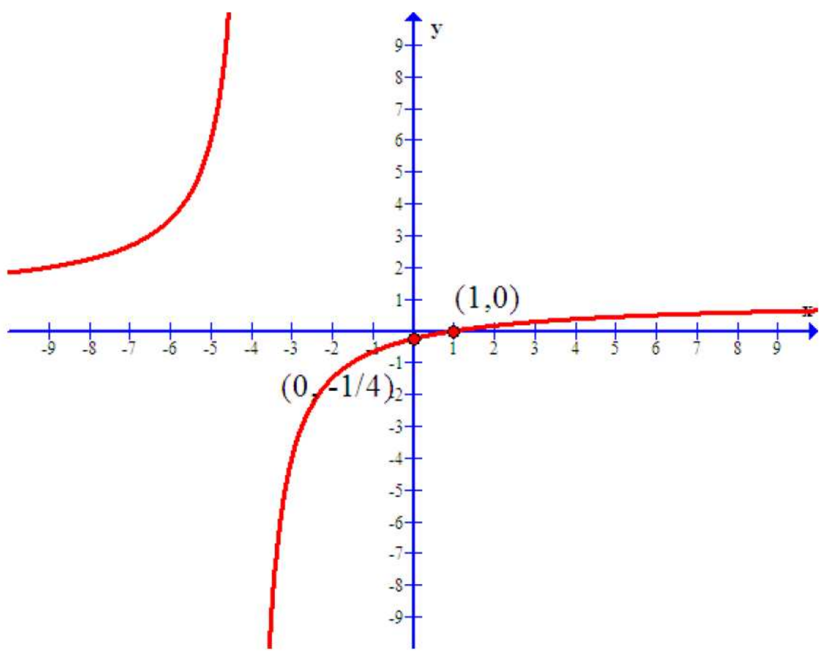
Math 1330 ONLINE
Sample Free Response Questions
Test 2
Solutions

1. There are no holes.
VA: To find the vertical asymptote, set the denominator equal to zero and solve for x. There's a vertical asymptote when $x = -4$.
HA: The degree of the numerator and the degree of the denominator are the same. The leading coefficient of the numerator is 1; the leading coefficient of the denominator is 1. To find the horizontal asymptote, make a fraction of the leading coefficients. The horizontal asymptote occurs when $y = 1$.
X int: To find the x intercept, set the numerator equal to zero and solve for x. There's an x intercept when $x = 1$.
Y int: To find the y intercept, compute $f(0)$, which is $\frac{-1}{4}$.
Graph:



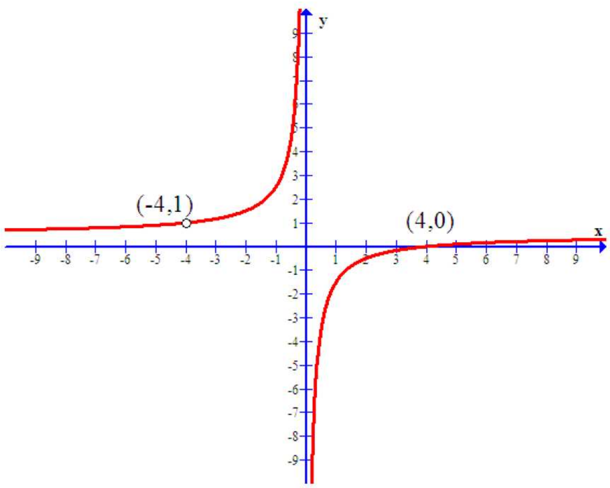
2. Factor completely: $f(x) = \frac{(x-4)(x+4)}{2x(x+4)}$. Reduce: $f(x) = \frac{(x-4)}{2x}, x \neq -4$
There's a hole in the graph when $x = -4$. To find the y value of the hole, sub -4 into what's left after you reduce: $\frac{-4-4}{2(-4)} = \frac{-8}{-8} = 1$. The hole occurs at (-4, 1).
Continue with $f(x) = \frac{(x-4)}{2x}, x \neq -4$.
VA: There's a vertical asymptote when $2x = 0$, or $x = 0$

HA: There's a horizontal asymptote at $y = \frac{1}{2}$

Xint: There's an x intercept when $x - 4 = 0$, or $x = 4$

Y int: Try to sub in 0 for x, but you'll get a zero in the denominator. There is no y intercept.

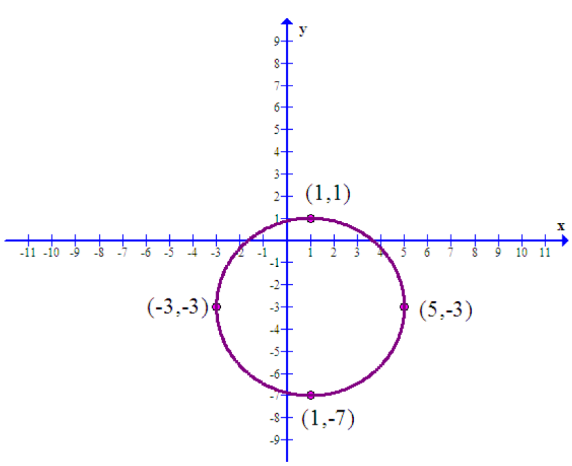
Graph:



3. Write in standard form:

$$x^2 + y^2 - 2x + 6y - 6 = 0$$
$$x^2 - 2x + y^2 + 6y = 6$$
$$x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$$
$$(x-1)^2 + (y+3)^2 = 16$$

Center is (1, -3); radius is 4.



4 (a) $\frac{x^2}{9}+\frac{y^2}{16}=1$

Center (0, 0).

Oriented vertically.

Vertices $(0,\pm 4)$.

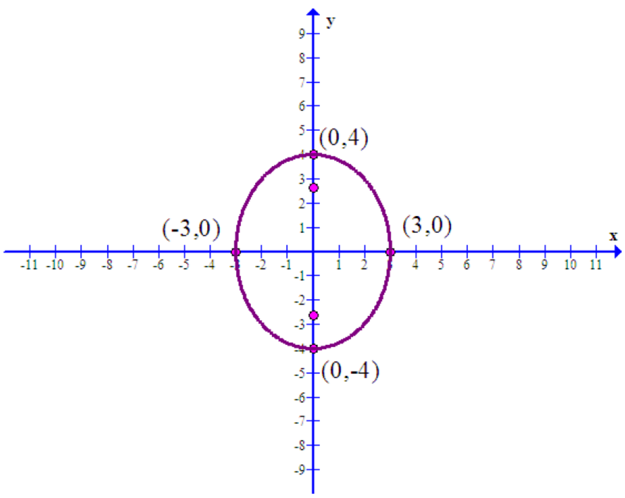
$a=4, b=3$ and $c=\sqrt{big-small}$, so $c=\sqrt{16-9}=\sqrt{7}$

Length of major axis is 8. Length of minor axis is 6.

Foci $\left(0,\pm\sqrt{7}\right)$.

Eccentricity = $\frac{\sqrt{7}}{4}$

Graph:



4 (b) $\frac{(x-3)^2}{25}+\frac{(y+2)^2}{4}=1$

Center (3, -2)

Oriented horizontally.

Vertices (8, -2) and (-2, -2).

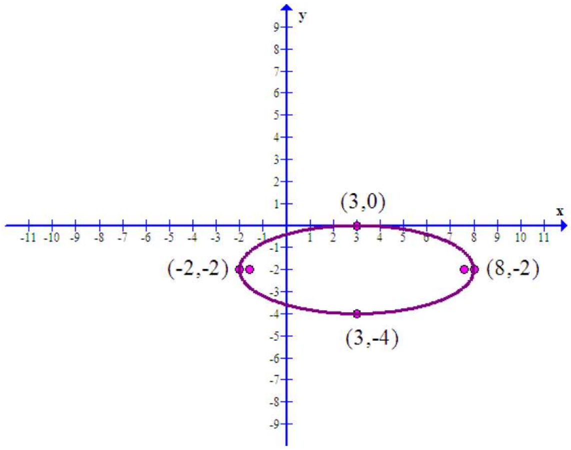
$a=5, b=2$ and $c=\sqrt{big-small}$, so $c=\sqrt{25-4}=\sqrt{21}$

Length of major axis is 10. Length of minor axis is 4.

Foci $\left(3\pm\sqrt{21},-2\right)$.

Eccentricity = $\frac{\sqrt{21}}{5}$

Graph:



5 (a) $(x-2)^2 = 8y$

Parabola opens up.

Vertex is (2, 0).

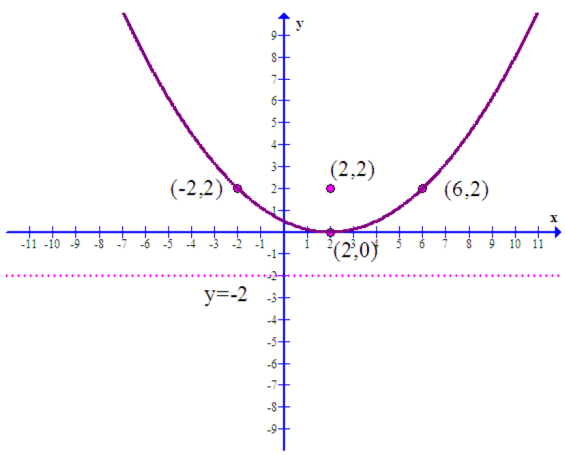
P = 2

Directrix is $y = -2$.

Focus is (2, 2).

Focal width is 8

Endpoints of focal chord are (6, 2) and (-2, 2)



5(b) $y^2 = -12x$

Parabola opens left.

Vertex is (0, 0)

P = -3

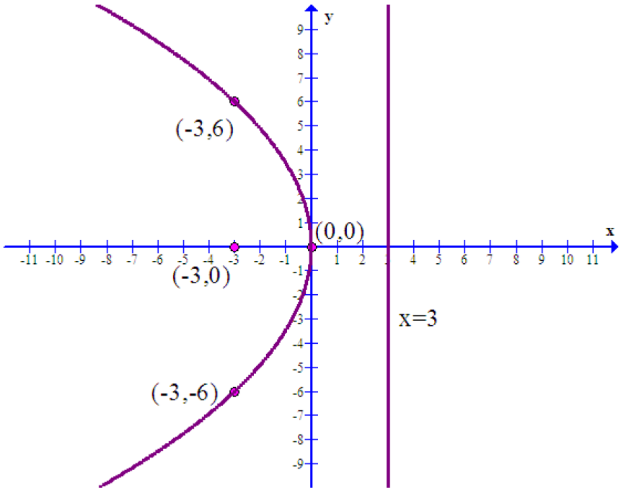
Directrix is $x = 3$

Focus is (-3, 0)

Focal width is 12

Endpoints of focal chord are (-3, 6) and (-3, -6)

Graph:



6(a) $\frac{(x-1)^2}{9} - \frac{(y+1)^2}{4} = 1$

Oriented horizontally.

Center (1, -1)

$a = 3, b = 2, c = \sqrt{big + small} = \sqrt{9+4} = \sqrt{13}$

Vertices: (4, -1) and (-2, -1)

Foci: $\left(1 \pm \sqrt{13}, -1\right)$

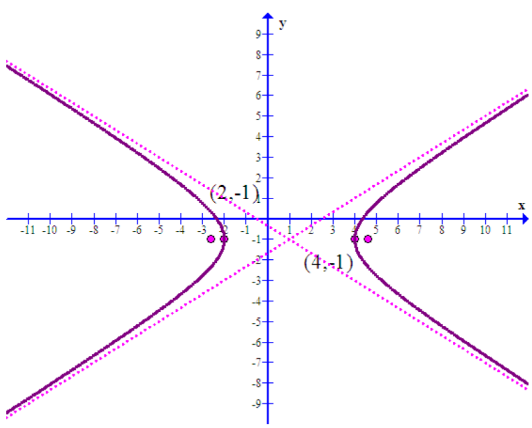
Length of the transverse axis = 6

Length of the conjugate axis = 4

Slopes of asymptotes are $\pm\sqrt{\frac{yden}{xden}} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

Equations of asymptotes are $y+1 = \pm\frac{2}{3}(x-1)$

Graph:



6(b) $\frac{y^2}{16} - \frac{x^2}{4} = 1$

Oriented vertically.

Center (0, 0)

$a = 4, b = 2, c = \sqrt{big + small} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

Vertices: $(0, \pm 4)$

Foci: $(0, \pm 2\sqrt{5})$

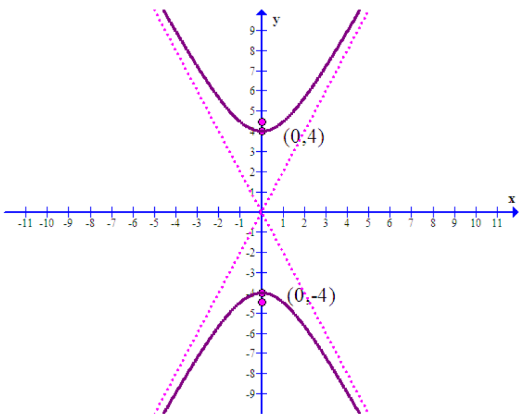
Length of the transverse axis = 8

Length of the conjugate axis = 4

Slopes of asymptotes are $\pm\sqrt{\frac{yden}{xden}} = \pm\sqrt{\frac{16}{4}} = \pm\frac{4}{2} = \pm 2$

Equations of asymptotes are $y = \pm 2x$

Graph:



7. $f(x) = x(x-2)^2(x+3)^2$

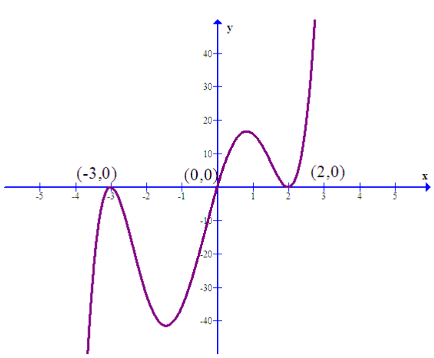
X intercepts are 0 (multiplicity 1), 2 (multiplicity 2) and -3(multiplicity 2)

Y intercept is $f(0) = 0(0-2)^2(0+3)^2 = 0$

Leading term is $x \cdot x^2 \cdot x^2 = x^5$ which has odd degree and a positive leading coefficient.

End behavior is ↙↗.

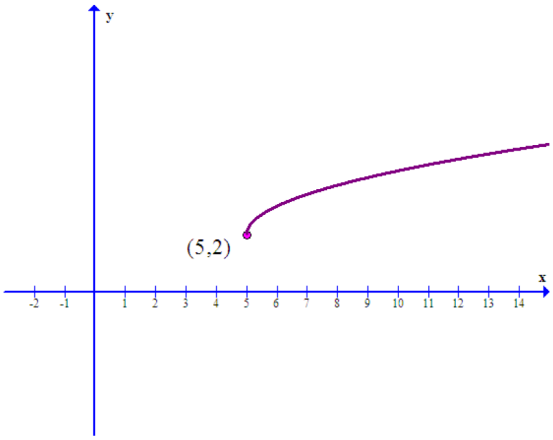
Graph:



8 (a) $f(x) = \sqrt{x-5} + 2$

Basic square root function shifted 5 units right and up 2. Key point (0, 0) translates to (5, 2).

Graph:

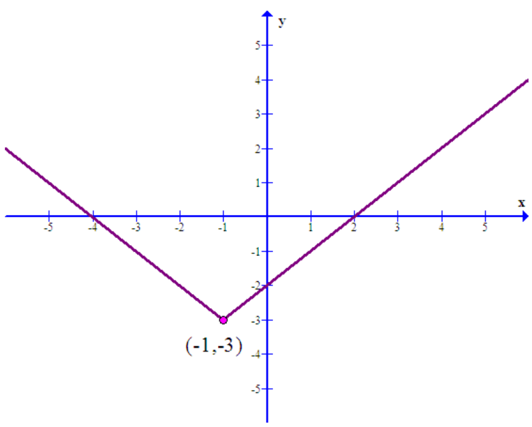


Range of the function is $[2, \infty)$. The function is one-to-one.

8 (b) $f(x) = |x+1| - 3$

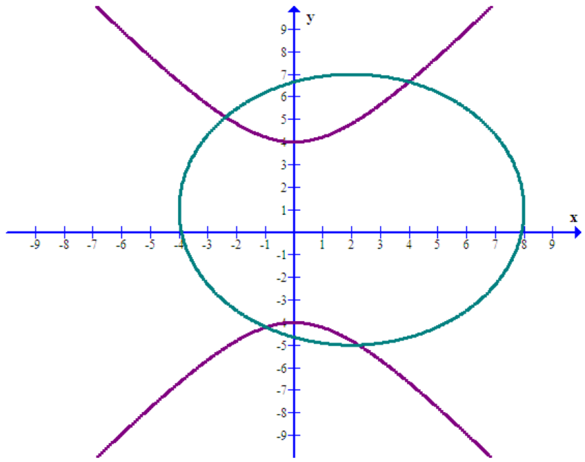
Basic absolute value function shifted one unit left and down 3. Key point (0, 0) translates to (-1, -3).

Graph:



Range of the function is $[-3, \infty)$. The function is not one-to-one.

9. The first equation is a vertically oriented hyperbola. Center at (0,0) Vertices (0, 4) and (0, -4). Points on the conjugate axis (-3, 0) and (3, 0). Draw the box and sketch.
The second equations is a circle with center at (2, 1) and radius 6. Graph it.



Count the number of intersection points. There are 4.

10. $$\begin{aligned} x^2 + y^2 &= 9 \\ 9x^2 - y^2 &= 1 \end{aligned}$$

Add the two equations together (note: in other problems, you may need to multiply one equation or both by constants to get the opposite coefficients that you need).

$$x^2 + y^2 = 9$$
$$9x^2 - y^2 = 1$$

$$10x^2 = 10$$
$$x^2 = 1$$
$$x = \pm 1$$

Now find the y values.

If $x = 1$,

$1^2 + y^2 = 9$

$1 + y^2 = 9$

$y^2 = 8$

$y = \pm\sqrt{8}$

$y = \pm 2\sqrt{2}$

If $x = -1$,

$(-1)^2 + y^2 = 9$

$1 + y^2 = 9$

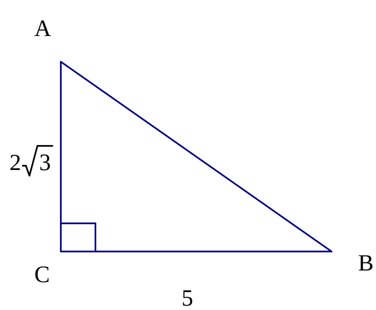
$y^2 = 8$

$y = \pm\sqrt{8}$

$y = \pm 2\sqrt{2}$

There are four solutions: $\left(1, 2\sqrt{2}\right), \left(1, -2\sqrt{2}\right), \left(-1, 2\sqrt{2}\right), \left(-1, -2\sqrt{2}\right)$.

11. Draw and label a right triangle. If $\tan B = \dfrac{2\sqrt{3}}{5}$, then the side opposite angle B is $2\sqrt{3}$ and the side adjacent to angle B is 5.



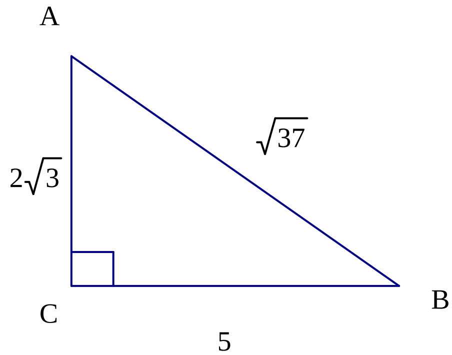
Find the length of AB using Pythagorean Theorem.

$\left(2\sqrt{3}\right)^2 + 5^2 = c^2$

$12 + 25 = c^2$

$37 = c^2$

$\sqrt{37} = c$



Now read off the six trig functions of angle A.

$$\sin A = \frac{5}{\sqrt{37}} = \frac{5\sqrt{37}}{37}$$

$$\cos A = \frac{2\sqrt{3}}{\sqrt{37}} = \frac{2\sqrt{3 \cdot 37}}{37} = \frac{2\sqrt{111}}{37}$$

$$\tan A = \frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{6}$$

$$\csc A = \frac{\sqrt{37}}{5}$$

$$\sec A = \frac{\sqrt{37}}{2\sqrt{3}} = \frac{\sqrt{111}}{6}$$

$$\cot A = \frac{2\sqrt{3}}{5}$$

Read off the six trig functions of angle B.

$$\sin B = \frac{2\sqrt{3}}{\sqrt{37}} = \frac{2\sqrt{3 \cdot 37}}{37} = \frac{2\sqrt{111}}{37}$$

$$\cos B = \frac{5}{\sqrt{37}} = \frac{5\sqrt{37}}{37}$$

$$\tan B = \frac{2\sqrt{3}}{5}$$

$$\csc B = \frac{\sqrt{37}}{2\sqrt{3}} = \frac{\sqrt{111}}{6}$$

$$\sec B = \frac{\sqrt{37}}{5}$$

$$\cot B = \frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{6}$$

12. (a) $\sin(300°)$

Angle is in quadrant 4. Sine is negative in quadrant 4. Reference angle is $360° - 300° = 60°$. So $\sin(300°) = -\sin(60°) = \frac{-\sqrt{3}}{2}$.

(b) $\tan\left(\frac{-\pi}{4}\right)$

Coterminal angle is $\frac{-\pi}{4} + 2\pi = \frac{7\pi}{4}$. Angle is in quadrant 4. Tangent is negative in quadrant 4. Reference angle is $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$. So $\tan\left(\frac{-\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$.

(c) $\cos(120°)$

Angle is in quadrant 2. Cosine is negative in quadrant 2. Reference angle is

$180° - 120° = 60°$. So $\cos(120°) = -\cos(60°) = \frac{-1}{2}$.

(d) $\sec(225°)$

Find $\cos(225°)$ first, then find its reciprocal. Angle is in quadrant 3. Cosine is negative in quadrant 3. Reference angle is $225° - 180° = 45°$. So $\cos(225°) = -\cos(45°) = \frac{-\sqrt{2}}{2}$. To find $\sec(225°)$, take the reciprocal of $\frac{-\sqrt{2}}{2}$.

$\sec(225°) = \frac{-2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$.

(e) $\sin\left(\frac{5\pi}{6}\right)$

Angle is in quadrant 2. Sine is positive in quadrant 2. Reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$. So $\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

(f) $\csc\left(\frac{4\pi}{3}\right)$

Find $\sin\left(\frac{4\pi}{3}\right)$ first, then find its reciprocal. Angle is in quadrant 3. Sine is negative in quadrant 3. Reference angle is $\frac{4\pi}{3}-\pi=\frac{\pi}{3}$. $\sin\left(\frac{4\pi}{3}\right)=-\sin\left(\frac{\pi}{3}\right)=\frac{-\sqrt{3}}{2}$. To find $\csc\left(\frac{4\pi}{3}\right)$, take the reciprocal. $\csc\left(\frac{4\pi}{3}\right)=-\frac{2}{\sqrt{3}}=\frac{-2\sqrt{3}}{3}$.

(g) $\cot(-45°)$

To find $\cot(-45°)$, start by finding $\tan(-45°)$. Find a positive coterminal angle. $-45°+360°=315°$. So find $\tan(315°)$. Angle is in quadrant 4. Tangent is negative in quadrant 4. Reference angle is $360°-315°=45°$. $\tan(315°)=-\tan(45°)=-1$. To find $\cot(-45°)$, take the reciprocal. $\cot(-45°)=\frac{1}{-1}=-1$.