Complete the table of values for $I\left(y\right)=\int_0^1 xyy'dx$.

y	$I\left(y\right)$
1	0
x	
x^2	
$\sqrt{x}$	
e^x	

Show that $h(x)=\begin{cases}(x-d)^2(x-e)^2 & ;d\le x\le e\\ 0 & ;otherwise\end{cases}$ is continuously differentiable on $[a,b]$ by examining its derivative at $x=d$ and $x=e$.

See if you can find the Euler-Lagrange equation for

$$I(y) = \int_0^{\frac{\pi}{2}} \underbrace{\left[y^2 - (y')^2 \right]}_{F(x,y,y')} dx$$

$$\{Hint:\ F_y - \frac{d}{dx}\left(F_{y'}\right) = 0.\}$$

Find all optimal solution candidates of

$$I(y) = \int_0^{\frac{\pi}{2}} \left[y^2 - (y')^2 \right] dx$$

Subject to

$$y(0) = 0 \quad \text{and} \quad y\left(\frac{\pi}{2}\right) = 1$$

{Hint: You need to solve $y'' + y = 0$, $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1.$*}*

If $\int_a^b g^2(x)\,dx = 0$, then g must be zero on $[a,b]$, and so

$\int_a^b f(x)g(x)\,dx = 0$. So the inequality is true in this case. Now suppose

that $\int_a^b g^2(x)\,dx \neq 0$, and consider $\int_a^b \left[f(x)-\lambda g(x)\right]^2 dx$.

$\int_a^b \left[f(x)-\lambda g(x)\right]^2 dx \geq 0$, and if we expand out the integrand, we get that

$$\int_a^b \left[f(x)-\lambda g(x)\right]^2 dx = \underbrace{\left[\int_a^b g^2(x)\,dx\right]\lambda^2 - 2\left[\int_a^b f(x)g(x)\,dx\right]\lambda + \int_a^b f^2(x)\,dx}_{\text{quadratic in } \lambda}$$

If this quadratic is always ≥ 0, then find a condition on its discriminant.

$$\int_0^{\frac{3\pi}{2}} \left[h_1(x)^2 - \left[h_1'(x) \right]^2 \right] dx = \int_0^{\frac{3\pi}{2}} \left[\sin^2 2x - 4\cos^2 2x \right] dx$$

$$=$$

$$\int_0^{\frac{3\pi}{2}} \left[h_2(x)^2 - \left[h_2'(x) \right]^2 \right] dx = \int_0^{\frac{3\pi}{2}} \left[x^2 \left(\frac{3\pi}{2} - x \right)^2 - \left(2x - \frac{3\pi}{2} \right)^2 \right] dx$$

$$=$$

See if you can find the Euler-Lagrange equation for

$$I(y) = \int_0^{\ln 2} \underbrace{\left[y^2 + (y')^2 \right]}_{F(x,y,y')} dx$$

{*Hint:* $F_y - \frac{d}{dx}\left(F_{y'}\right) = 0$.}

Find all optimal solution candidates of

$$I(y) = \int_{0}^{\ln 2} \left[y^2 + (y')^2 \right] dx$$

Subject to

$$y(0) = 0 \quad \text{and} \quad y(\ln 2) = \frac{3}{4}$$

{Hint: You need to solve $y'' - y = 0$, $y(0) = 0$, $y(\ln 2) = \frac{3}{4}$.*}*

See if you can determine if it's a maximum or minimum.

$$I\left(\sinh x+h(x)\right)=\int_0^{\ln 2}\left[\left[\sinh x+h(x)\right]^2+\left[\cosh x+h'(x)\right]^2\right]dx$$

$$=\underbrace{\int_0^{\ln 2}\left(\sinh^2 x+\cosh^2 x\right)dx}_{I(\sinh x)}+2\int_0^{\ln 2}\left(h(x)\sinh x+h'(x)\cosh x\right)dx$$

$$+\int_0^{\ln 2}\left[h^2(x)+\left[h'(x)\right]^2\right]dx$$

See if you can find the Euler-Lagrange equation for

$$I\left(y\right)=\int_a^b \underbrace{\sqrt{1+\left(y'\right)^2}}_{F\left(x,y,y'\right)}\,dx$$

$\{Hint\colon\ F_y-\dfrac{d}{dx}\left(F_{y'}\right)=0.\}$

Find all optimal solution candidates of

$$I(y) = \int_a^b \sqrt{1 + (y')^2}\, dx$$

Subject to

$$y(a) = c \quad \text{and} \quad y(b) = d$$

{*Hint: Solve* $y' = K$,$y(a) = c$, $y(b) = d$.}

Show that the only solution of

$$h''(x) = 0$$

$$h(a) = h(z) = 0$$

is $h(x) = 0$ on $[a, z]$.

See if you can find the Euler-Lagrange equation for

$$I(y) = \int_0^1 \underbrace{\left[(y')^2 + 12xy \right]}_{F(x,y,y')} dx$$

$\{Hint\!: \; F_y - \dfrac{d}{dx}\left(F_{y'}\right) = 0.\}$

Find all optimal solution candidates of

$$I(y) = \int_0^1 \left[(y')^2 + 12xy \right] dx$$

Subject to

$$y(0) = 0 \text{ and } y(1) = 1$$

{Hint: Solve $y'' = 6x$, $y(0) = 0$, $y(1) = 1$.*}*

See if you can determine if it's a maximum or minimum?

{*Hint:* $I\left(x^3+h\right)=\int_0^1\left[\left(3x^2+h'\right)^2+12x\left(x^3+h\right)\right]dx$.}

$$I(y)=\int_0^1 xyy'dx$$

$F(x,y,y')=xyy'$, so $F_y=xy'$ and $F_{y'}=xy$. See what happens when you try to solve $F_y-\frac{d}{dx}F_{y'}=0$ subject to $y(0)=0$ and $y(1)=1$.

The large rings:

Immerse both rings into the soap solution and carefully remove them. Position them above the measuring stick and gently pull them apart trying to keep the rings perpendicular to the measuring stick. At the start, the catenoid itself will act as a framework for the soap film, and you will need someone else to break the disk of soap film that forms in the center. Once this disk is broken, the catenoid you see will be the minimal one. Try to record the separation distance at which the catenoid makes a transition into the two disks.

Transition distance:________________________________

The medium rings:

Immerse both rings into the soap solution and carefully remove them. Position them above the measuring stick and gently pull them apart trying to keep the rings perpendicular to the measuring stick. At the start, the catenoid itself will act as a framework for the soap film, and you will need someone else to break the disk of soap film that forms in the center. Once this disk is broken, the catenoid you see will be the minimal one. Try to record the separation distance at which the catenoid makes a transition into the two disks.

Transition distance:_________________________________

The small rings:

Immerse both rings into the soap solution and carefully remove them. Position them above the measuring stick and gently pull them apart trying to keep the rings perpendicular to the measuring stick. At the start, the catenoid itself will act as a framework for the soap film, and you will need someone else to break the disk of soap film that forms in the center. Once this disk is broken, the catenoid you see will be the minimal one. Try to record the separation distance at which the catenoid makes a transition into the two disks.

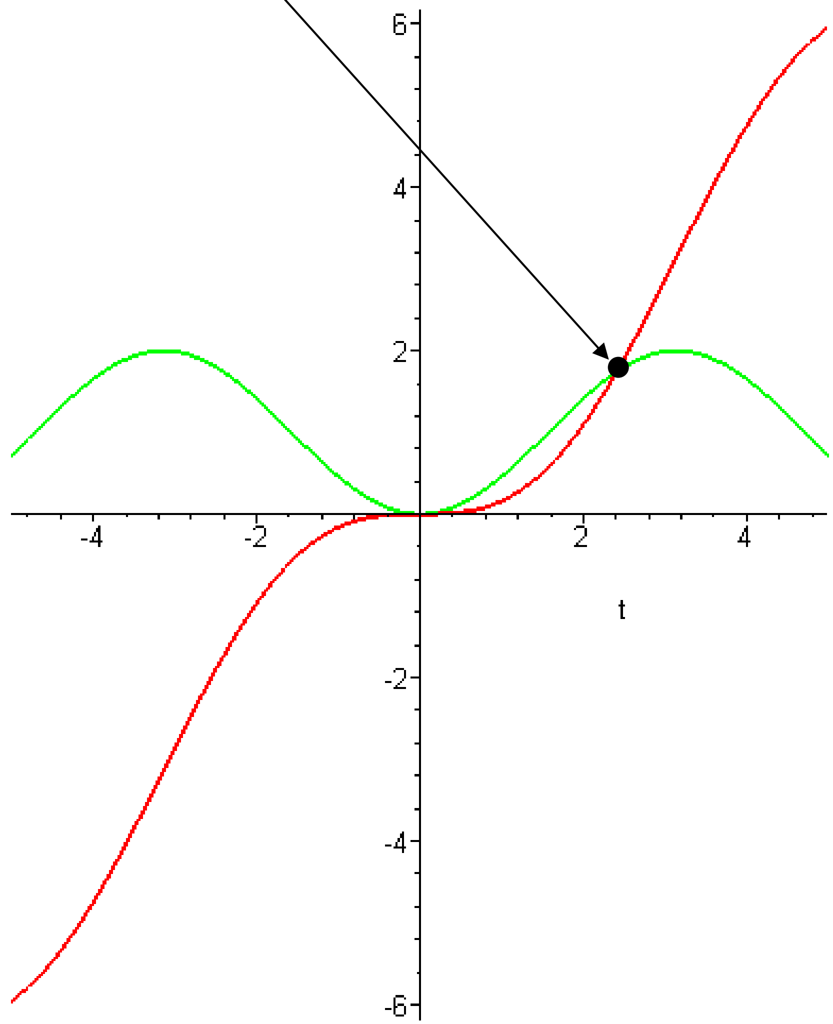
Transition distance:________________________________

1. If $dx = \dfrac{1}{C^2}\sin^2\left(\tfrac{t}{2}\right)dt$, then $x = \dfrac{1}{C^2}\int \sin^2\left(\tfrac{t}{2}\right)dt$. Use the trigonometric identity $\sin^2\theta = \dfrac{1}{2}\left(1-\cos 2\theta\right)$ to get the result.

2. Solve for k if $x = \dfrac{1}{2C^2}\left(t-\sin t\right)+k$ and $x\left(0\right)=a$.

The t-coordinate of the intersection point of the curves $t - \sin t$ and $1 - \cos t$ between 2 and 3

is approximately ______________.



If $g(x,y)=0$, and $\frac{\partial g}{\partial y}\neq 0$ on $g(x,y)=0$, then differentiate both sides with respect to x to get $\frac{\partial g}{\partial x}\cdot\frac{dx}{dx}+\frac{\partial g}{\partial y}\cdot\frac{dy}{dx}=0$ and solve for $\frac{dy}{dx}$.

If you substitute $y = x$ into $x^2 + y^2 + xy - 9 = 0$, you get $x^2 + x^2 + x^2 - 9 = 0$, so find the intersection points.

If you substitute $y = -x$ into $x^2 + y^2 + xy - 9 = 0$, you get $x^2 + x^2 - x^2 - 9 = 0$, so find the intersection points.

See if you can approximately solve

$$2\tan^{-1}\left(-\frac{1}{2c}\right)\sqrt{c^2+\tfrac{1}{4}}=\frac{5}{4}$$

$c \approx$ ______________________

Again, the solution curve is a circular arc or a line segment that passes through the points $(0,0)$ and $(1,0)$. Again, the line segment can be eliminated, so the solution curve will have the form $y = \sqrt{r^2 - \left(x - \frac{1}{2}\right)^2} + c$. The endpoint conditions lead to $c = -\sqrt{r^2 - \frac{1}{4}}$, and you need to solve $2\tan^{-1}\left(-\frac{1}{2c}\right)\sqrt{c^2 + \frac{1}{4}} = \frac{\pi}{2}$, which has a limiting solution. Determine $\lim_{c \to 0^-}\left[2\tan^{-1}\left(-\frac{1}{2c}\right)\sqrt{c^2 + \frac{1}{4}}\right]$, to get the solution, and solve the optimization problem.