UH Math Circle

March 24, 2012

Introductions

Warm up… What does this do?

A simple game.

Flipping coins.

What Does This Do?

What do you think is going on below?

							4	**-5**	**15**
1	2	-1	1	4	3	1	2	-2	5
				4	8	-4	4		
					-5	5	-2	-2	5
					-5	-10	5	-5	
						15	-7	3	5
						15	30	-15	15
							-37	18	-10

Repeat the Process…

$$\frac{x^4 - 2x^3 + 3x - 1}{x^2 + 2x - 2} =$$

A Simple Game

Suppose we decide to play a game with a fair coin. On each turn in the game, we flip a coin, and we record either H to T, depending upon whether heads or tails occurs. Then we continue to flip and record the results until our first flip occurs again, ending the turn. At this point we record the number of flips of the opposite type that occurred and we receive a point for each of these occurrences.

Here are some examples of flips and point values:

TT	HH	THT	HTH	THHT	HTTH	HTTTH	THHHT
0	0	1	1	2	2	3	3

What is the expected point value of a turn? i.e. what is the average number of points we can expect in any given turn?

Project Problem – Flipping Coins

Suppose a fair coin is flipped 100 times and the outcomes are recorded in order.

- Try this. We'll talk about what you observe.
- Some *streaks* occurred when you flipped your coin 100 times. Make a list of the difference *streaks*.
- Is there a way to simulate more "trials" in a way that does not take up so much time?

Exploring with TI Basic

We can use the TI calculator to simulate 100 experiments of flipping a coin 100 times, where we keep track of the maximum streak length in each experiment.

The program on the right uses J to keep track of the experiment. In each experiment, a list of length 100 containing random 0's and 1's is stored in L1. Then L2 is created to keep track of the streak lengths. Finally, the maximum value of the streaks for the experiment is recorded in L3(J). We display J at each step to keep track of the progress.

```
For(J,1,100)
randInt(0,1,100) → L₁
1 → L₂(1)
For(K,2,100)
If L₁(K) = L₁(K-1)
Then
L₂(K-1)+1 → L₂(K)
Else
1 → L₂(K)
End
End
max(L₂) → L₃(J)
Disp J
End
```

Using *Hand* Calculations

We will try to determine the probability of observing a streak of at least 5 heads in 100 flips of a fair coin.

- How is this question different from the question of determining the probability of observing a streak of at least 5 in 100 flips of a fair coin?
- How is the probability of observing a steak of at least 5 heads in 100 flips related to the probability of not observing a streak of at least 5 heads in 100 flips?
- How many different ways are there to flip a coin 100 times?
- *Recursion trick…* Let n be a positive integer, and define S_n to be the set of flips of n coins where we do not observe a streak of at least 5 heads. Give the number of elements in each of S_1, S_2, S_3, S_4 and S_5.
- Suppose $n > 5$. What are the possible endings of flips of n coins where we do not observe a streak of at least 5 heads? How many of each of these occur?
- Use the outcome above to give the number of elements in S_6, S_7, S_8, S_9 and S_{10}. Does it appear possible to give the number of elements in S_{100}?
- Can you use this information to find the probability of observing a streak of at least 5 heads in 100 flips of a fair coin?