14 questions   11 MC   66 pts
3 FR   34 pts

Lesson 9 – 1st part of Lesson 19

Math 1314 ONLINE

Review for Test 2

Review Example 1:   Marginal analysis

L9

A company produces noise-canceling headphones. Management of the company has determined that the total daily cost of producing *x* headsets can be modeled by the function $C(x) = 0.0001x^3 - 0.03x^2 + 135x + 15{,}000$.

(A) Find the marginal cost function. $C'(x)$

(B) Use the marginal cost function to approximate the actual cost of producing the 51st, 71st, 101st, 141st and 201st headsets.

~~(C) Interpret your results.~~

$C'(50) = \$132.75$

$C'(70) = \$132.27$

$C'(100) = \$132^{-}$

$C'(140) = \$132.48$

$C'(200) = \$135^{-}$

Review Example 2: $R(x) = xp$

A company estimates that the demand for its product can be expressed as $p = -.04x + 800$, where *p* denotes the unit price and *x* denotes the quantity demanded.

→ (A) Find the revenue function. $R(x) = x(-0.04x + 800)$

(B) Then find the marginal revenue function.

(C) Use the marginal revenue function to approximate the actual revenue realized on the sale of the 4001st item.

B. $R'(x)$

C. $R'(4000) = 480$ $\boxed{\$480}$

Marginal analysis

Review Example 3:

A company estimates that the cost to produce x of its products is given by the function $C(x) = 100x + 200,000$ and the demand function is given by $p = 400 - 0.02x$.

(A) Find the profit function.

(B) Then find the marginal profit function. $P'(x)$

(C) Use the marginal profit function to compute the actual profit realized on the sale of the 1001st unit and the 2001st unit.

$P(x) = R(x) - C(x)$ $\quad$ $R(x) = x(400 - 0.02x)$

A $P(x) = x(400 - 0.02x) - (100x + 200000)$

B. GGB

C. $P'(1000) = 260$ $\quad$ \$260 on 1001st item

$P'(2000) = 220$ $\quad$ \$220 on 2001st item

Review Example 4:

A company produces office furniture. Its management estimates that the total annual cost for producing x of its top selling executive desks is given by the function $C(x) = 400x + 500,000$.

(A) Find the average cost function.

(B) What is the average cost of producing 3000 desks?

(C) Find the marginal average cost function.

(D) Find the marginal average cost when 3000 desks are produced.

~~(E) What happens to $\overline{C}(x)$ when x is very large?~~

* A. $\bar{C}(x) = \frac{C(x)}{x} = \frac{400x + 500000}{x}$

* B. $\bar{C}(3000) = 566.67$

enter $C(x) = \ldots$

type $A(x) = C(x)/x$ = avg cost function

C. $\bar{C}'(x) = -\frac{500000}{x^2}$

D $\bar{C}'(2999) = -0.06$ ← cost is decreasing

$$E(p) = \frac{-p \cdot f'(p)}{f(p)}$$

Review Example 5:

Suppose $p = -0.04x + 150$.

(A) Find the elasticity of demand. (formula)

(B) Find $E(50)$ and interpret the results.

(C) Find $E(100)$ and interpret the results.

(D) If the unit price is \$50, will raising the price result in an increase in revenues or a decrease in revenues?

(E) If the unit price is \$100, will raising the price result in an increase in revenues or a decrease in revenues?

C $E(100) = \frac{2500}{1250} = 2$

demand is elastic

$-p \cdot (-25) = 25p$

A. solve p for x

$p = -0.04x + 150$

$\frac{1p - 150}{-0.04} = \frac{-\cancel{0.04}x}{-\cancel{0.04}}$

$\boxed{-25p + 3750 = x = f(p)}$

$f'(p) = -25$

**

$$\boxed{E(p) = \frac{25p}{3750 - 25p}}$$

$E(p) = \frac{-pf'(p)}{f(p)}$

B $E(50) = \frac{1250}{2500}$

$E(50) = \frac{1}{2}$ demand is inelastic

$E(p) > 1$

$E(p) = 1$

$E(p) < 1$

Review Example 6:

Exponential models — Lesson 11

A biologist wants to study the growth of a certain strain of bacteria. She starts with a culture containing 25,000 bacteria. After three hours, the number of bacteria has grown to 63,000. ① How many bacteria will be present in the culture 6 hours after she started her study? ② What will be the rate of growth 6 hours after she started her study? Assume the population grows exponentially and the growth is uninhibited.

identify 2 pts $(0, 25000)$ $(3, 63000)$

fitexp $f(x) = 25000e^{0.3081x}$

① $f(6) = 158760$ $\boxed{158760 \text{ bacteria}}$

② $f'(6) \approx 48912$ $\boxed{48912 \text{ bacteria/hour}}$

Review Example 7:

decay

The half-life of a substance is 26 hours. If 120 mg of the substance is present at the beginning of an experiment,① how much is left 49 hours after the experiment starts?② At what rate is the amount of the substance changing after 49 hours?

write down 2 ordered pairs (0, 120)

(26, 60)

write exp. model $f(x) = 120e^{-0.0267x}$

① FV $f(49) = 32.4979$ $\boxed{32.5 \text{ mg}}$

② ROC $f'(49) = -0.8664$

$\boxed{\text{decreasing at the rate of } 0.8664 \text{ mg / hour}}$

Review Example 8:

Suppose your company's HR department determines that an employee will be able to assemble $Q(t) = 50 - 30e^{-0.5t}$ products per day, t months after the employee starts working on the assembly line.

(A) How many units can a new employee assemble as s/he starts the first day at work?

(B) How many units should an employee be able to assemble after one month at work? After two months at work? After six months at work?

(C) How many units should an experienced worker be able to assemble?

A. $Q(0) = 20$ 20 units FV

B. $Q(1) = \ldots$ EV

$Q(2) = \ldots$ FV

$Q(6) = \ldots$ FV

C. $\lim_{t \to \infty} Q(t) = 50$ $\boxed{50 \text{ units}}$

Logistic ~~Review~~ Example 9:

Suppose the percent of US homes with cable TV access is given by the function $P(t) = \frac{98}{1+34.76e^{-0.62t}}$ where t is given in years since 1990 and P is given as a percent.

(A) What percent of American homes had cable TV in 1990? In 2000? EV $P(0)$ $P(10)$

(B) What is the limiting value? 98

(C) At what rate was the percent changing in 2000? ROC $P'(10)$

(D) What percent of American homes currently have cable TV according to this model?

EV $P(23)$ 2013 − 1990

Review Example 10:

Analyze the function: $f(x)=\frac{3}{2}x^4-2x^3+12x+2$

from f

domain $(-\infty, \infty)$

asymptotes none

x int $(-1.5695, 0)$ $(-0.1675, 0)$

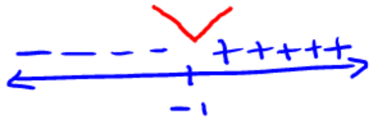
y int $f(0)=2$ $(0, 2)$

from f'

critical numbers $f'(x)=0$

$x=-1$

number line



f is increasing on $(-1, \infty)$

f is decreasing on $(-\infty, -1)$

relative minimum $(-1, -4.5)$

no relative maxima

$f(-1) = -4.5$

from f''

$f''(x)=0$ f'' ; root[poly]

$x=0$, $x=.6667$

f is concave up on $(-\infty, 0) \cup (.6667, \infty)$

f is concave down on $(0, .6667)$

inflection points $(0, 2)$

$(.6667, 9.7037)$

Lesson 13

Review Example 11:

Analyze the function $f(x) = x\ln(x^3+3)$.

from f

domain $(-1.4422, \infty)$ $x^3+3>0$

asymptotes $x=-1.4422$ $x^3>-3$

xint $(0,0)$ $(-1.2599,0)$ $x^3=-3$

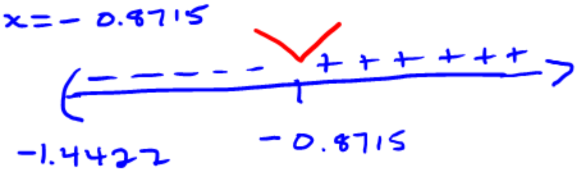
yint $(0,0)$ $x=\sqrt[3]{-3}$

$x=-1.4422$

from f'

critical # $x=-0.8715$

number line



f is increasing on $(-0.8715, \infty)$

f is decreasing on $(-1.4422, -0.8715)$

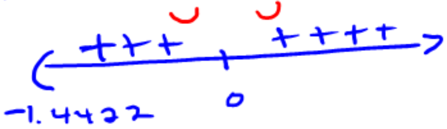
f has a relative minimum $(-0.8715, -.7402)$

f has no relative maxima

from f''

$f''(x)=0$ at $x=0$

number line

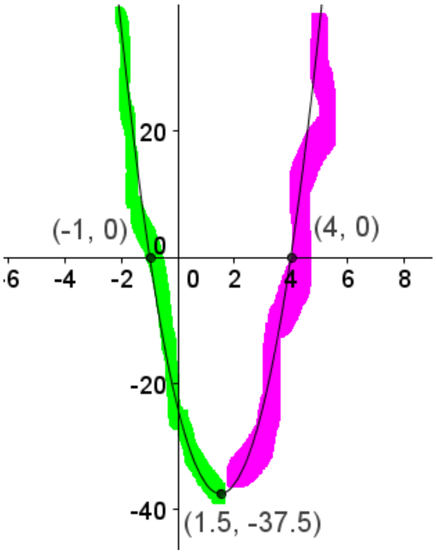


f is conc up on $(-1.4422, 0) \cup (0, \infty)$

f has no inflection points

Review Example 12:

The graph given below is the *first derivative* of a function, *f*. Find the interval(s) on which the function is increasing, the interval(s) on which the function is decreasing, the *x* coordinate of each relative extremum (and state whether it is a relative maximum or a relative minimum), the interval(s) on which the function is concave upward, the interval(s) on which the function is concave downward and the *x* coordinate of any inflection points.



∧ ∨

f' ++++ | - - - - - | ++++

-1 4

f is inc on $(-\infty, -1) \cup (4, \infty)$

f is dec on $(-1, 4)$

relative max at $x = -1$

relative min at $x = 4$

f'' - - - - | ++++

1.5

f is conc ↓ $(-\infty, 1.5)$

f is conc ↑ $(1.5, \infty)$

inflection pt $x = 1.5$

Review Example 13: Lesson 15

Find any relative extrema using the Second Derivative Test if $f(x)=\frac{1}{3}x^3-2x^2-5x-10$.

① c.n. $f'(x)=0$

$f'(x)=x^2-4x-5$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$x-5=0$ $x+1=0$

$x=5$ $x=-1$

$f''(x)=2x-4$

$f''(5)=6$

$f''(-1)=-6$

② Find $f''(5)=6 \overset{\text{min}}{\cup}$

$f''(-1)=-6 \underset{\text{max}}{\cap}$

③ results f has rel min (5, -43.3333)

f has rel max (-1, 7.3333)

Review Example 14:

Find the absolute extrema of the function $f(x)=\sqrt{x}\left(x^3-4\right)^2$ on [0.5, 1].

critical numbers

$f'(x)=0$ $x=0.6751$, ~~1.5874~~

Not in interval

function values

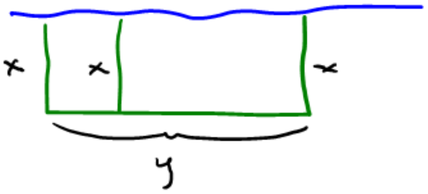
x	f(x)
.5	10.6177
.6751	11.2016
1	9

abs max is 11.2016

abs min is 9

Review Example 15: Lesson 15

Suppose you wish to fence in a rectangular-shaped pasture that lies along the straight edge of a river. You will divide the pasture into two parts by means of a fence that runs perpendicular to the river and parallel to two of the sides of the pasture. You have 1500 meters of fencing to use, and you wish to fence in the maximum possible area. The side along the river will not be fenced. Determine the dimensions of the pasture that will provide the maximum area. What is that area?



$3x+y=1500$

$y=1500-3x$

objective max. area

$A=xy$

① $f(x) = \boxed{A(x)=x(1500-3x)}$

in GGB

$A'(x)$

find roots

② c.n. $x=250$

③ test $A''(250)=-6$ $\cap$ max

$x=250$

$y=750$

$\boxed{187500\ m^2}$ $\boxed{250m \text{ by } 750m}$

Review Example 16:

If you cut away equal squares from all four corners of a piece of cardboard and fold up the sides, you will make a box with no top. Suppose you start with a piece of cardboard the measures 4 feet by 5 feet. Find the dimensions of the box that will give a maximum volume.

L15

$V(x) = x(5-2x)(4-2x)$

c.n.

etc

objective: max volume

critical numbers $x=0.7362,\ 2.2638$

2nd deriv test

$V''(0.7362) = -18.3312$ $\cap$ max

~~$V''(2.2638) = 18.3303$~~ $\cup$ min

(generates a min)

dimensions $x=.7362$

$5-2x = 3.5276$

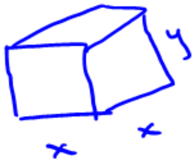
$4-2x = 4.5276$

3.5276 ft by 4.5276 ft by 0.7362ft

no top

Review Example 17:

Suppose you want to minimize the amount of material that is needed to ~~by~~ construct a open ~~closed~~ box. The box will have a square base, and must have a volume of 100 cubic inches. What are the dimensions of the box that will use the smallest amount of material to construct?



SA = bottom + 4 sides

$SA = x^2 + 4xy$ ← minimize this

$V = x^2 y$

$100 = x^2 y$

$\frac{100}{x^2} = y$

$A(x) = x^2 + 4x \cdot \frac{100}{x^2}$

optimize
c.n.
test

c.n. $x = 5.848$

2nd deriv test $A''(5.848) = 6$ ∪ min

dimensions are 5.849 in by 5.84 in by 2.924 in

answer $x = 5.848$ in

$y = \frac{100}{(5.848)^2} = 2.924$

L16

$f(x) = 2x^2 + 3$ [0,8] n=4
by hand

Review Example 18:

Suppose $f(x) = 1.68x^3 - 2.59x^2 + 4.6x + 5.71$. Approximate the area between the curve and the x axis on the interval [2.08, 5.16] using

L17
(A) Riemann sums, right endpoints and 31 rectangles
(B) Riemann sums, left endpoints and 28 rectangles
(C) Riemann sums, midpoints and 42 rectangles
(D) Upper sums and 35 rectangles
(E) Lower sums and 25 rectangles

1
0
.5

RectangleSum[ ]

UpperSum[ ]
LowesSum[ ]

Then find the exact area.

$\int_{2.08}^{5.16} f(x)$

Integral[f, 2.08, 5.16]

A. 256.5475
B. 238.5497
C. 247.8967
D. 255.5532
E. 237.4385
Exact area = 247.9183

$\int_0^2 (2x-7)\,dx$

$\frac{2x^2}{2} - 7x \Big|_0^2 = x^2 - 7x\Big|_0^2 = (2^2 - 7(2)) - (0^2 - 7 \cdot 0)$

$= 4 - 14 - 0 = \boxed{-10}$

$$\int \frac{4}{\sqrt{x^3}} \, dx$$

$$\int \frac{4}{(x^3)^{1/2}} \, dx$$

$$\int \frac{4}{x^{3/2}} \, dx$$

$$\int 4x^{-3/2} \, dx$$

$$\frac{4x^{-3/2+1}}{-3/2+1} = \frac{4x^{-1/2}}{-\frac{1}{2}}$$

$$= -\frac{2}{1} \cdot 4x^{-1/2}$$

$$= -8x^{-1/2}$$

$$= -\frac{8}{x^{1/2}}$$

$$= \boxed{\frac{-8}{\sqrt{x}}}$$