**Math 1314**
Lesson 2

Course Overview

What is calculus?

Calculus is the study of change, particularly, how things change over time. It gives us a framework for measuring change using some fairly simple models.

The formal body of information that makes up a calculus course was developed in the 17th century by two mathematicians, each working separately: Isaac Newton and Gottfried Wilheim Leibniz. Newton' method of "fluxions" studied how things change. Leibniz did similar work at about the same time, and contributed additionally some of the notation we use to this day.

We will address two basic questions in this course. First, how can we find the slope of a line that is tangent to an arbitrary curve at a given point? The graph below shows a cubic function and the line tangent to the function at the point where $x = -1$
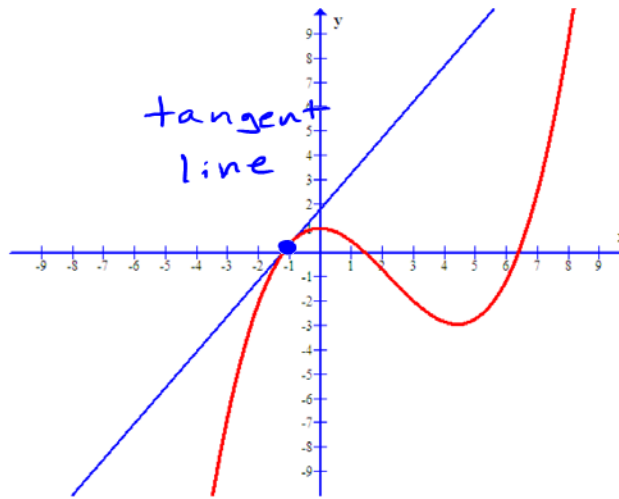

We are interested in finding the slope of this line, because it gives the rate at which the y value of the function is changing. You already have some experience working with slope. You have found the slope of a line in earlier coursework using a formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. This gives a rate of change: it gives the change in y over the change in x. It tells us how much y changes for a computed change in x. We will use this idea when we learn how to find the slope of a tangent line. *Differential calculus* treats problems of this type.

The second question we will address is the area question: how can we find the area of a region? If the region is a familiar one, such as a triangle, rectangle or circle, we can use a formula from

$$A = \tfrac{1}{2}bh$$

geometry to find the area. We will, however, be interested in finding the area of an arbitrary region, such as the shaded area in the graph below:
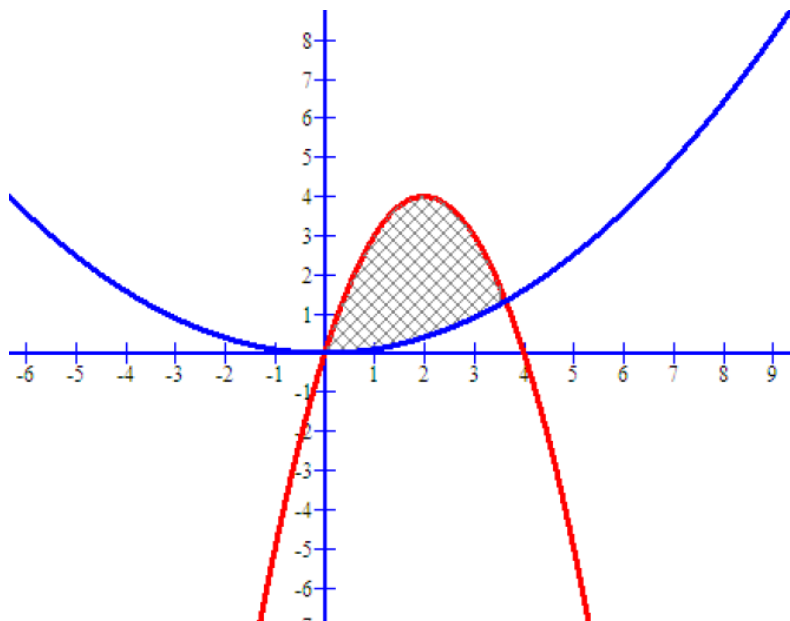
① limits
② derivatives
③ integrals

The shape of this region does not conform to the formulas we know, so we will need another method for finding this area. We will use *integral calculus* to solve problems of this type.

Both of these areas of inquiry rely on the concept of a *limit.* We will begin our study of calculus by looking at limits. Then we will investigate the types of problems we can solve using differential calculus and integral calculus.

A major focus of this course will be the use of technology in the teaching and learning of calculus. We will be less concerned with paper and pencil computations in this course, and more interested in solving problems. Our main tool will be GeoGebra, a free software, that you will need to download and use on a daily basis in this course.

POPPER 1: C   ← — Input it now!!

Lesson 2: An Introduction to GeoGebra

GeoGebra (GGB) is a free software package that we will use throughout the semester. The program can be used as a simple calculator, and it can also be used to perform some fairly sophisticated calculus operations. In this lesson, you will learn your way around the GGB display, and you will also learn to do some basic graphing.

You will need to download GGB onto your computer. Go to www.geogebra.org and follow the instructions for downloading the program. If you are using a computer where you cannot download the program, choose the Applet option on the download page.

This is what you will see when you open GGB.
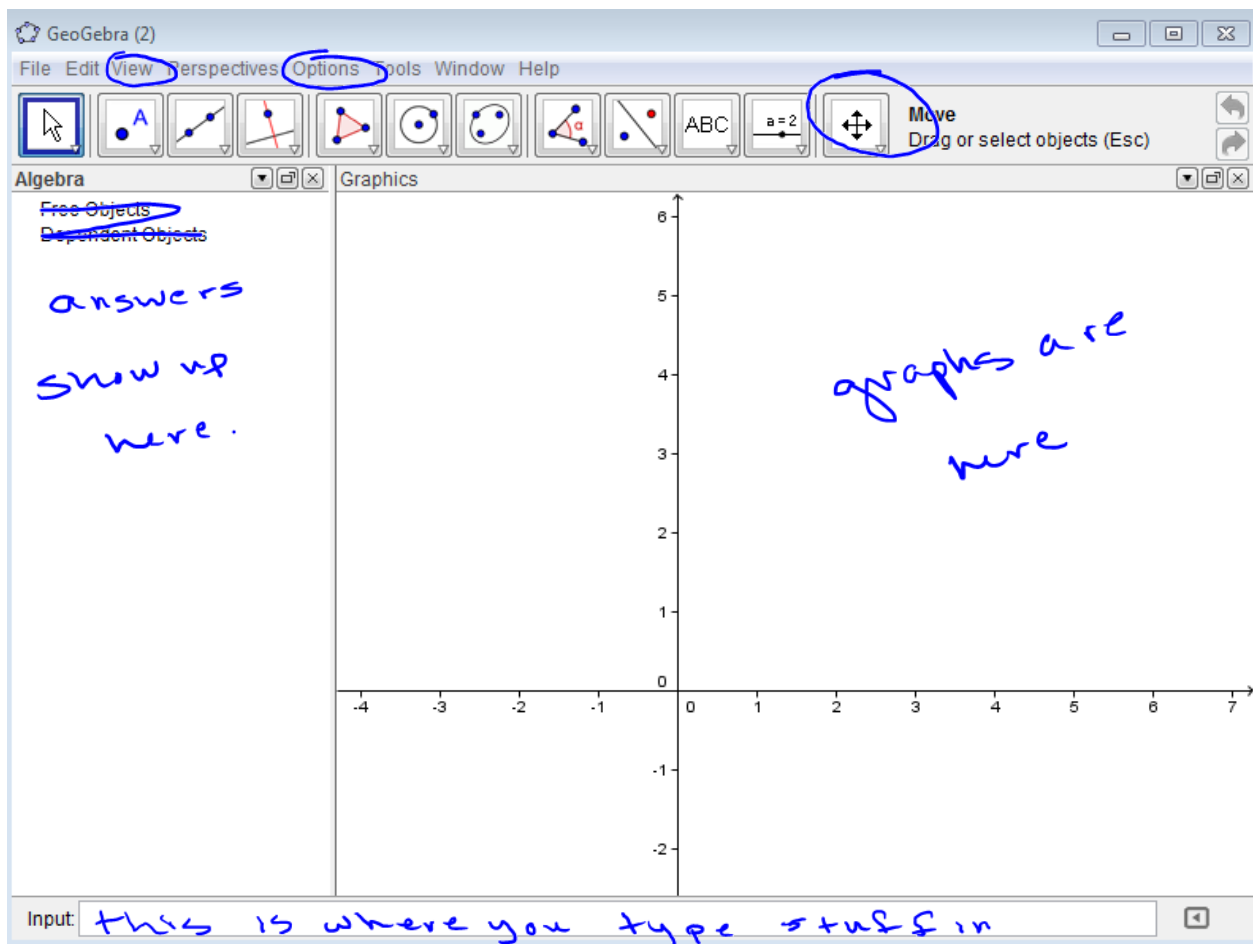
The GGB display consists of four parts: the menus/icons at the top, the algebra window at the left, the graphics window at the right and the input line at the bottom.

You can resize the graphics and algebra windows by putting your cursor on the vertical line between them until you see a double-headed arrow. Then drag the line to the left or the right depending on which part you wish to enlarge.

You will enter functions or expressions using the input line. Put the cursor in the input line and start typing. You can give your function a name, such as $f(x)$. Use the caret key ("^") to raise the variable to a power. You can use $\text{sqrt}(f(x))$ to indicate the square root of a function. You will need to make liberal use of parentheses when entering rational functions.

You can adjust the number of decimal places displayed by selecting "Options," then clicking on "Rounding" and selecting the number of decimal places you wish to display. The standard for this class is 4 decimal places, although you will sometimes round to fewer or more.

You can adjust the font size by selecting "Options," then clicking on "Font" and selecting the size that you'd like.

POPPER 2: Write an equation of the line in slope-intercept form with slope -2 and y intercept 9.

    A. $y = 9x - 2$ B. $y = 2x - 9$ C. $y = 2x + 9$ D. $y = -2x + 9$

$$y = mx + b \qquad\qquad y = -2x + 9$$

Example 1: Compute $8*4^2 - 3*28 \quad = 44$

Example 2: Compute $2(3 - 5*2^2) - 7(-3)^3 \quad = 155$

Example 3: Compute $\dfrac{-3^2 + 4(7 - 2^2)}{(2 - 3^2)^2 - 9} \quad = 0.075$

Example 4: Compute $\sqrt{5e^2 - 3(-2)^3}$

$$\text{sqrt}(5e^2 - 3(-2)^3) = 7.8067$$

Example 5: Evaluate $x^2 y^3 - 3xy + 4y^2$ if $x = -2$ and $y = -1$.

$$(-2)^2 (-1)^3 - 3(-2)(-1) + 4(-1)^2$$
$$4(-1) - 6 + 4 = -10 + 4 = \boxed{-6}$$

Tables

You can make a table, similar to a table in Excel. To do this, enable the spreadsheet view. Click on "View" and enable "Spreadsheet." You can use the menus, or you can learn the shortcuts, which are displayed on the menus.

Example 6: Suppose $f(x) = -1.875x^2 + 12.3x - 7.164$. Use GGB to create a table of values that starts at $x = 1.63$ and has an increment of 0.26. Create a list of ordered pairs. What is the sixth function value listed in the table?

Step 1: Create the list of x values.

How do you create a formula in GGB?

In one cell, type # #.
In cell below it type
= a1 + .26

How do you replicate your formula?

	A
1	1.63
2	1.89
3	2.15
4	2.41
5	2.67
6	2.93
7	3.19
8	3.45

Click on 2nd cell and drag to replicate

Step 2: Create the list of y values.

In b1, type = f(a1)
press enter
click è, drag to replicate

A	B	C
1.63	7.9033	
1.89	9.3853	
2.15	10.6138	
2.41	11.5888	
2.67	12.3103	
2.93	12.7783	
3.19	12.9928	

Step 3: Create the list of ordered pairs.

In C_1 type $=(a_i, b_i)$
click & drag to
replicate

How do you create a list in GGB?

Use Create List tool button

File Edit View Options Tools \
▸ Algebra
▱ Function

Select create

usheet
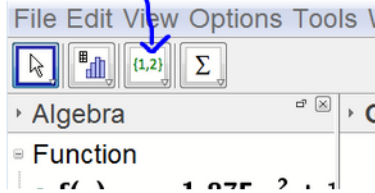
	B	C	D
.63	7.9033	(1.63, 7.9033)	
.89	9.3853	(1.89, 9.3853)	
.15	10.6138	(2.15, 10.6138)	
.41	11.5888	(2.41, 11.5888)	
.67	12.3103	(2.67, 12.3103)	
.93	12.7783	(2.93, 12.7783)	
.19	12.9928	(3.19, 12.9928)	
.45	12.9538	(3.45, 12.9538)	
.71	12.6613	(3.71, 12.6613)	

▱ List
  ○ list1 = {(1.63, 7.9033), (1.89, 9.3853), (2.15, 10.6138), (2.41, 1

Example 7: Evaluate $C(x) = 0.0003x^3 + 1.8x^2 - .12x + 23,135$ if $x = 100$, $x = 200$, $x = 300$, $x = 400$, $x = 500$, $x = 600$, $x = 700$, and $x = 800$.

Cell 1 value is   100

Increment is   100

Spreadsheet

	A	B	C	D
1	100	41423	(100, 41423)	
2	200	97511	(200, 97511)	
3	300	193199	(300, 193199)	
4	400	330287	(400, 330287)	
5	500	510575	(500, 510575)	
6	600	735863	(600, 735863)	
7	700	1007951	(700, 1007951)	
8	800	1328639	(800, 1328639)	
9				

Problems like Example 7 will usually be framed in terms of a problem you might be called upon to solve at some point in your career.

Example 8: A company estimates that the total costs associated with producing *x* units of one of its products can give computed using the model $C(x) = 0.0003x^3 + 1.8x^2 - .12x + 23,135$ where x is the number of items produced and $C(x)$ is the total cost of producing x items. Use the model to approximate the total costs associated with producing 600 items.

$ 735,863

$$-2p + 400 = x$$

POPPER 4:

Solve for x: $p = 200 - 0.5x$

$$\frac{1p - 200}{-0.5} = \frac{-0.5x}{-0.5}$$

A. $x = 40 - 5p$ B. $x = 400 - 2p$ C. $x = 100 - \frac{1}{2}p$ D. $x = 400 - \frac{1}{2}p$

Example 9: Make a table of values that gives the function values of $f(x) = 2x^2 - 8x + 11$ when x = -2, x = 0, x = 1, x = 5 and x = 12.

Should we use the same kind of table as in the other examples?

No

How else could we find these values?

enter function in input line

▸ **Algebra**
▱ Function
    ◦ f(x) = 2 x² − 8 x + 11
▱ Number
    ◦ a = 35
    ◦ b = 11
    ◦ c = 5
    ◦ d = 21
    ◦ e = 203

input 1 line
$\left\{\begin{array}{l} f(-2) \\ f(0) \\ f(1) \\ f(5) \\ f(12) \end{array}\right.$

Example 10: The path of a small rocket is modeled by the function $h(t) = -16t^2 + 128t + 12$ where initial velocity is 128 feet per second and initial height is 12 feet. The model gives the height of the rocket in feet, t seconds after launch. Find the height of the rocket 2 seconds after launch, 4 seconds after launch, 5 seconds after launch and 8 seconds after launch.

○ g = 204
○ i = 268
○ j = 252
○ k = 12

after 2 sec   204 ft
after 4 sec   268 ft
after 5 sec   252 ft
after 8 sec   12 ft

6
7
8
9
10

A
B
C
D
E

POPPER 6: Suppose $h(t) = -16t^2 + 128t + 12$. Find $h(3)$.

A. 252   B. 540   C. 348   D. 249

Graphing

Example 11: Graph the function $f(x) = -2x - 3$.

Step 1: Enter the function in the input line.

Step 2: Resize the graphics window, if needed, to get a good view of the function.

Methods for resizing

How do you return to a standard viewing window?

Example 12: Graph the function $g(x) = x^3 - 2x^2 - 9x + 18$ and resize the graphics window, if needed, to find a good view of the function.

How do you resize the graphics window?

Example 13: Graph the function $h(x) = -3x^3 + 3x^2 + 96x - 180$ and find an appropriate viewing window.

Most of the work we will do in GGB will require the use of commands that are built into the program.

Where can you access a list of commands?

Finding Some Features of a Graphed Function

You can find the zeros (also called roots or x intercepts) of a function using GGB.

Example 14: Suppose $g(x) = x^3 - 2x^2 - 9x + 18$. Find the zeros of the polynomial function.

How do we know this is a polynomial function?

Step 1: Enter the function in the input line. Graph and resize if needed.

Step 2: Find the zeros of the polynomial function.

What's the command?

Example 15: Suppose $f(x) = \dfrac{x^2 - 3x - 5}{x^2 - 3}$. Find the zeros of the function.

Is this a polynomial function? How can we tell?

Step 1: Enter the function in the input line. Graph and resize if needed.

Step 2: Find the zeros of the function.

What's the command?

Note that we need different options on the Root command menu, depending on the function we have to work with.

Good things to know when working with GGB:

To obtain a new window in GGB, press Control N or click on the File menu and select new.

To deactivate (or reactivate) an equation or value in the algebra window, click on the blue (or open) circle to the left of the item.

If you made an error entering the function, right-click on the function in the algebra window and then select Object Properties. The right hand side of the function is shown on the Value line. You can place your cursor on that line and use the left and right arrow keys to move the cursor to the location of the error. Then backspace or use the delete key to delete the error and use your keyboard to correct it.

The relative extrema of a function are the high points and low points of the graph of a function, when compared to other points that are close to the relative extremum. A relative maximum will be higher than the points near it, and a relative minimum will be lower than the points near it.
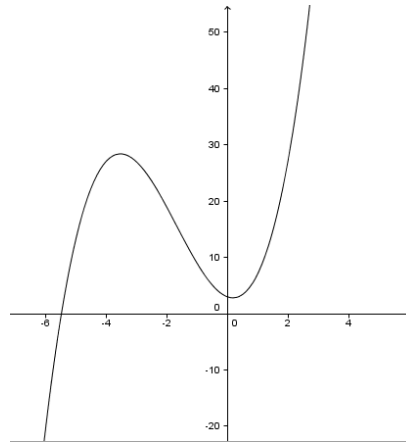
GGB will help you find these points.

Example 16: Suppose $g(x) = x^3 + 5x^2 - 2x + 3$. Find any relative extrema.

Is this a polynomial function?

Step 1: Enter the function in the input line. Graph and resize, if needed.

Step 2: Find the relative extrema.

What's the command?

Example 17: Suppose $h(x) = x - 3x^{\frac{1}{3}}$. Find any relative extrema.

　　　　Is this a polynomial function?

Step 1: Enter the function in the input line. Graph and resize, if needed.

Step 2: Find the relative extrema.

　　　　What's the command?

This is similar to the method used on a graphing calculator to find maxima and minima.

You can find the point(s) where two functions intersect by using the Intersect command in GGB.

Example 18: Find any points where $f(x) = 1.45x^2 - 7.2x - 1.6$ and $g(x) = 2.84x - 1.29$ intersect.

Step 1: Enter the functions one by one in the input line. Graph and resize if necessary to find all points of intersection.

Step 2: Find intersection points.

　　　　What's the command?

Example 19: Find any points where $f(x) = x - \dfrac{6}{x}$ and $g(x) = 4e^{-2x} + \dfrac{1}{2}x$ intersect.

Step 1: Enter the functions one by one in the input line. Graph and resize if necessary to view all points of intersection.

Step 2: Find intersection points.

Does Intersect[<Object, Object>] return all points of intersection?

Why not?

What's the command?

POPPERS 8 – 10

Suppose $f(x) = x^3 - 12x^2 + 7x + 5$.

Match the quantity described in column A with the value in column B

Column A	Column B
8. A zero of the function	A. 7.6968
9. The x coordinate of the relative minimum	B. 11.1443
10. The y coordinate of the relative maximum	C. 1.0684
	D. 0.3032
	E. 6.4071