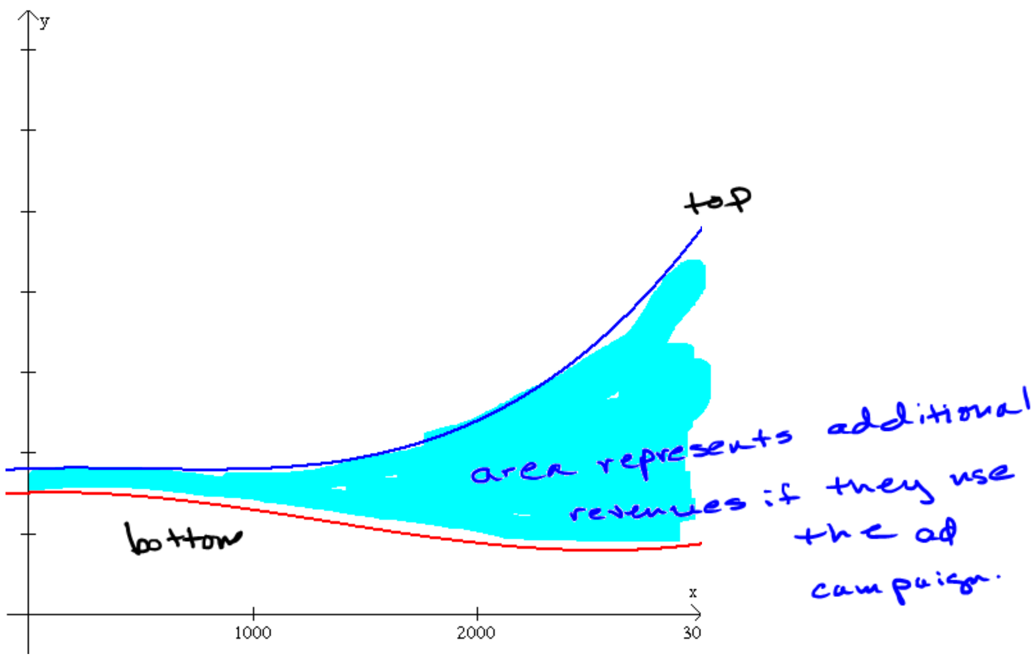
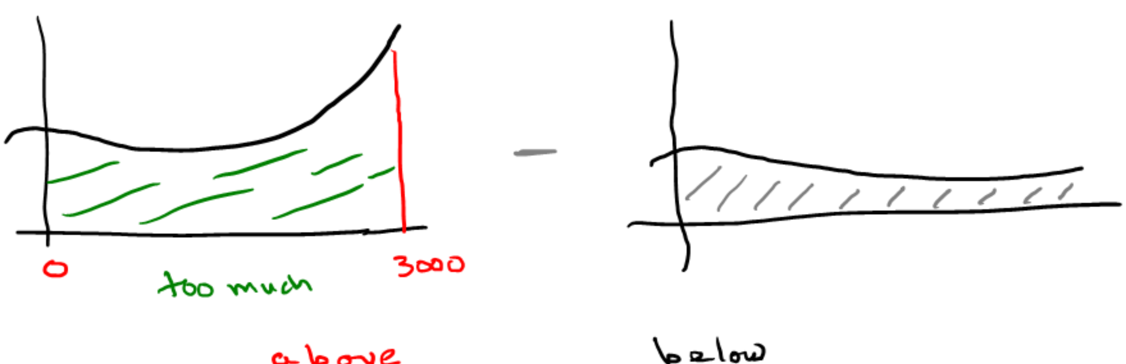
Math 1314
Lesson 20: Area Between Two Curves

Suppose the graph below represents two different models for the growth of a company's revenues. The bottom function projects change in revenues if no additional advertising is purchased. The top function projects change in revenues if the company begins an aggressive new advertising campaign. The area in between the two curves will tell us how much additional revenue the company will realize if they undertake the new ad campaign:



Here's the basic method for finding this difference:



$$\int_a^b (\text{top function} - \text{bottom function})\,dx$$

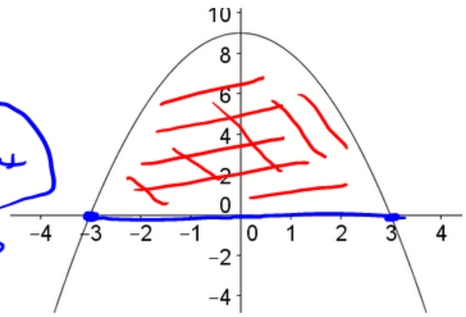
We can compute the area between the two curves. The general "formula" is

$$\int_a^b (\text{top function - bottom function})dx$$

$g(x) = 0$
the x axis

Example 1: Find the area of the region between the graph of $f(x) = 9 - x^2$ and from x = -3 to x = 3.

Must graph !!



Definite Integral:

simplified

$$\int_{-3}^{3} [(9 - x^2) - 0]\,dx = \boxed{\int_{-3}^{3} (9 - x^2)\,dx}$$

$g(x)=0$

GGB Command:

top, bottom

IntegralBetween[function, function, start, end]

Inputs:

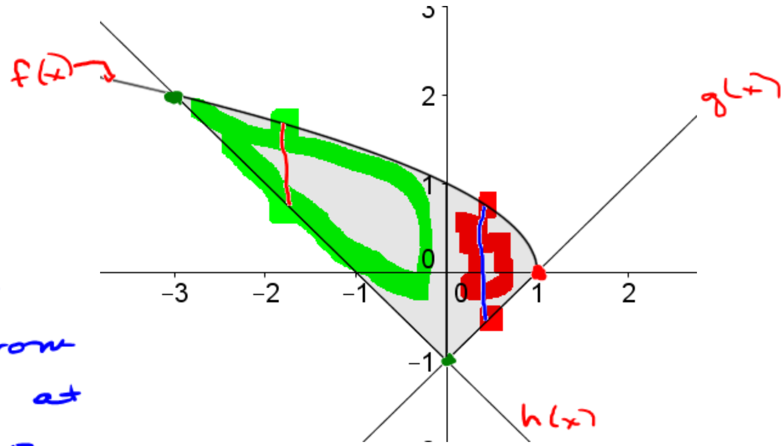
IntegralBetween[f, g, -3, 3]

Answer:

$\boxed{36}$

Example 2: Find the area between the functions $f(x)=\sqrt{1-x}$, $g(x)=x-1$, and $h(x)=-x-1$. The graphs of the three functions are shown below.

bottom function changes from h to g at x = 0



set up 2 definite integrals

Definite Integral:

$$\int_{-3}^{0}\left[\sqrt{1-x}-(-x-1)\right]dx+\int_{0}^{1}\left[\sqrt{1-x}-(x-1)\right]dx$$

$$\int_{-3}^{0}\left(\sqrt{1-x}+x+1\right)dx+\int_{0}^{1}\left(\sqrt{1-x}-x+1\right)dx$$

GGB Command:

IntegralBetween[function, function, start, end]

Inputs:

IntegralBetween[f, h, -3, 0] + IntegralBetween[f, g, 0, 1]

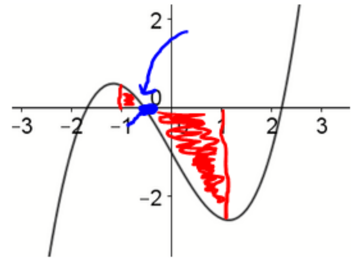
Answer: $\boxed{4.3333}$

Example 3: Find the area between the functions ~~Find the area between the functio~~n

$g(x) = 0$ $[-1, 1]$

$f(x) = \frac{1}{2}x^3 - 2x - 1$ and the x axis on the interval $-1 \le x \le 1$.

graph it first.



$x = -0.5392$

Set it up (2 integrals)

$$\int_{-1}^{-0.5392} \left[\left(\frac{1}{2}x^3 - 2x - 1 \right) - 0 \right] dx +$$

$$\int_{-0.5392}^{1} \left[0 - \left(\frac{1}{2}x^3 - 2x - 1 \right) \right] dx$$

Definite Integral:

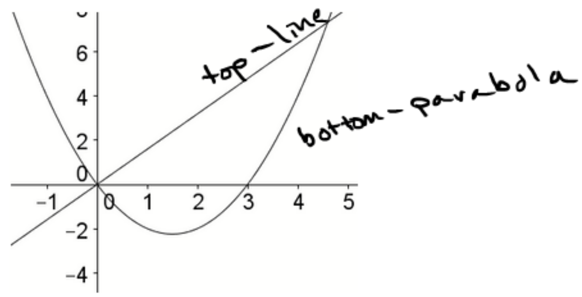
GGB Command: IntegralBetween

Inputs:

IntegralBetween[f, g, -1, -.5392] + IntegralBetween[g, f, -.5392, 1]

Answer: $\boxed{2.2481}$

Example 4: Find the area between the functions $f(x) = x^2 - 3x$ and $g(x) = 1.6x$.



Points of Intersection: $(0,0)$ $(4.6, 7.36)$

Definite Integral:

$$\int_0^{4.6} \left[1.6x - (x^2 - 3x)\right] dx = \int_0^{4.6} (4.6x - x^2)\, dx$$

GGB Command:

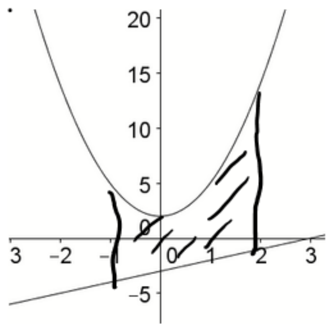
IntegralBetween

Inputs:

IntegralBetween[g, f, 0, 4.6]

Answer: $\boxed{16.2227}$

Example 5: Find the area between the functions $f(x) = 3x^2 + 2$ and $g(x) = x - 3$. on the interval $[-1, 2]$



~~**Points of Intersection:**~~

Definite Integral: $\int_{-1}^{2} [(3x^2+2) - (x-3)]\, dx = \int_{-1}^{2} (3x^2 - x + 5)\, dx$

GGB Command:

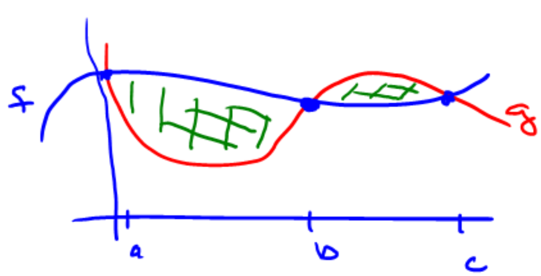
Inputs:

IntegralBetween[f, g, -1, 2] =

Answer:

$\boxed{22.5}$

Recap



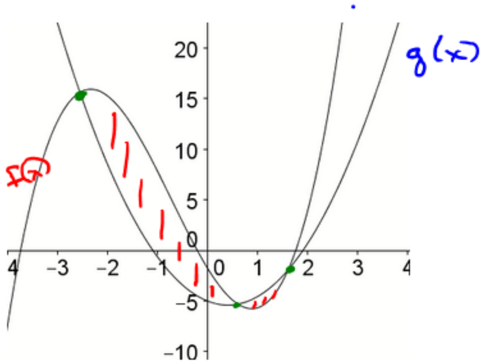
$\int_a^b (\text{top} - \text{bottom})\, dx$

$\int_a^b [f(x) - g(x)]\, dx + \int_b^c [g(x) - f(x)]\, dx$

Among CASA testers MC Avg was 59.01/66 ≈ 89.4

Example 6: Find the area that is completely enclosed by $f(x)=1.3x^3+2.8x^2-8.1x-1.7$ and $g(x)=2.4x^2-2x-5$.

Graph: Required



Set up: $\int_{-2.5442}^{.6155} \left[(1.3x^3+2.8x^2-8.1x-1.7)-(2.4x^2-2x-5)\right] dx + \int_{.6155}^{1.621} \left[(2.4x^2-2x-5)-(1.3x^3+2.8x^2-8.1x-1.7)\right] dx$

Points of Intersection: Intersect[object, object]

$x = -2.5442,\ 0.6155,\ 1.6210$

GGB Command:

IntegralBetween (2 times)

Inputs:

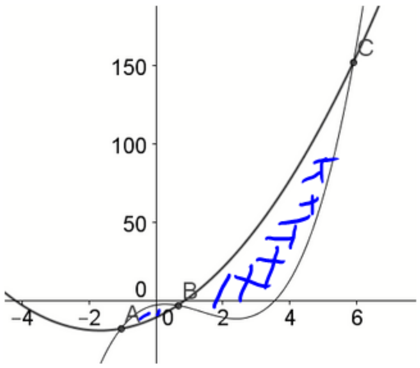
IntegralBetween[f, g, −2.5442, 0.6155] + IntegralBetween[g, f, 0.6155, 1.621]

Answer:

$\boxed{18.4769}$

Example 7: Find the area of the region that is completely enclosed by the graphs of the functions $f(x) = 2x^3 - 8x^2 + 4x - 3$ and $g(x) = 3x^2 + 10x - 11$.

Graph: Required !!



Set up:

$$\int_{-1.044}^{.6501} \left[(2x^3 - 8x^2 + 4x - 3) - (3x^2 + 10x - 11)\right] dx \; +$$

$$\int_{.6501}^{5.8939} \left[(3x^2 + 10x - 11) - (2x^3 - 8x^2 + 4x - 3)\right] dx$$

Points of Intersection: Intersect[f, g]

$x = -1.044,\ 0.6501,\ 5.8939$

GGB Command: IntegralBetween (2 times)

Inputs:

IntegralBetween[f, g, -1.044, 0.6501] + IntegralBetween[g, f, .6501, 5.8939]

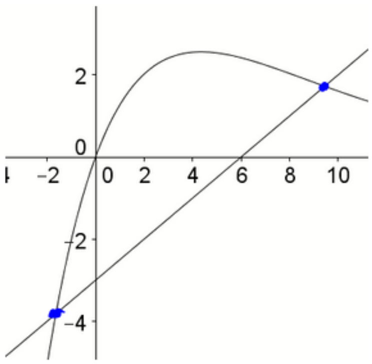
Answer:

$\boxed{217.3048}$

Example 8: Find the area of the region that is completely enclosed by the graphs of the functions $f(x) = 1.6xe^{-0.23x}$ and $g(x) = 0.5x - 3$.

f is not a polynomial

Graph:



b/c f is not a polynomial

Points of Intersection: Intersect[function, function, start, end]

x = -1.6377, 9.4435

GGB Command:

IntegralBetween

Inputs:

IntegralBetween[f, g, -1.6377, 9.4435]

Answer:

$\boxed{28.1639}$

units

Example 9: Without any effort to curb population growth, a government estimates that its population will grow at the rate of $60e^{.02t}$ thousand people per year. However, they believe that an education program will alter the growth rate to $-t^2+60$ thousand people per year over the next 5 years. How many fewer people would there be in the country if the education program is implemented and is successful?

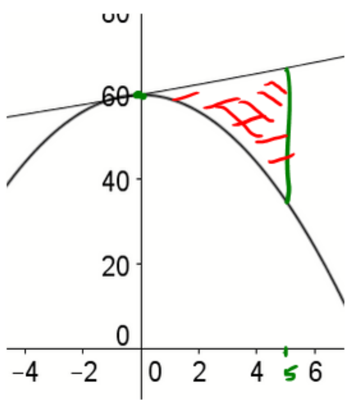
[0,5]

$f(t) = 60e^{.02t}$

$g(t) = -t^2+60$

(no need to find pts of intersection)

Graph: Required!!



Definite Integral: $\int_0^5 \left[(60e^{.02t}) - (-t^2+60)\right] dt$

GGB Command: IntegralBetween

Inputs:

IntegralBetween[f, g, 0, 5]

Answer:

57.1794

$\boxed{57.1794 \text{ thousand people}}$

$\boxed{57179 \text{ people}}$

units

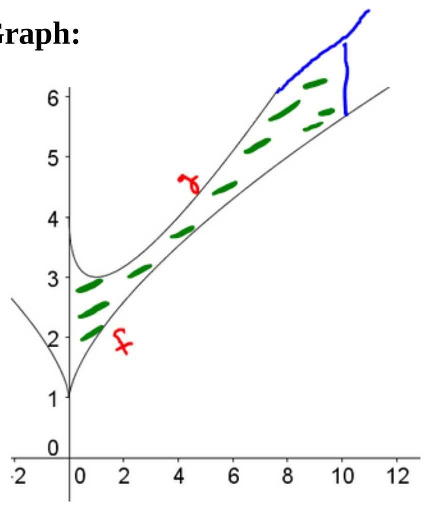
Example 10: The management of a hotel chain expects its profits to grow at the rate of $1+t^{\frac{2}{3}}$ million dollars per year t years from now. If they renovate some of their existing hotels and acquire some new ones, their profits would grow at the rate of $t-2\sqrt{t}+4$ million dollars per year. Find the additional profits the company could expect over the next ten years if they proceed with their renovation and acquisition plans.

[0, 10]

$f(t) = 1 + t^{2/3}$

$g(t) = t - 2\sqrt{t} + 4$

Graph:



Definite Integral: $\int_0^{10} \left[(t - 2\sqrt{t} + 4) - (1 + t^{2/3}) \right] dt$

GGB Command: IntegralBetween

Inputs: IntegralBetween[g, f, 0, 10]

Answer: 9.9868

$\boxed{\$9.99 \text{ million}}$