\sqrt{81}$ ______ 9

46. -5 ______ $-\sqrt{25}$

47. 5.32 ______ $\frac{53}{10}$

48. $\frac{7}{100}$ ______ $0.0\overline{7}$

49. $\frac{1}{3}$ ______ $\frac{1}{4}$

50. $\frac{1}{6}$ ______ $\frac{1}{5}$

51. $-\frac{1}{3}$ ______ $-\frac{1}{4}$

52. $-\frac{1}{6}$ ______ $-\frac{1}{5}$

53. $\sqrt{15}$ ______ 4

54. 7 ______ $\sqrt{49}$

55. -3 ______ $-\sqrt{9}$

56. $\sqrt{29}$ ______ 5

Answer the following.

57. Find the additive inverse of the following numbers. If undefined, write "undefined."
(a) 3 **(b)** -4 **(c)** 1
(d) $-\frac{2}{3}$ **(e)** $2\frac{3}{7}$

58. Find the multiplicative inverse of the following numbers. If undefined, write "undefined."
(a) 3 **(b)** -4 **(c)** 1
(d) $-\frac{2}{3}$ **(e)** $2\frac{3}{7}$

59. Find the multiplicative inverse of the following numbers. If undefined, write "undefined."
(a) -2 **(b)** $\frac{5}{9}$ **(c)** 0
(d) $1\frac{3}{5}$ **(e)** -1

60. Find the additive inverse of the following numbers. If undefined, write "undefined."
(a) -2 **(b)** $\frac{5}{9}$ **(c)** 0
(d) $1\frac{3}{5}$ **(e)** -1

61. Place the correct number in each of the following blanks:
(a) The sum of a number and its additive inverse is _____. (Fill in the correct number.)
(b) The product of a number and its multiplicative inverse is _____. (Fill in the correct number.)

62. Another name for the multiplicative inverse is the ____________________.

Order the numbers in each set from least to greatest and plot them on a number line.
(Hint: Use the approximations $\sqrt{2} \approx 1.41$ and $\sqrt{3} \approx 1.73$.)

63. $\left\{-1,\ -\sqrt{2},\ 0.\overline{4},\ \frac{0}{5},\ -\frac{9}{4},\ \sqrt{0.49}\right\}$

64. $\left\{-\sqrt{3},\ 1,\ 0.65,\ \frac{2}{3},\ -1.\overline{5},\ \sqrt{0.64}\right\}$