UH Math Circle

November 5, 2011

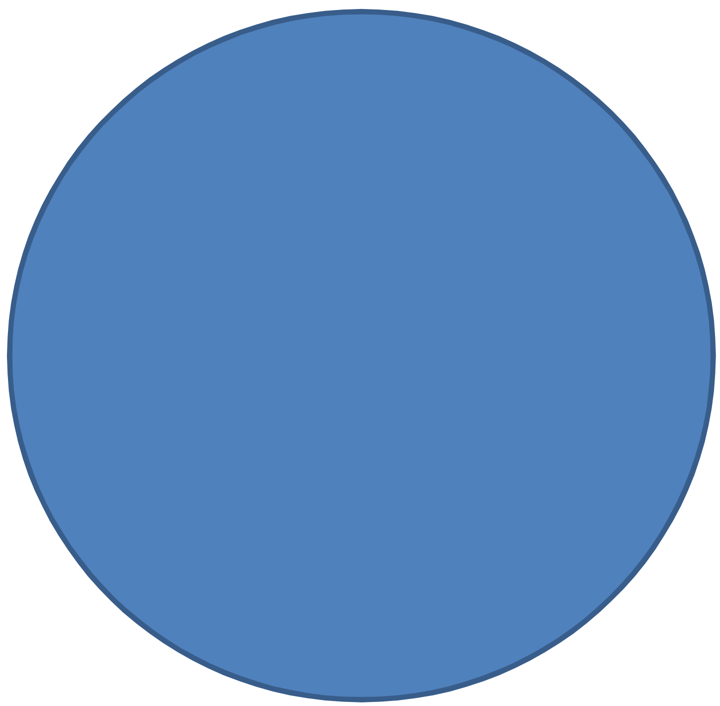
Today

- Introductions
- Food for Thought – Old and New
- What does this do? – Old and New
- Project problem. – Old and New

Food for Thought – I

(first presented on October 1st)

Can you divide a circular disk into two or more congruent pieces so that at least one of the pieces does not touch the center of the circle?



Food for Thought - II

(first presented on October 1st)

Pick a value in the first row. Then move forward that number from left to right and top to bottom. Keep going until you cannot complete a process.

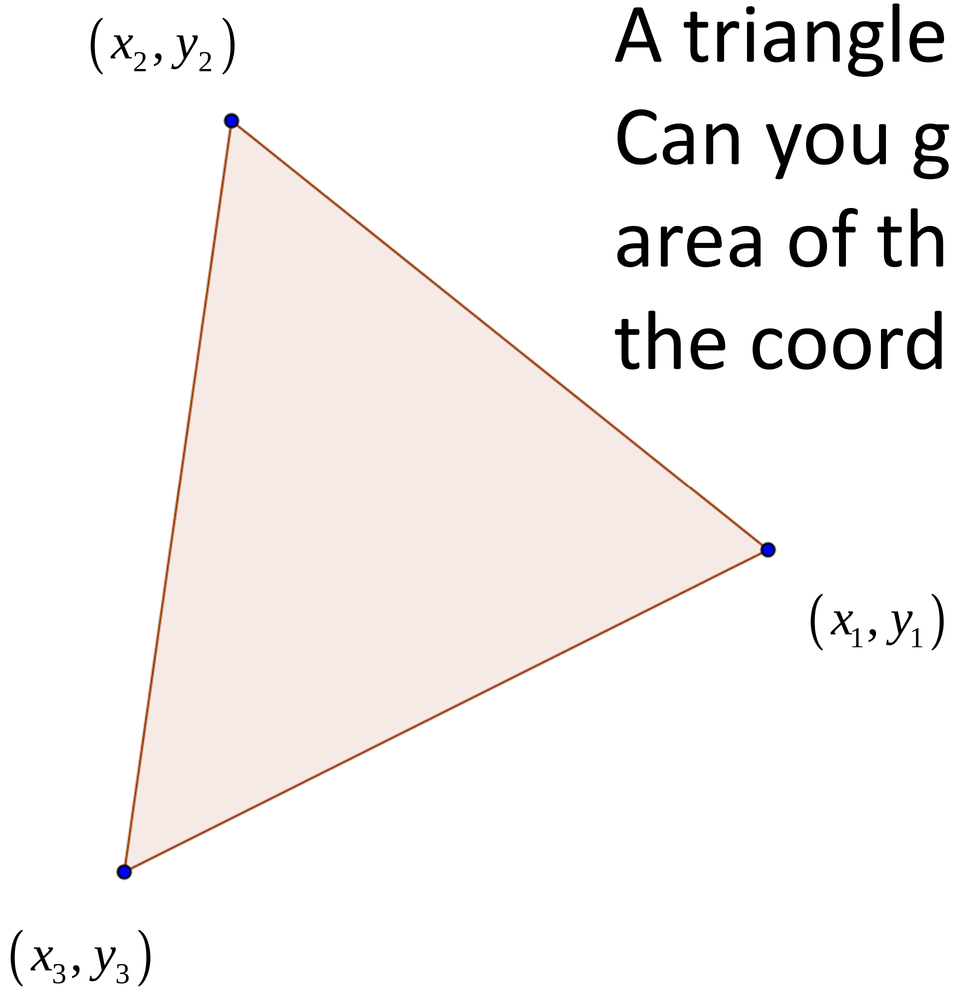
4	1	5	3	3	5	2	4
3	2	2	5	1	5	2	5
2	4	2	1	3	4	2	3
3	5	4	3	2	3	3	3
1	1	1	3	5	5	5	5
1	2	1	5	5	5	3	3

In this case, you will always land on the 4th entry in the last row. Something similar will happen nearly EVERY time a list of numbers is generated in a random manner.

Question: Can you create a grid where the last value can be different depending upon where you start?

Food for Thought – III

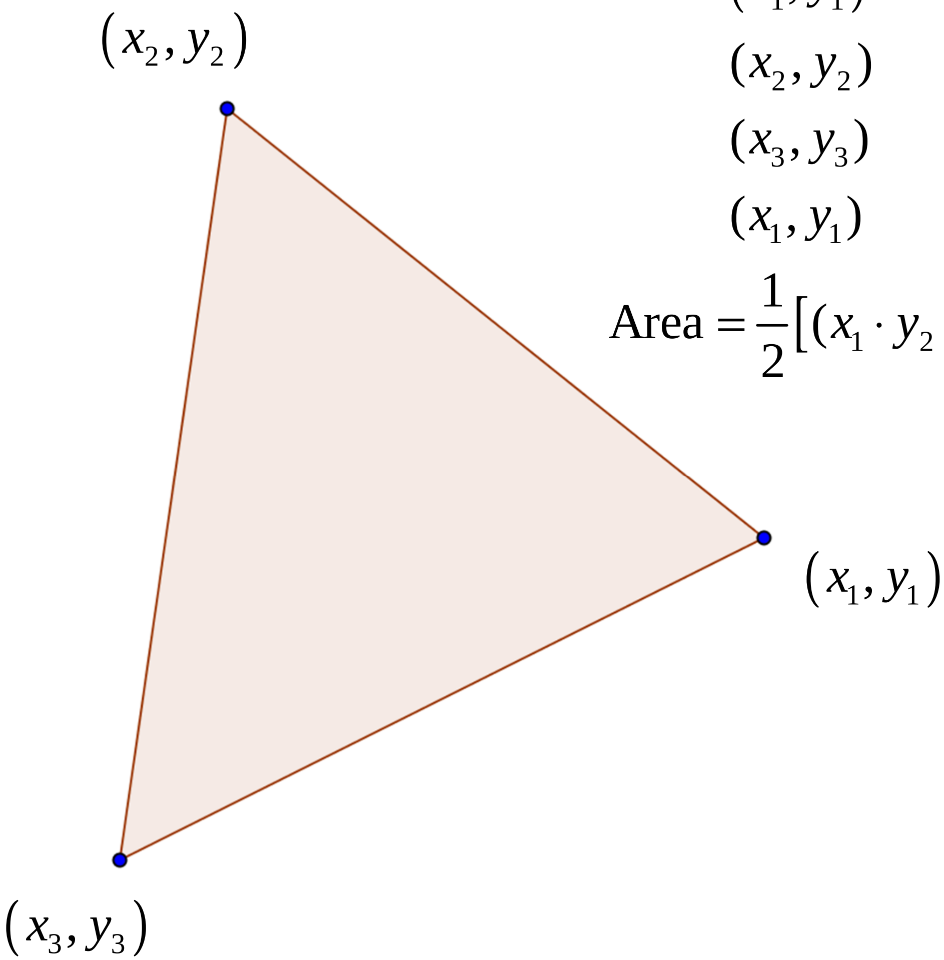
(first presented on October 1st)



A triangle is shown on the left. Can you give a formula for the area of the triangle in terms of the coordinates of the vertices?

Areas of Triangles - FYI

List the vertices ***counter clockwise*** from first to first. It does not matter which vertex you list as "first".



(x_1, y_1)

(x_2, y_2)

(x_3, y_3)

(x_1, y_1)

Notice the cross multiplication in the formula below from point to point.

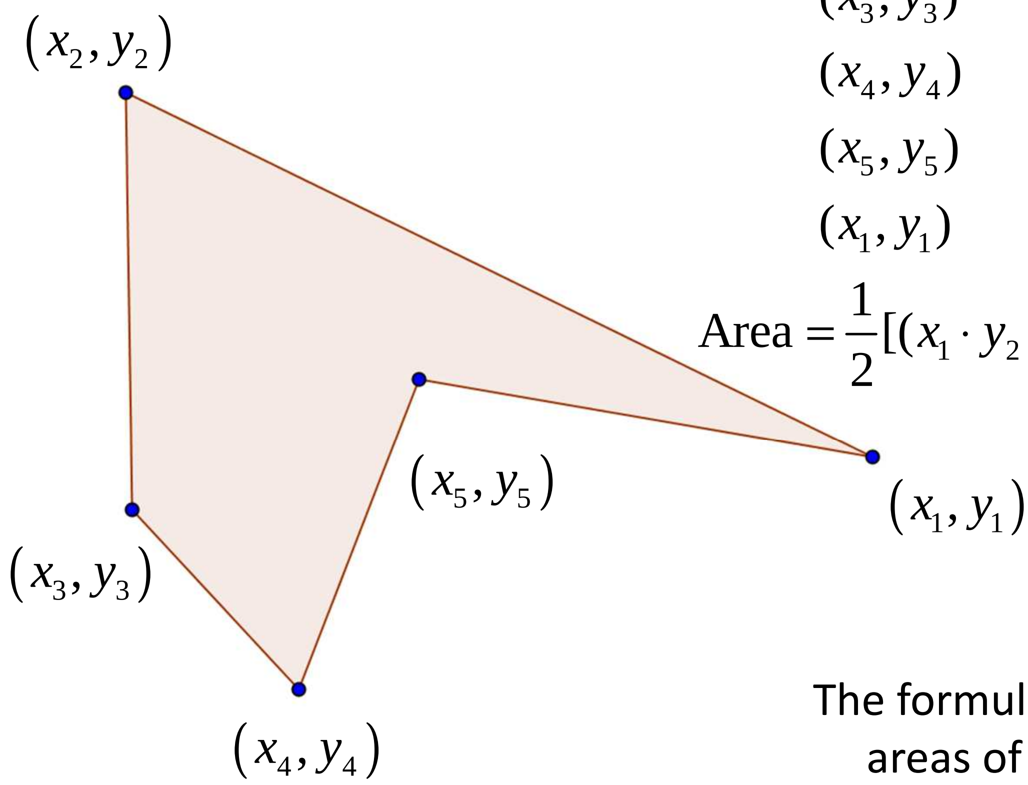
$$\text{Area} = \frac{1}{2}\left[(x_1 \cdot y_2 - y_1 \cdot x_2) + (x_2 \cdot y_3 - y_2 \cdot x_3) + (x_3 \cdot y_1 - y_3 \cdot x_1)\right]$$

Areas of Polygons - FYI

List the vertices ***counter clockwise*** from first to first. It does not matter which vertex you list as "first".

$$(x_1, y_1)$$
$$(x_2, y_2)$$
$$(x_3, y_3)$$
$$(x_4, y_4)$$
$$(x_5, y_5)$$
$$(x_1, y_1)$$

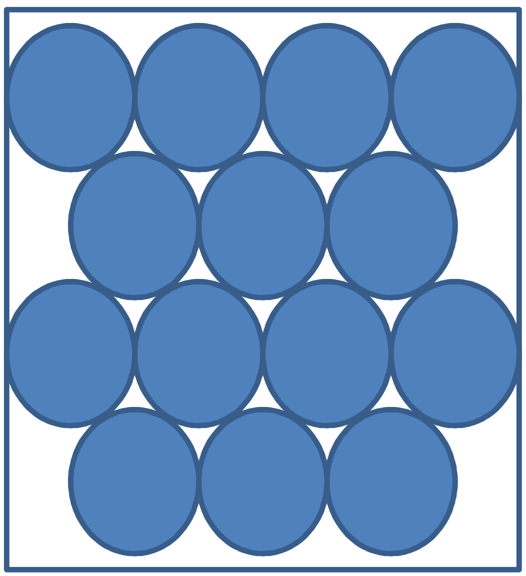
Notice the cross multiplication in the formula below from point to point.



$$\text{Area} = \frac{1}{2}[(x_1 \cdot y_2 - y_1 \cdot x_2) + (x_2 \cdot y_3 - y_2 \cdot x_3) + (x_3 \cdot y_4 - y_3 \cdot x_4) + (x_4 \cdot y_5 - y_4 \cdot x_5) + (x_5 \cdot y_1 - y_5 \cdot x_1)]$$

The formula can be generalized to give formulas for areas of polygons with an arbitrary number of vertices.

Food For Thought – *New*



A square is shown on the left with side length 1. Many small circles, all having the same radius, are placed in the square so that their interiors do not overlap. Explore the area of the portion of the square that is not in any of the circles.

What is the smallest that this area can be if we only use 1 circle?

What happens when we only use 2 circles? 3 circles? 4 circles?

Can this area be made as small as you like by using smaller circles?

What Does This Do?

(from October 1)

Suppose a and b are natural numbers with $a > b$. Follow the pseudo code below to find the output.

```
Let m be the largest positive integer so
that a ≥ mb, and let r be the nonnegative
integer so that a = mb + r.

Do While r > 0
    a = b
    b = r
    Let m be the largest positive integer so
    that a ≥ mb, and let r be the nonnegative
    integer so that a = mb + r.
Loop
Display b
```

We discovered that this algorithm finds the gcd of *a* and *b*.

What Does This Do? – *New*

Suppose $a > 0$, and let $x_0 = a$.

For each positive integer k define

$$x_k = \frac{x_{k-1}}{2} + \frac{a}{2x_{k-1}}$$

What happens to the x_k values when k gets large?

Project Problem

(from October 1)

Suppose 2 distinct lines are drawn on a sheet of paper. How many regions can be formed?

Repeat with 3 lines.

Repeat with 4 lines.

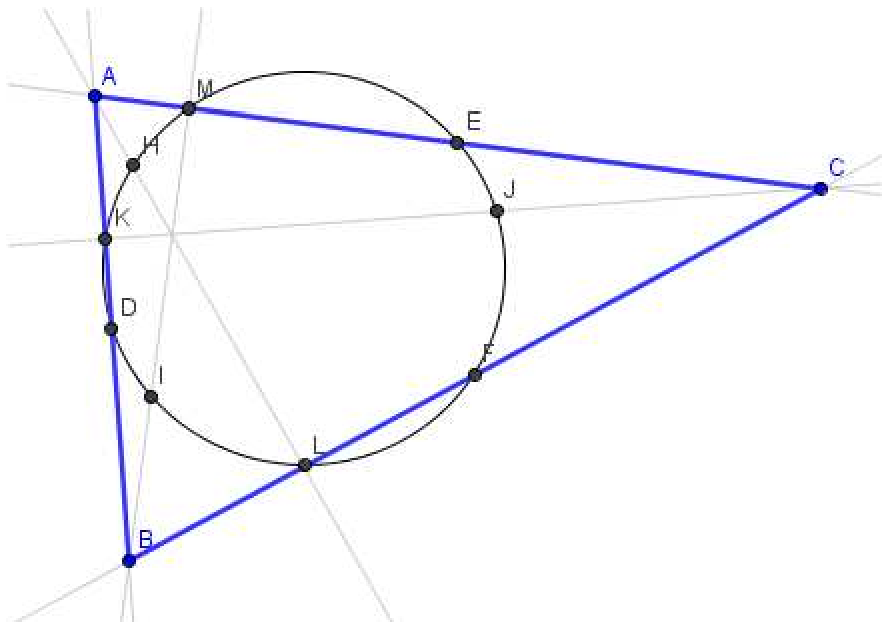
Can you generalize your answer?

Repeat the project with “lines” replaced by “circles”.

Project Problem – *New*

Feuerbach 9 Point Circle

Without using Google… Can you prove that the what appears to happen, actually happens?



Java Applet demonstration of the Feuerbach 9 Point Circle

http://www.math.uh.edu/~jmorgan/Michelle/

Additional Help…

Video Demonstration with GeoGebra at
http://www.math.uh.edu/~jmorgan/Rice/Feuerbach/Feuerbach/Feuerbach.html

GeoGebra is available at http://www.GeoGebra.org