Lessons 22-24

$f(x) = x^2 - 4x + 7$

$f(x,y) = 3xy + 7x^2 + xy^4 + 9$

Math 1314
Lesson 22: Functions of Several Variables

~~In this, the last unit of the semester, we'll find that the calculator will not be particularly useful, except for doing some basic computation.~~

In this section, we will consider functions of more than one variable. You are already familiar with some examples of these.

$P = 2x + 2y$

$$P(x, y) = 2x + 2y$$
$$A(P, i, t) = P(1 + i)^t$$

These formulas are functions of several variables. We have just never called them that before. We will, for the most part, limit our discussion to functions of two variables.

Functions of Two Variables

Definition: A real valued function of two variables, f, consists of a set A of ordered pairs of real numbers (x, y) called the domain of the function, and a rule that associates with each ordered pair in the domain of f one and only one real number, denoted by $z = f(x, y)$.

We'll start by evaluating a function of two variables at some given ordered pairs.

(1) **Example 1**: Suppose $f(x, y) = 3x^2 y - 4xy + 6$. Compute $f(0,0)$, $f(2,-1)$ and $f(-1,-3)$.

GGB Command:

$f(x, y)$

$f(0,0) = 6$

$f(2,-1) = -6$

$f(2, -1)$

$f(-1,-3) = -3$

$f(-1, -3)$

Example 2: The volume of a cylindrical tank with radius r and height h is given by the formula $V = f(r,h) = \pi r^2 h$. Find the volume of a tank with radius 6 feet and height 20 feet.

$V = f(6,20) = 2261.9467 \text{ ft}^3$ $\quad f(r,h) = \pi * r\wedge 2\, h$

More likely

$V = f(6,20) = \pi \cdot 6^2 \cdot 20 = \pi * 36 * 20 = 720\pi \text{ ft}^3$

Example 3: Suppose you borrow money to buy a house. You agree to a 30-year loan with equal payments. This formula gives the amount of the reduction in principal of the loan (given in dollars) after making i payments on a loan of A dollars amortized over t years.

$$B = f(A,r,t,i) = A\left[\frac{\left(1+\frac{r}{12}\right)^{i}-1}{\left(1+\frac{r}{12}\right)^{12t}-1}\right]$$

12 / 30 / 360

.06

Suppose you borrowed \$180,000 at 6% annual interest. How much has the balance of the loan gone down after making 60 payments? ~~How much is the remaining balance of the loan after 20 years of payments?~~

$f(180000, .06, 30, 60) = 180000\left[\frac{(1+\frac{.06}{12})^{60}-1}{(1+\frac{.06}{12})^{360}-1}\right]$

numerator $1.3489 - 1$

denom $6.0226 - 1$

final $180000\,(.3489/5.0226) = 12502.16$ $\quad \$\,12502^{16}$

Example 4: Use the table shown below to find the monthly payment on a loan when \$125000 is financed for 30 years at 5.25% interest.

$f(125000, 5.25, 30)$

Monthly Payments in Dollars at 5.25% APR

Amt Financed	10 yrs	15 yrs	20 yrs	25 yrs	30 yrs
\$50,000	618	445	376	329	300
\$75,000	928	683	564	494	449
\$100,000	1237	911	751	659	599
\$125,000	1546	1138	939	823	749
\$150,000	1855	1366	1127	988	899
\$175,000	2164	1594	1315	1153	1049
\$200,000	2475	1821	1503	1317	1198

\$ 749

$f(x) = x^2 - 8x + 3$ $(-\infty, \infty)$

$g(x) = \frac{x-1}{x-3}$ $x \neq 3$ denom $\neq 0$

$h(x) = \sqrt{x-4}$ $x - 4 \geq 0$, $x \geq 4$

(2) **Domain**

You may be asked to find the domain of a function of two variables.

Example 5: Find the domain: $f(x,y) = 5x^2 + 6y^2$

everything works!!

→ $\{(x,y) \mid x, y \text{ are real numbers}\}$

Example 6: Find the domain: $f(x,y) = \frac{x+5y}{2x-y}$ ← denom $\neq 0$

$2x - y \neq 0$

$2x \neq y$

$\{(x,y) \mid y \neq 2x\}$

$f(x,y) = \frac{2x+y}{y}$

$y \neq 0$

$\{(x,y) \mid x \text{ is any real number}, y \neq 0\}$

Example 7: Find the domain: $f(x,y) = \frac{5y}{2x^2 + 3y^2}$ ← denom $\neq 0$

$2x^2 + 3y^2 \neq 0$

the only restriction is that $(0,0)$ can't be in the domain

$\{(x,y) \mid \text{both } x \text{ & } y \text{ are not zero}\}$

Example 8: Find the domain: $f(x,y) = \sqrt{8x^2 - 4y}$

radicand can't be $-$ must be ≥ 0

$8x^2 - 4y \geq 0$

$\frac{8x^2}{4} \geq \frac{4y}{4}$

$2x^2 \geq y$

$y \leq 2x^2$

$\{(x,y) \mid y \leq 2x^2\}$

Graphing functions in space is quite difficult. You will not need to do this. Here are a couple of examples of graphs of functions of two variables. We can't graph these on the calculator either. These are usually done by computer.